Alfurat Al-Awsat Technical University
Technical College / Al-Najaf
Department : Building & Construction Technology Engineering
Subject: Theory of Structures
Class: Third year
Lecturer : Professor Dr. Hakim Alkurayshi

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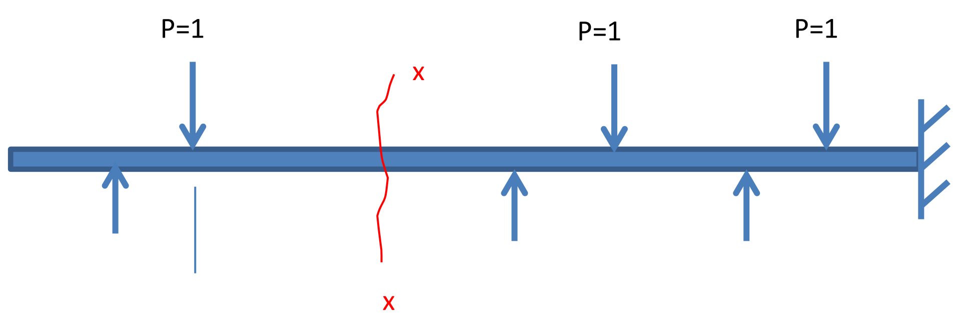
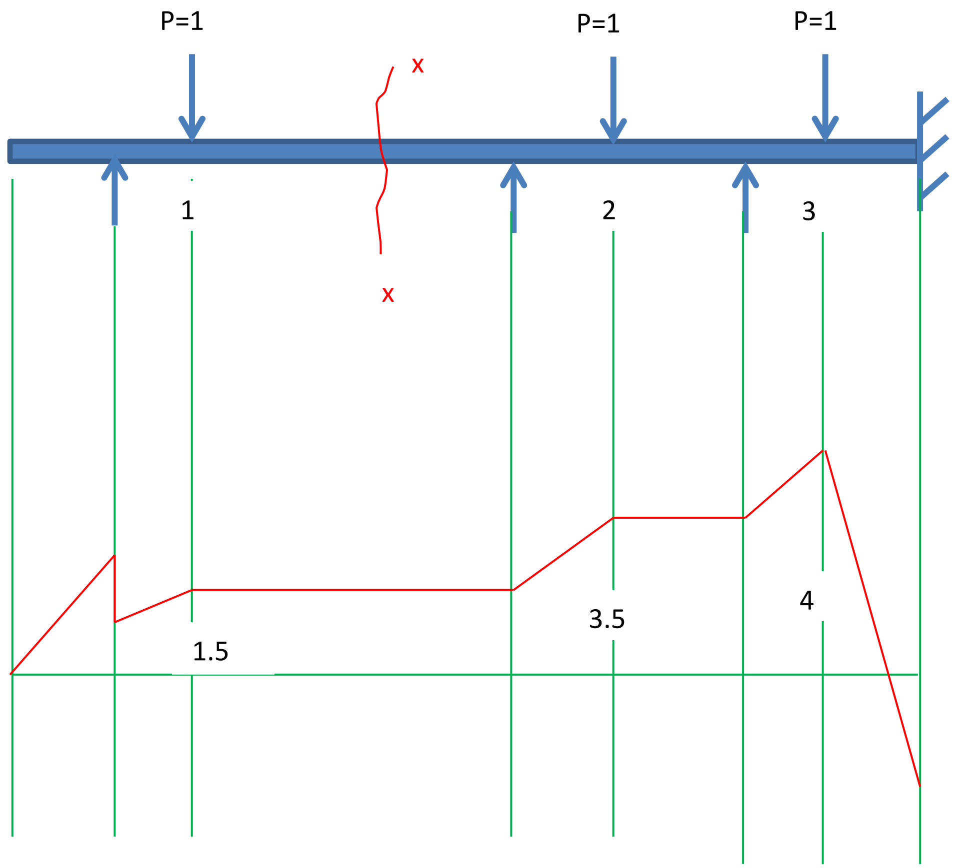
Lecture (9) Influence lines in beams and trusses

Definition : The influence line (or influence diagram) in a beam is a curve drawn on the beam where the ordinate at a point gives the magnitude of the influence in a specified section when a unit load reaches that point.

