



# Theory of Structure

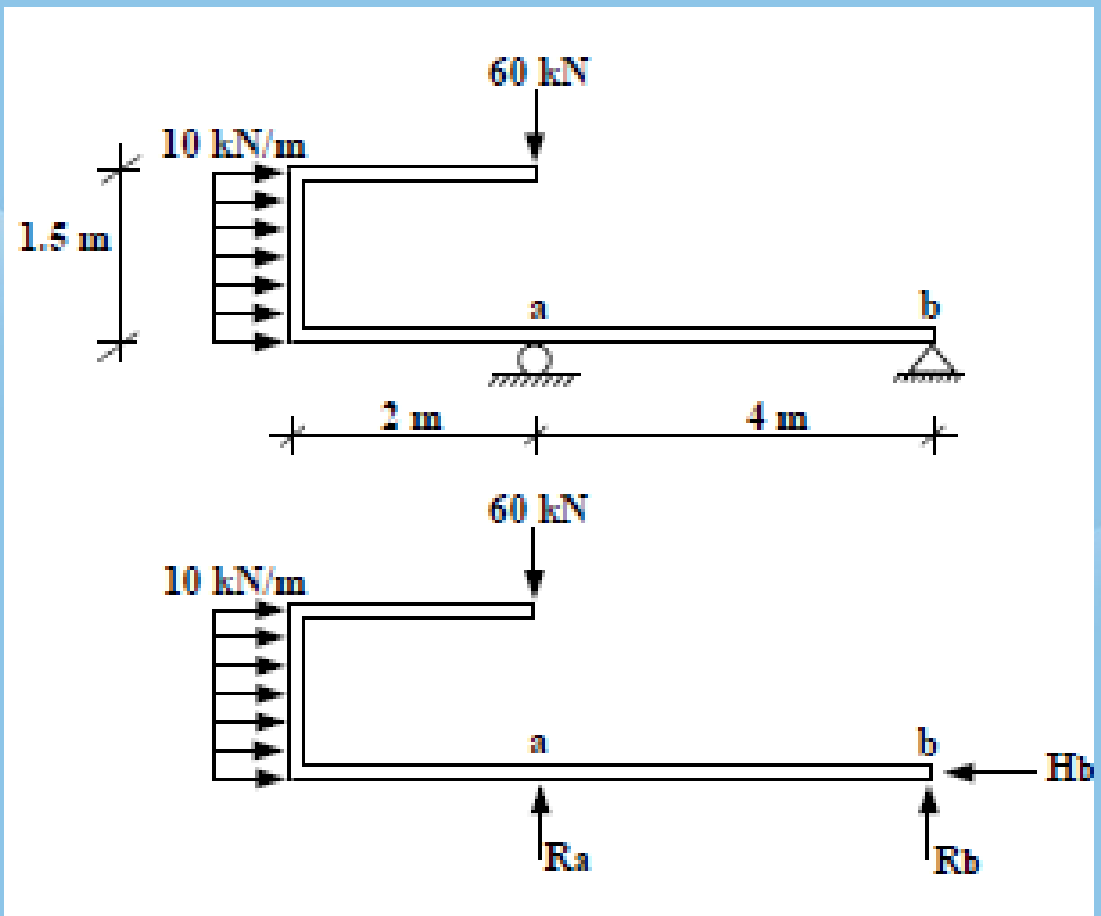
## Lec.2 (Stability and Determinacy)



- **Stability and Determinacy of Structures**
- Stability
- The stability of a structure can be divided into
- (i) external stability
- (ii) internal stability
- A structure is **externally stable** if the supports can provide the required number of independent reaction components for static equilibrium of the structure.

