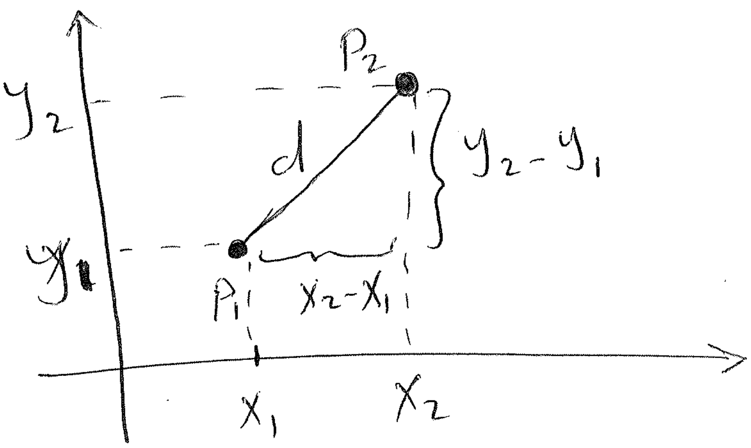
المحاضرة الأولى

* The distance between two points:

Let any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane as shown in Figure below. Then the distance (d) between the point P_1 and the point P_2 is calculated using Pythagorean theorem.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: $P_1(2, 2)$, $P_2(5, 6)$

$x_1, y_1 \qquad x_2, y_2$

$$d = \sqrt{(5-2)^2 + (6-2)^2}$$

$$d = \sqrt{3^2 + 4^2}$$

$$d = \sqrt{9+16}$$

$$d = \sqrt{25}$$

$$d = 5 \text{ units}$$

(A)

* Slope of the Line :

Let the Line (L) is not parallel to the y-axis and let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ any two distance point on $\underline{\underline{L}}$.

then the slope (m) of L is given by :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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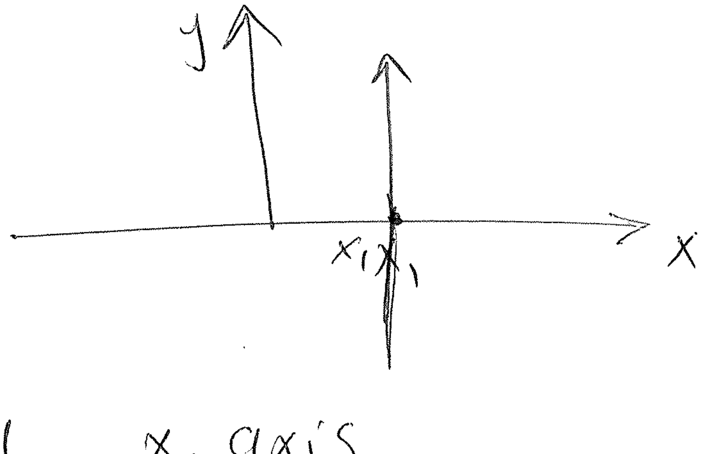
* Equation of the straight Line

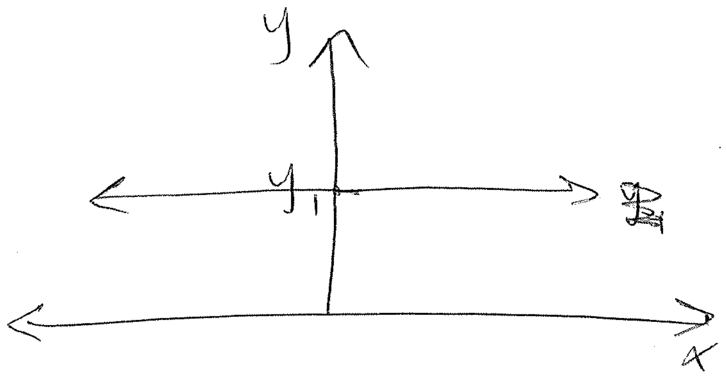
① Line Parallel to y-axis

$x = x_1$



② Line Parallel to x-axis

$y = y_1$



* The equation of a line by two points:

$P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and Line (L) and let another point $P(x, y)$ on L, the we can write the the equation of the line as follow:

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-y_1}{x-x_1} = m$$

$$y-y_1 = m(x-x_1)$$

Example
Find the equation of the line passes through (2, 5) and (6, -3)
$P(x_1, y_1)$ $P_2(x_2, y_2)$

$$\frac{y-5}{x-2} = \frac{-3-5}{6-2}$$

$$\frac{y-5}{x-2} = \frac{-8}{4}$$

$$y-5 = -2(x-2)$$

$$y-5 = -2x+4$$

$$y+2x-9=0$$

©

* The equation of the line by the point and the slope.