The influence in a specified section may be the shearing force , the bending moment , the deflection …………………etc.

Consider this beam :





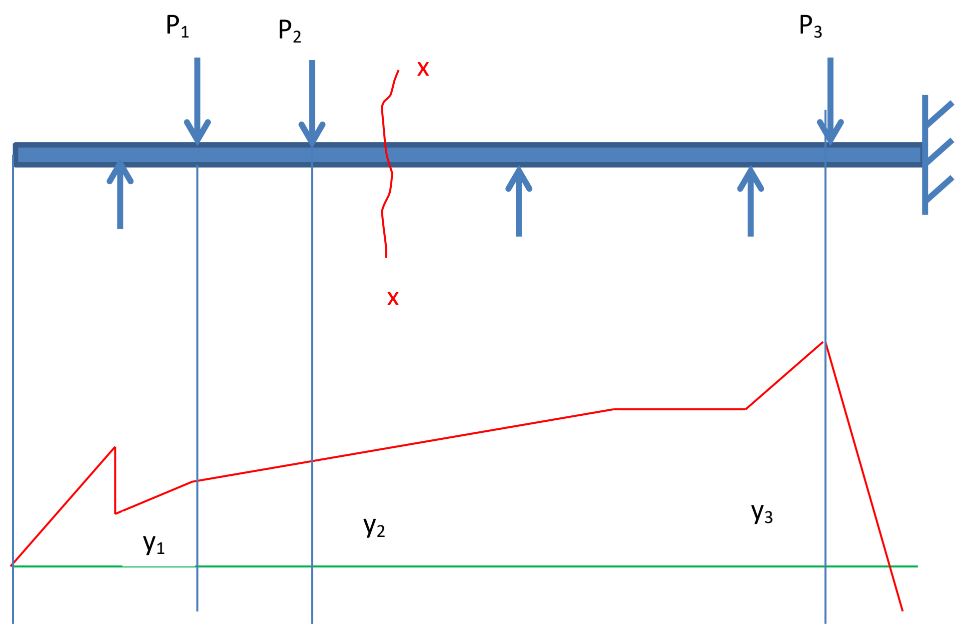
لنفرض أن هذا الخط البياني يمثل قيم قوق القص عند المقطع س-س فنقرأه بالشكل التالي أن قيمة قوة القص في هذا المقطع تساوي 1.5 حينما يكون الحمل على الجسربقيمة 1 عند النقطة 1 وتكون قيمة قوة القص 3.5 في هذا المقطع حينما يكون الحمل الذي يساوي 1 عند النقطة 2 وهكذا

A unit load is used for all influence lines. When a load P moves on a beam ,the magnitude of the influence at a specified section will be P.y. This is because the magnitude of the influence at section x-x is directly proportional to the load.

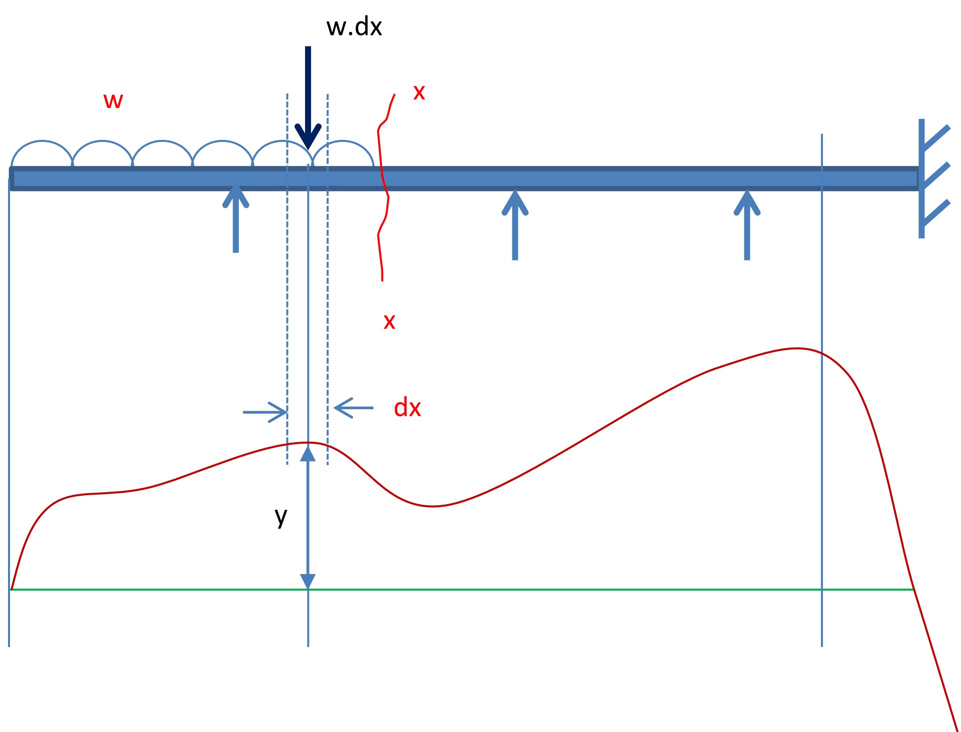
Several concentrated loads:

Use the principle of superposition , the total magnitude of the influence at section x-x will be ,

$$I = P_1 . y_1 + P_2 . y_2 + P_3 . y_3 + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$



Uniform load on a beam



Divide the uniform load into infinite number of small loads{ w.dx at position x}. The magnitude of the influence at section x-x due to (w.dx) will be :

$$dI = y.(w.dx)$$

Summing,

$$I = \sum_{0}^{\infty} dI = \int dI = \int y.(w.dx)$$

$$= w \int y.dx \quad = w\,[Area\ under\ the\ uniform\ load]$$

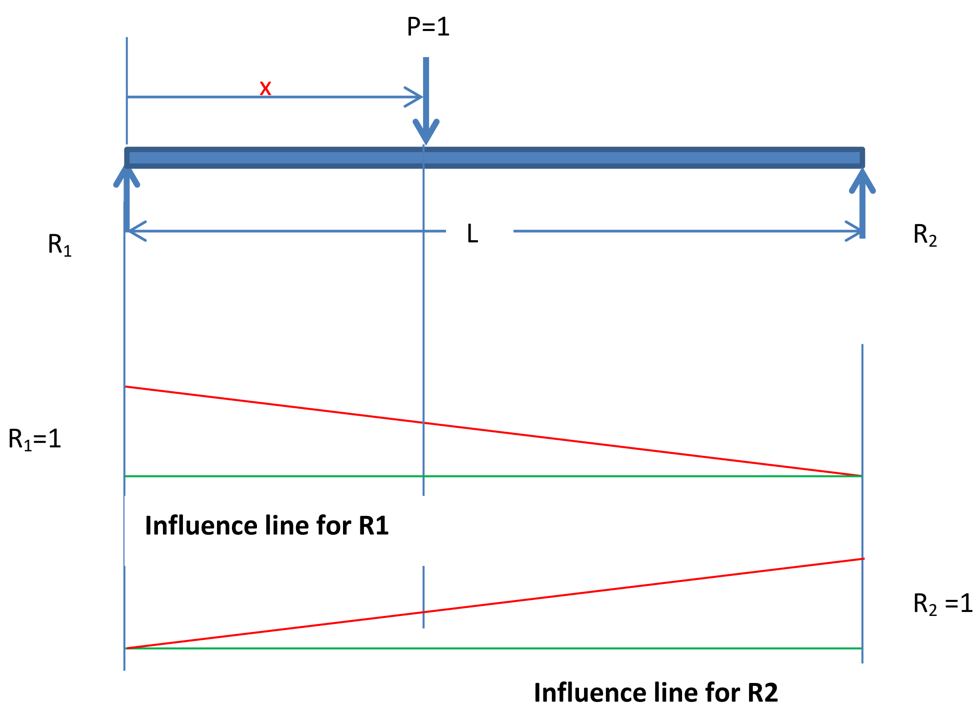
<u>Influence line (I.L.)for reactions in simply supported beams</u>

(1) Only simply supported beams.