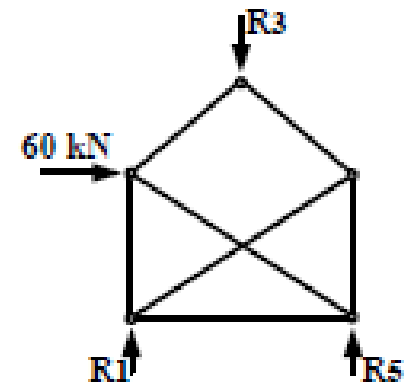
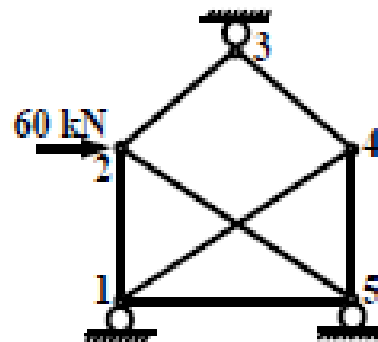


- Stable structure

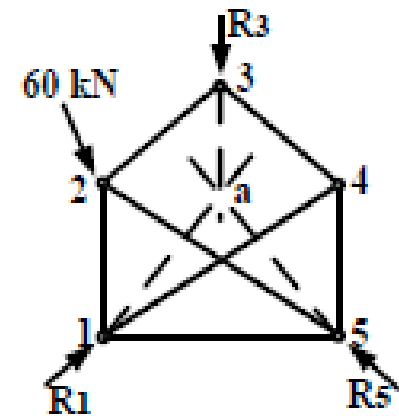
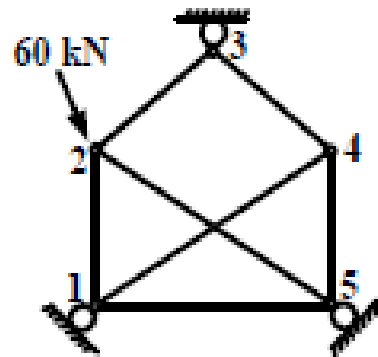




- Unstable system



(a)



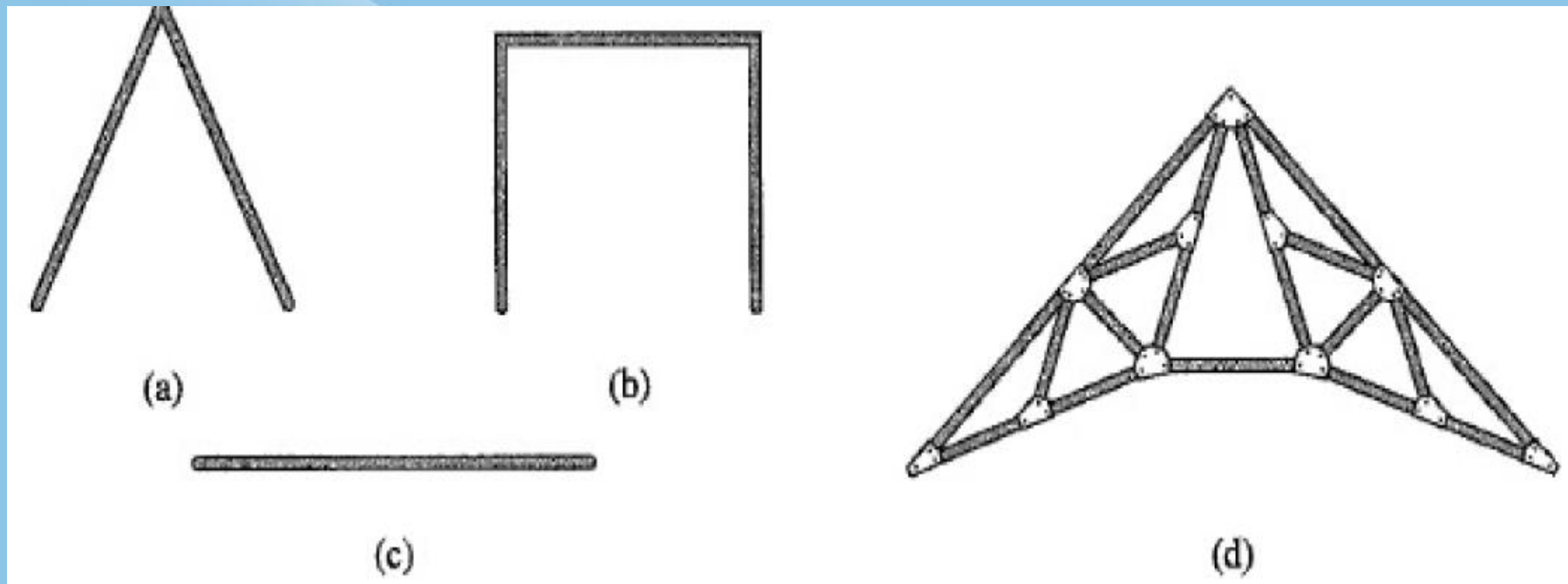
(b)



- for stability, the three reactive forces should be:  
(i) non-parallel and (ii) non concurrent.
- In other words, the stability of a structure is determined not only by the number of reaction components but also by their arrangement.
- **Internal Stability**
- A structure is considered to be internally stable, or rigid, if:
- It maintains its shape and remains a rigid body when detached from the supports.