If we know the slope (m) of L.
And that the point $P_1(x_1, y_1)$ of the line L. Let $P(x, y)$ ~~a~~ to be any ~~$~~ point on L. Then the equation of the line is given by:

$$y - y_1 = m(x - x_1)$$

Example: Find the equation of the line passes through $(2, 4)$ and whose slope is $\frac{1}{2}$

solution

$$y - 4 = \frac{1}{2}(x - 2)$$

$$y = 0.5\cancel{x} - 1 + 4$$

$$y = \frac{x}{2} + 3$$

$$y - \frac{x}{2} \mp 3 = 0$$

D

أحمد سويد

Math I

١/١٢/٢٠٢٠
المحاضرة الأولى

الاقتران

Functions

الدوال

* A function from a set D to a set Y is a rule that assigns a unique element $f(x) \in Y$ to each element $x \in D$.

* The Set D of all possible input values to a function is called the domain of the function.

* The set of all values of $f(x)$ as x varies throughout D is called the range of the function.

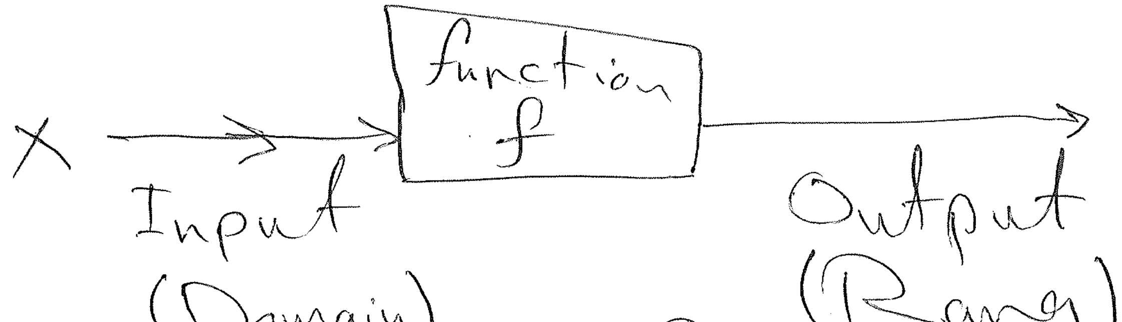
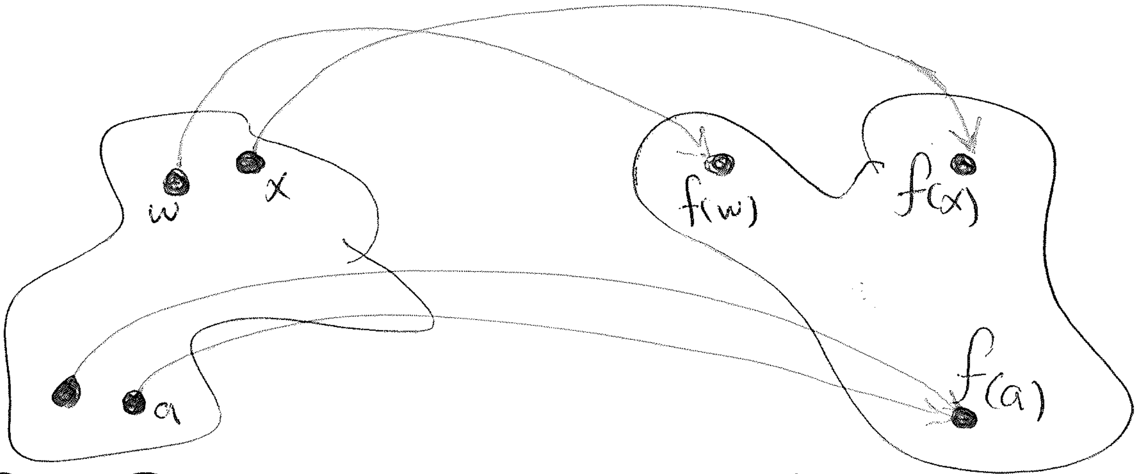


Fig. 1
A diagram showing Funct- as a kind of mach



D: Domain Set

Y: Range Set

Fig. 2: An Arrow Diagram showing a function

Example (1): Verify the Real domains and Real ranges of these functions

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{x - 4}$	$[4, \infty)$	$[0, \infty)$
$y = \sqrt{x^2 - 1}$	$(-\infty, \infty)$	$(-\infty, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