Let the unit load P=1 be at distance x from R1 . Then

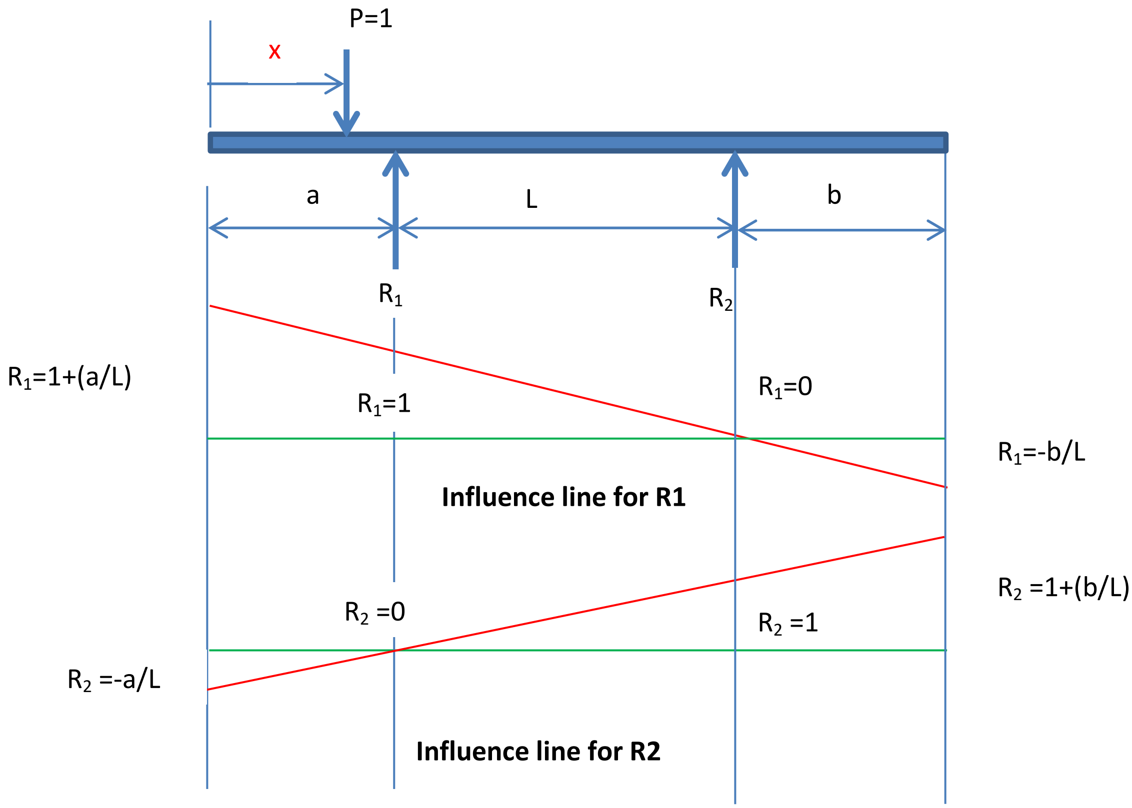
$$R_1 = \frac{P.(L-x)}{L} = 1 - \frac{x}{L}$$

$$R_2 = \frac{P.x}{L} = \frac{x}{L}$$



(2) Simply supported beam with overhangs

Let the unit load P=1 be at distance x from left end . Then

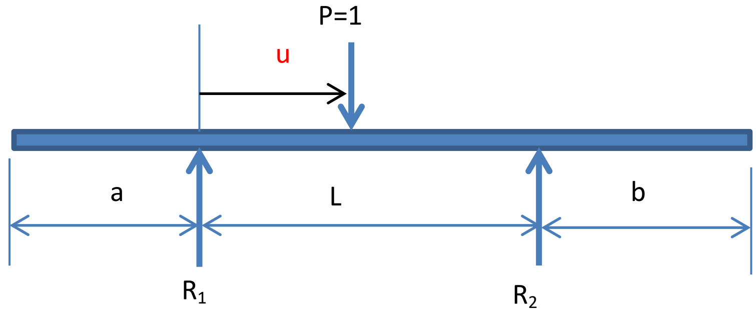


$$R_1 = \frac{P.(L + a - x)}{L} = \mathbf{1} + \frac{\boldsymbol{a}}{\boldsymbol{L}} - \frac{\boldsymbol{x}}{\boldsymbol{L}}$$