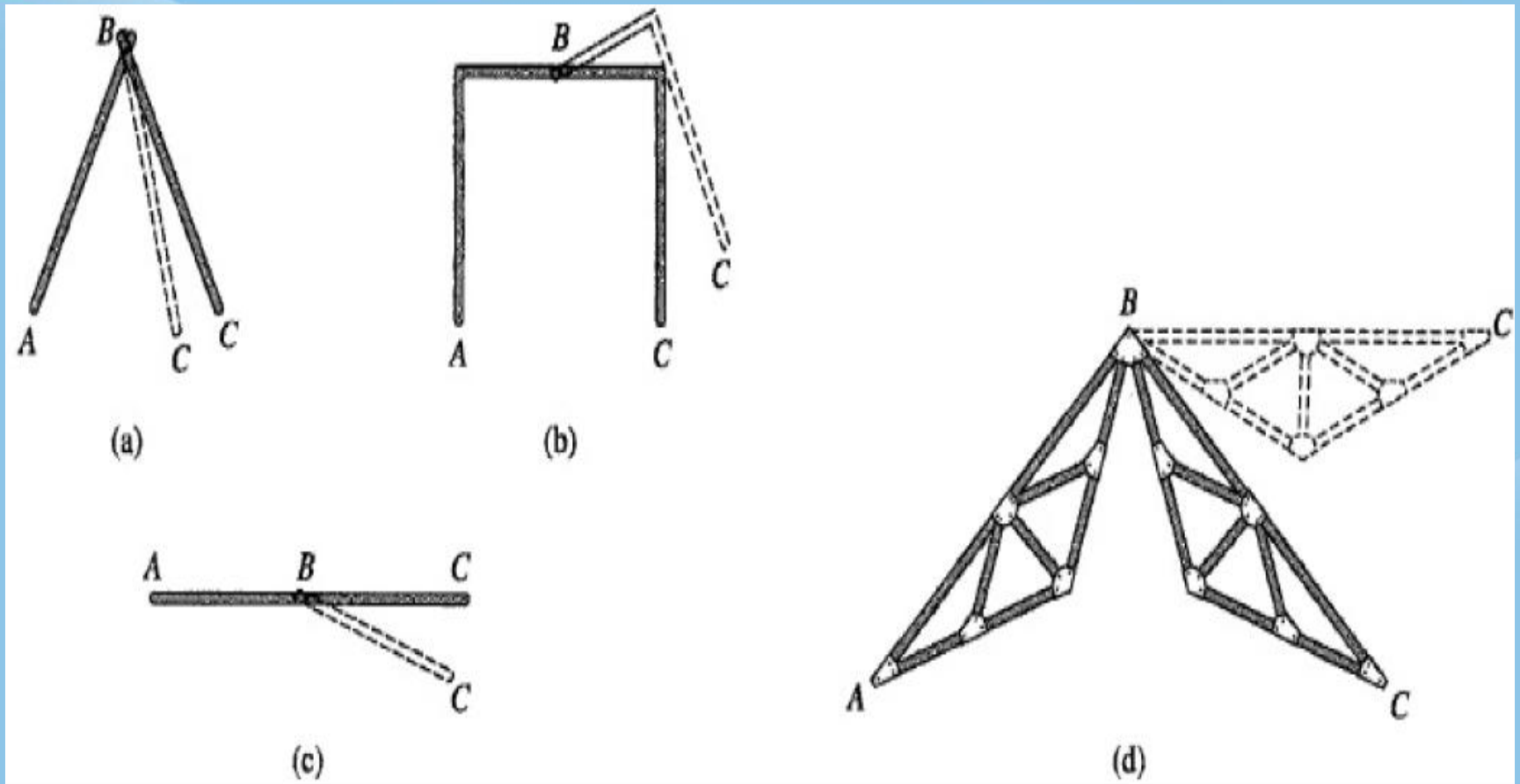
- Conversely, a structure is termed internally unstable (or non rigid) if:
- It cannot maintain its shape and may undergo large displacements under small disturbances when not supported externally.
- Some examples of internally stable structures are shown in Fig.