1. Algebriac Functions

Linear Algebriac Functions

Example: $f(x) = 5x + 3$

$f(x, w) = 2x - w + 10$

Other Algebriac Functions:

$$\left.\begin{array}{l} f(x) = 7x^3 - 2x^2 + x - 2 \\ f(x) = x^2 - 1 \end{array}\right\} \text{polynomials function}$$

Not Linear

Functions Types

(iii) Absolute Value Function

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example: Solve $|x+9| = 5$

Solution:

$$x + 9 = \pm 5$$

$x + 9 = 5$

$x = -4$

$x + 9 = -5$

$x = -14$

$\therefore \boxed{x = -4}$ and $\boxed{x = -14}$

* Absolute Value Properties:

① $|-a| = |a|$

② $|a \cdot b| = |a|\,|b|$

③ $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$

④ $|a+b| \leq |a| + |b|$

* Absolute Value and Intervals

if a is any positive number then:

① $|x| = a \longrightarrow x = \pm a$

② $|x| \leq a \longrightarrow -a \leq x \leq a$

③ $|x| \geq a \longrightarrow x \geq a$ or

$x \leq -a$

Example: Solve the inequality and show the solution on the real line.

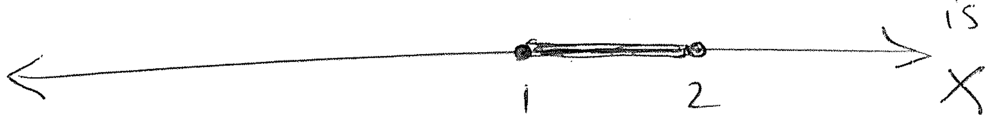
a.) $|2x-3| \leq 1$

solution: $-1 \leq 2x - 3 \leq 1$

$2 \leq 2x \leq 4$

$1 \leq x \leq 2$

The solution set is $[1,2]$



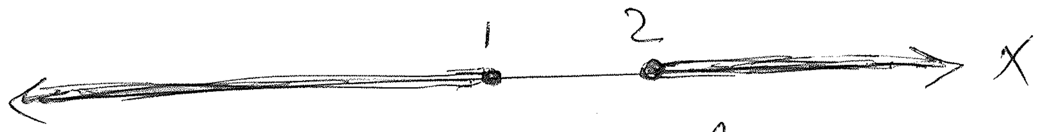
Example: Solve the inequality and show the solution on the real line

b.) $|2x-3| \geq 1$

Solution: $2x-3 \geq 1$ or $2x-3 \leq -1$

$2x \geq 4$ $\qquad$ $2x \leq 2$

$x \geq 2$ $\qquad$ $x \leq 1$



The solution set is

$(-\infty, 1] \cup [2, \infty)$

(iii) Peicewise-Defined Functions

This function contains different Formulas on different parts of its domain.

Example: دالة متشعبة

$$|x| = \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$$

Example:

$$f(x) = \begin{cases} -x , & x < 0 \\ x^2 , & 0 \leq x \leq 1 \\ 1 , & x > 1 \end{cases}$$

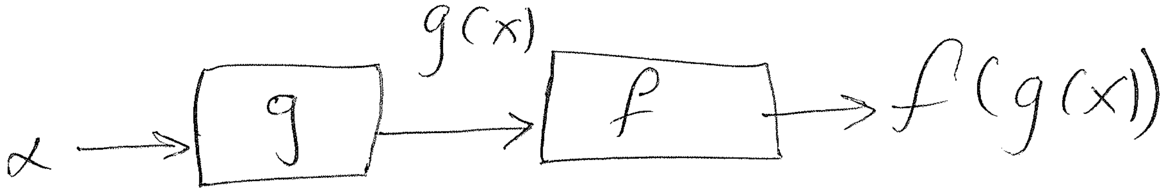
IV. Composite Functions:

~~Example:~~

$$f(x) \xrightarrow{g} g(x)$$

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$



Example: Let $f(x) = x^2 + 3$ and $g = \sqrt{x}$

Find a.) $(f \circ g)(x)$ b.) $(g \circ f)(x)$

Solution:

a.) $(f \circ g)(x) = \cancel{(\sqrt{x})} \; f(g(x))$

$$= (g(x))^2 + 3$$

$$= (\sqrt{x})^2 + 3$$

$$= x + 3$$

b.) $(g \circ f)(x) = g(f(x))$

$$= \sqrt{f(x)}$$

$$= \sqrt{x^2 + 3}$$

Example: Express $h(x) = (x-4)^5$ as a composition of two functions.