$$R_2 = \frac{-P.(a - x)}{L} = -\frac{a}{L} + \frac{x}{L}$$

$$This\ is\ for\ 0 \le x \le a$$

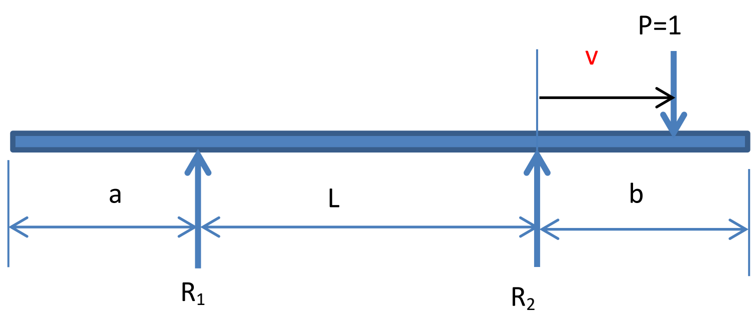
Now let the unit load P=1 be at distance u from R1 where $0 \leq u \leq L$,then ,



$$R_1 = \frac{P.(L-u)}{L} = \mathbf{1} - \frac{\boldsymbol{u}}{\boldsymbol{L}}$$

$$R_2 = \frac{P.u}{L} = \frac{u}{L}$$

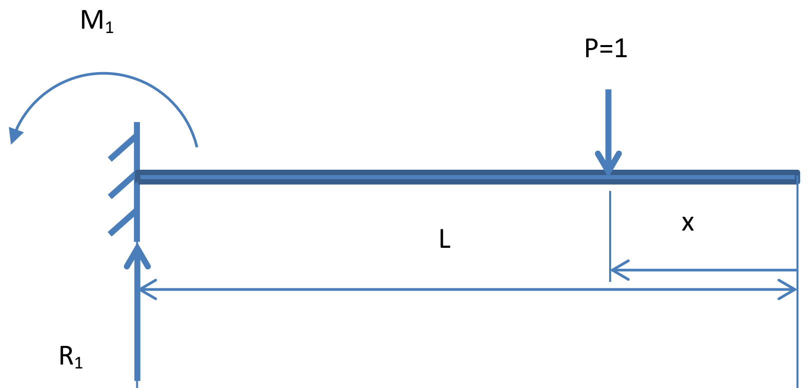
Finally let the unit load P=1 be at distance v from R2 where $0 \leq v \leq b$



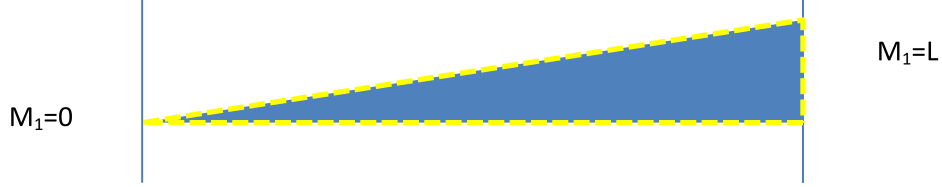
$$R_1 = \frac{-Pv}{L} = -\frac{\boldsymbol{v}}{\boldsymbol{L}}$$

$$R_2 = \frac{P.(L+v)}{L} = \mathbf{1} + \frac{\boldsymbol{v}}{\boldsymbol{L}}$$

(3) Influence lines for reactions in a cantilever



Influence line for R_1

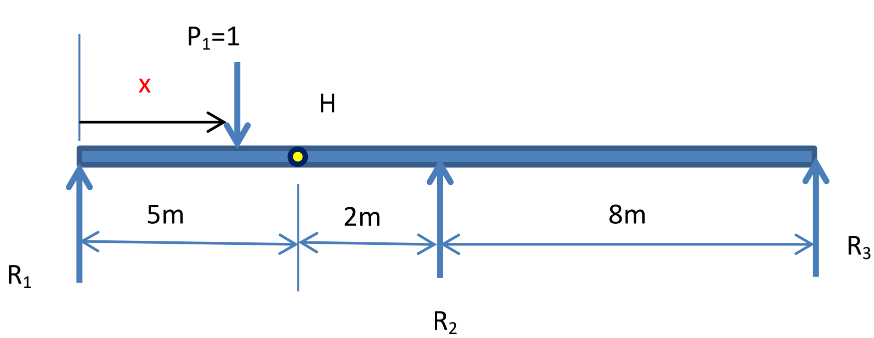


Influence line for M_1

(3) Influence lines for reactions in hinged beams

Draw I.Ls. for all reactions :