Internally stable



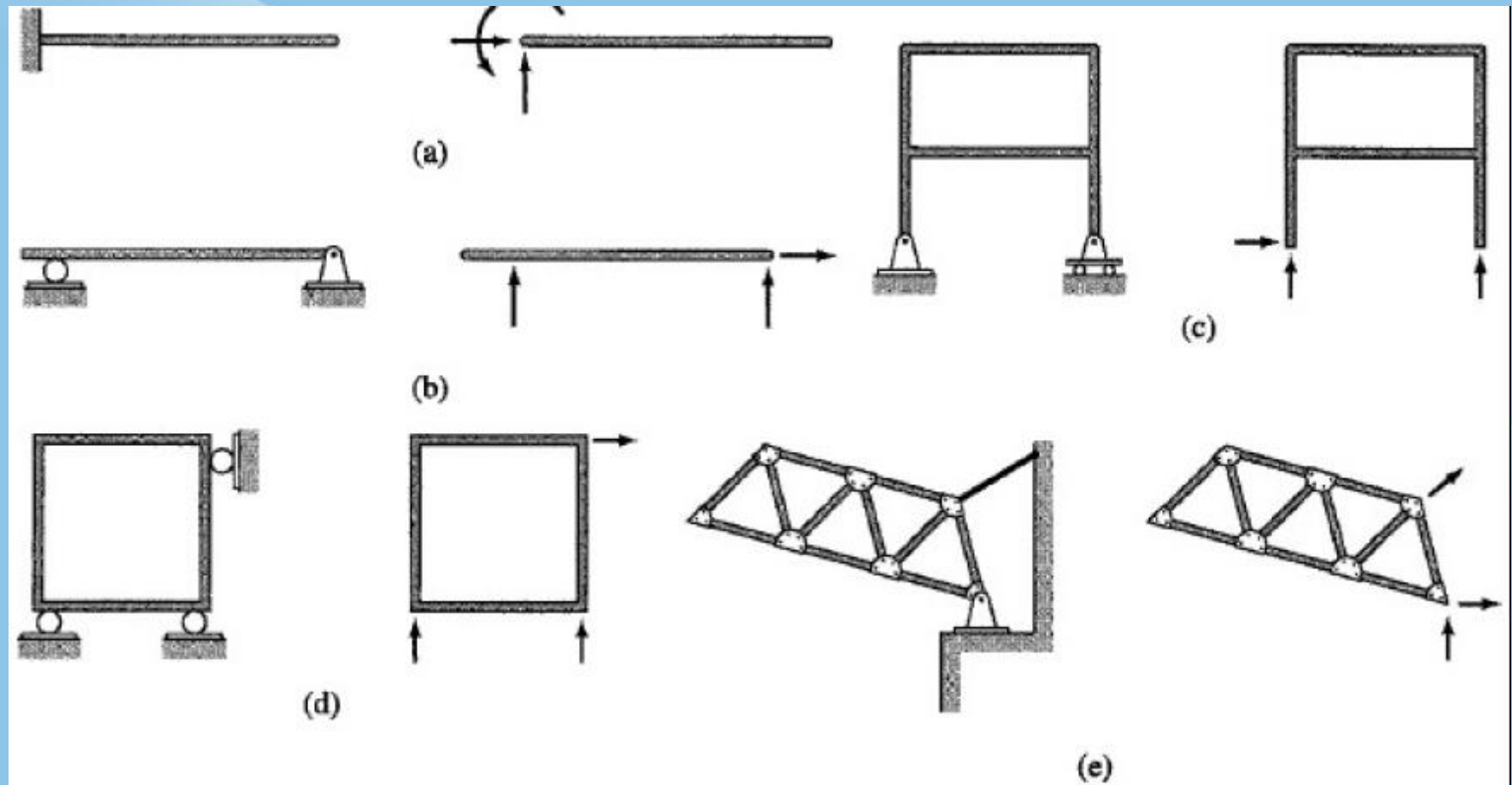
Fig. below shows some examples of internally unstable structures. A careful look at these structures indicates that each structure is composed of two rigid parts, AB and BC, connected by a hinged joint B, which cannot prevent the rotation of one part with respect to the other.