$R_1+R_2+R_3=1$



Solution:

First let a unit load P_1=1

be at distance x_1 from R1,

where $0 \le x_1 \le 5$m.Then ,

B.M.$_H$=0

$$R_1x5 - P_1(5 - x_1) = 0$$

$$R_1 = 1 - \frac{x_1}{5}$$

$To\ find\ R_2, use\ \sum M = 0\ about\ R_3$,

$$R_2x8 + R_1x15 - P_1(15 - x_1) = 0$$

$$R_2x8 + (\frac{5-x_1}{5})x15 - (15 - x_1) = 0$$

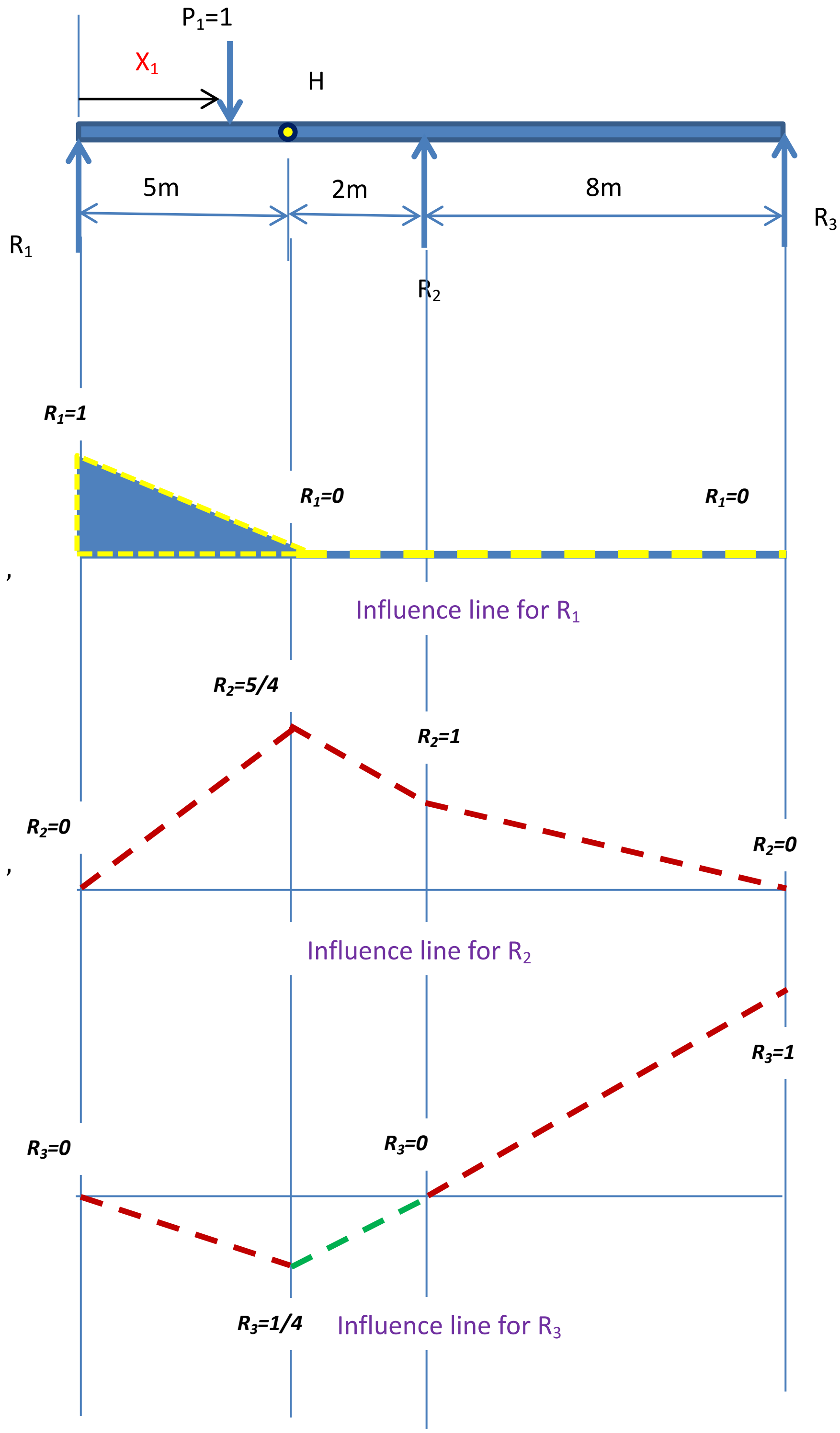
$$R_2 = \frac{x_1}{4}$$

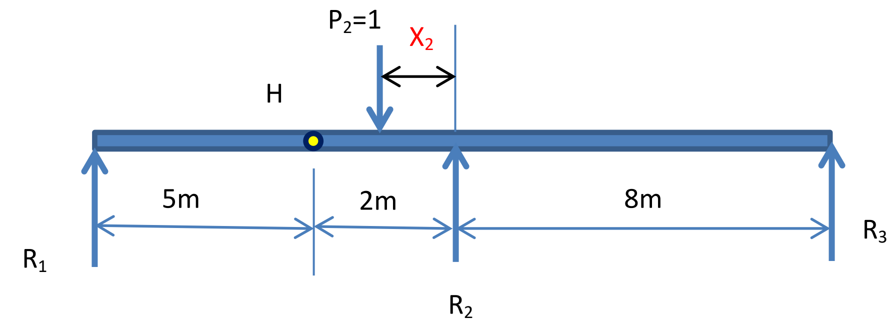
$To\ find\ R_3, use\ \sum M = 0\ about\ R_1$,

$$R_3x15 + R_2x7 - P_1(x_1) = 0$$

$$15R_3 + \frac{x_1}{4}x7 - x_1 = 0$$