- Internally un stable



- **Statically Determinate Structures**
- If the internal forces in the members of a structure as well as its reactions can be found by the equations of static equilibrium, it is known as a statically determinate structure. shown in Figure below





- **Statically Indeterminate Structures**
- If the internal forces in the members of a structure as well as its reactions cannot be found out by the equations of static equilibrium, it is known as a statically indeterminate structure.
- These structures are also known as hyperstatic structures.



- Degree of Indeterminacy of Trusses
- Every member of a truss carries one internal force, viz, an axial force ( $m$ ) .
- the reactions that they supply to the structure will give the additional unknowns ( $r$ ).
- the total number of unknowns of a truss will be  $(m+r)$
- Now if the truss is in equilibrium, every isolated portion must be in equilibrium.
- For a truss having  $j$  joints, each joint yields two equations of static equilibrium.



- the degree of static indeterminacy  $D_t$  of a plane truss may be written as:-

$$D_t = (m + r) - 2j$$

- Also a criterion may be established to specify the
- stability and determinacy of a truss as:

(i) If  $m + r < 2j$  the truss is unstable

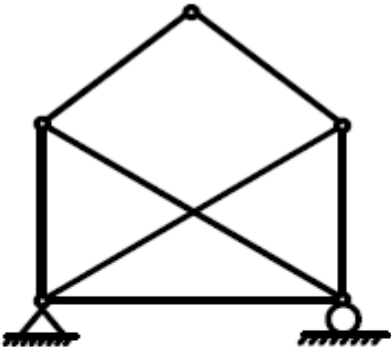
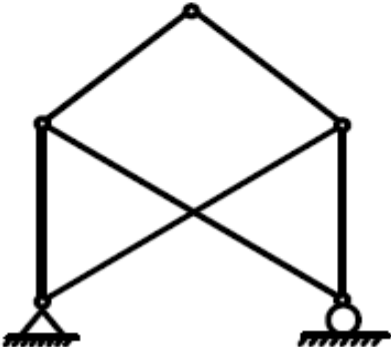
(ii) If  $m + r = 2j$  the truss is statically determinate

(iii) If  $m + r > 2j$  the truss is statically indeterminate

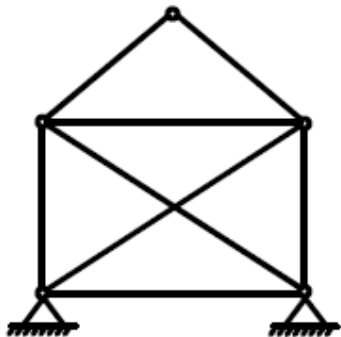
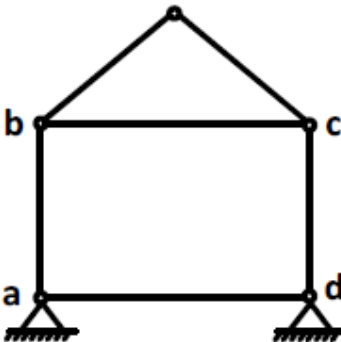


- the satisfaction of condition  $m + r \geq 2j$  does not ensure a stable truss. As previously explained (check interior and exterior stability).
- Example, Determine the degree of static indeterminacy and the type of trusses.



| Truss                                                                              | m | r | j | $m+r \geq 2j$ | Classification                    |
|------------------------------------------------------------------------------------|---|---|---|---------------|-----------------------------------|
|   | 7 | 3 | 5 | $10=10$       | Stable and statically determinate |
|  | 6 | 3 | 5 | $9 < 10$      | Unstable                          |



| Truss                                                                              | m | r | j | $m+r-2j$  | Classification                                                                            |
|------------------------------------------------------------------------------------|---|---|---|-----------|-------------------------------------------------------------------------------------------|
|   | 8 | 4 | 5 | $12 > 10$ | Statically indeterminate, the degree of static indeterminacy is 2                         |
|  | 6 | 4 | 5 | $10 = 10$ | Unstable (internal geometric instability due to lack of lateral resistance in panel abcd) |



- **Degree of Indeterminacy of Rigid Frames**
- Every member( $m$ ) of a rigid plane frame carries three unknowns
- internal forces, viz, an axial force, a shearing force, and a bending moment.
- the independent reaction components replacing the supports is  $r$ .
- the total number of unknowns in the frame will be:  $3m + r$
- each isolated rigid joint there are three equations of static equilibrium



- if the total number of rigid joints is  $j$ , then  $3j$  independent equations may be written for the entire frame.
- the degree of static indeterminacy  $D_f$  of a frame with all joints as rigid may be written as

$$D_f = (3m + r) - 3j$$

- Each hinged joint provides  $(n-1)$  equations, where  $n$  is the number of members meeting at that joint.
- If there are  $k$  hinged joints in the frame, a total of  $c = \sum^k (n - 1)$  independent equations exist, and the degree of static indeterminacy is given by

$$D_f = (3m + r) - (3j + c)$$

$C =$  (هنا تمثل عدد الأعضاء الإنشائية الملتقية عند المفصل مطروح منها 1)

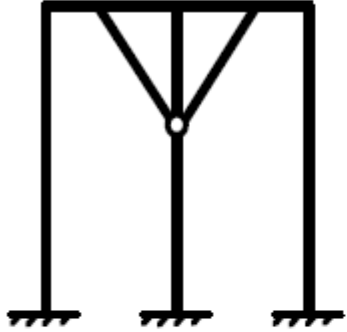
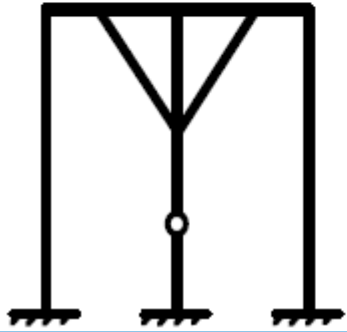


- the stability and determinacy of a frame as:

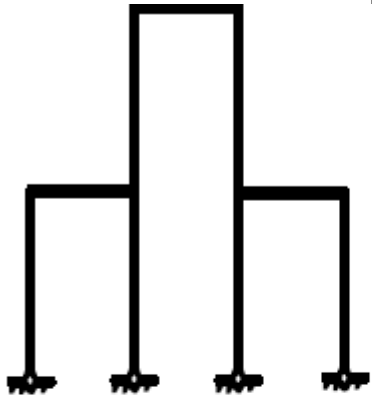
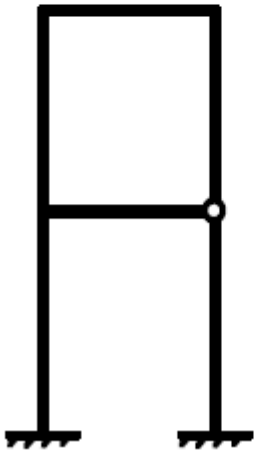
- |       |    |                   |                                                                     |
|-------|----|-------------------|---------------------------------------------------------------------|
| (i)   | If | $3m + r < 3j + c$ | the frame is unstable                                               |
| (ii)  | If | $3m + r = 3j + c$ | the frame is statically determinate provided that it is also stable |
| (iii) | If | $3m + r > 3j + c$ | the frame is statically indeterminate                               |



- Example, Determine the degree of static indeterminacy and the type of frames.

| Truss                                                                               | m  | r | j  | c | $3m+r \geq 3j+c$ | Classification                       |
|-------------------------------------------------------------------------------------|----|---|----|---|------------------|--------------------------------------|
|    | 10 | 9 | 9  | 3 | $39 > 30$        | Statically indeterminate, $(D_f=9)$  |
|  | 11 | 9 | 10 | 1 | $42 > 31$        | Statically indeterminate, $(D_f=11)$ |



| Truss                                                                              | m | r | j  | c | $3m+r \geq 3j+c$ | Classification                        |
|------------------------------------------------------------------------------------|---|---|----|---|------------------|---------------------------------------|
|   | 9 | 8 | 10 | 0 | $35 > 30$        | Statically indeterminate, $(D_f = 5)$ |
|  | 6 | 6 | 6  | 2 | $24 > 20$        | Statically indeterminate, $(D_f = 4)$ |