$$R_3 = -\frac{x_1}{20}$$





Next ,let P_2 be at x_2 from R_2 where $0 \le x_2 \le 2$m.Then

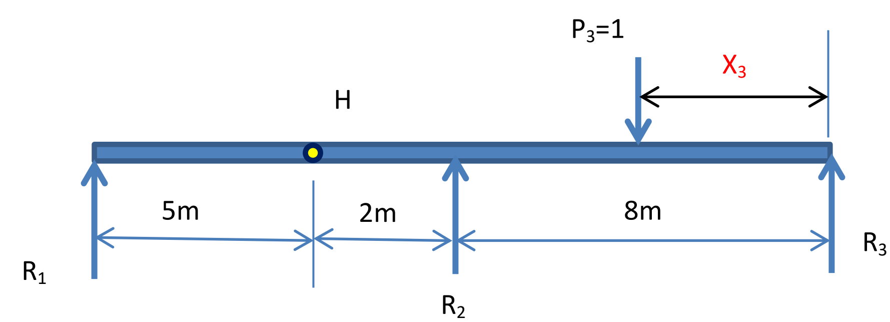
R_1x5=0 [B.M.=0 at H] .... R_1=0

$R_2x8 + R_1x15 - P_2(x_2+8) = 0$....................$R_2 = \frac{x_2}{8} + 1$

$To\ find\ R_3, use \sum M = 0\ about\ R_2$,

$R_3x8 - R_1x7 + P_2(x_2) = 0$............................$R_3 = -\frac{x_2}{8}$

Finally ,let P_3 be at x_3 from R_3 where $0 \le x_3 \le 8$m.Then



R_1x5=0 [B.M.=0 at H] .... R_1=0

$R_2 = \frac{x_3}{8}$, $[\sum M = 0\ at\ R_3]$

$R_3 = 1 - \frac{x_3}{8}$, $[\sum M = 0\ at\ R_2]$