~~one~~ solution is:

$f(x) = x^5$, $g(x) = (x-4)$

$$h(x) = f(g(x))$$

$$= (g(x))^5$$

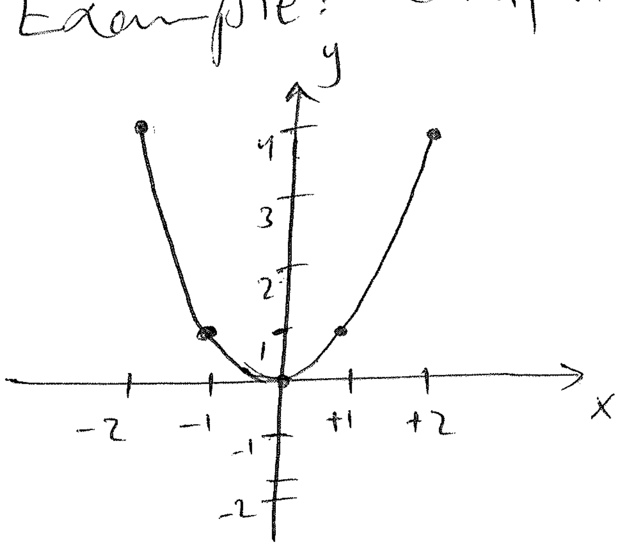
$$= (x-4)^5$$

Graphs ~~the~~ of functions:

To graph a function we carry out these steps:

1. Make table of of pairs for the function.
2. Plot enough points to learn the slope of the graph.
3. Complete the sketch by connecting the points.

Example: 1. Graph $y = x^2$ for the domain $-2 \leq x \leq 2$

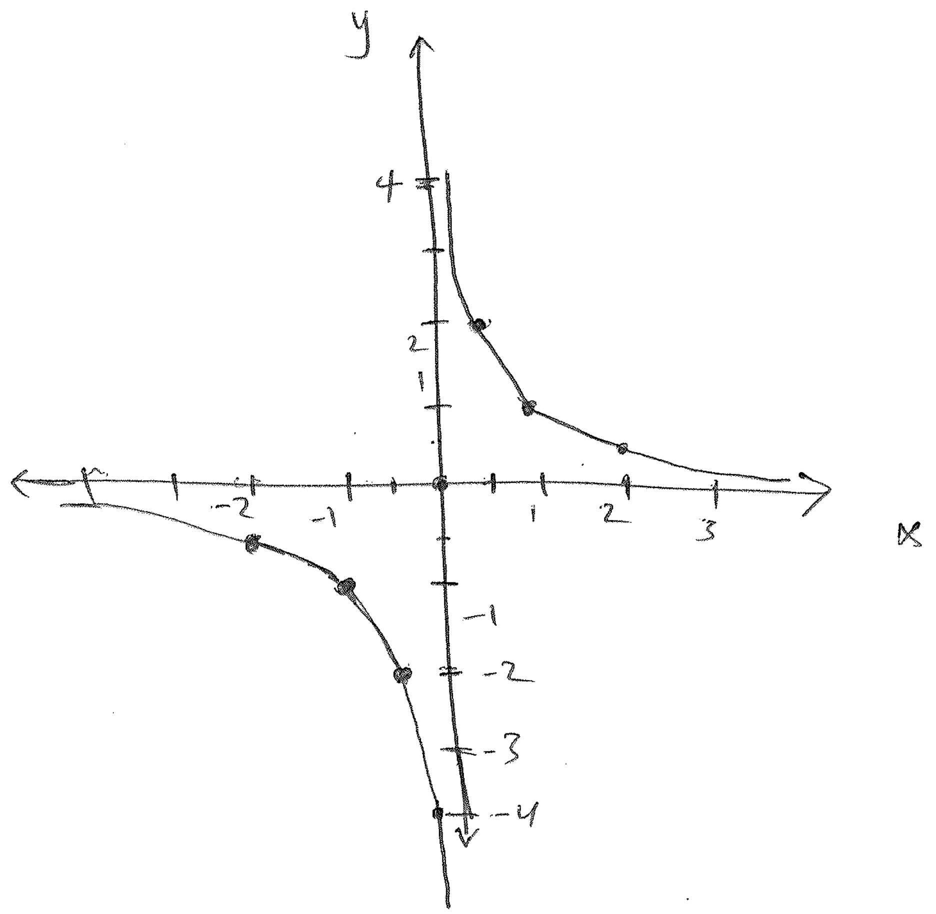


x	y
-2	4
-1	1
-0.5	0.25
0	0
0.5	0.25
1	1
2	4

نضيفها أيضاً

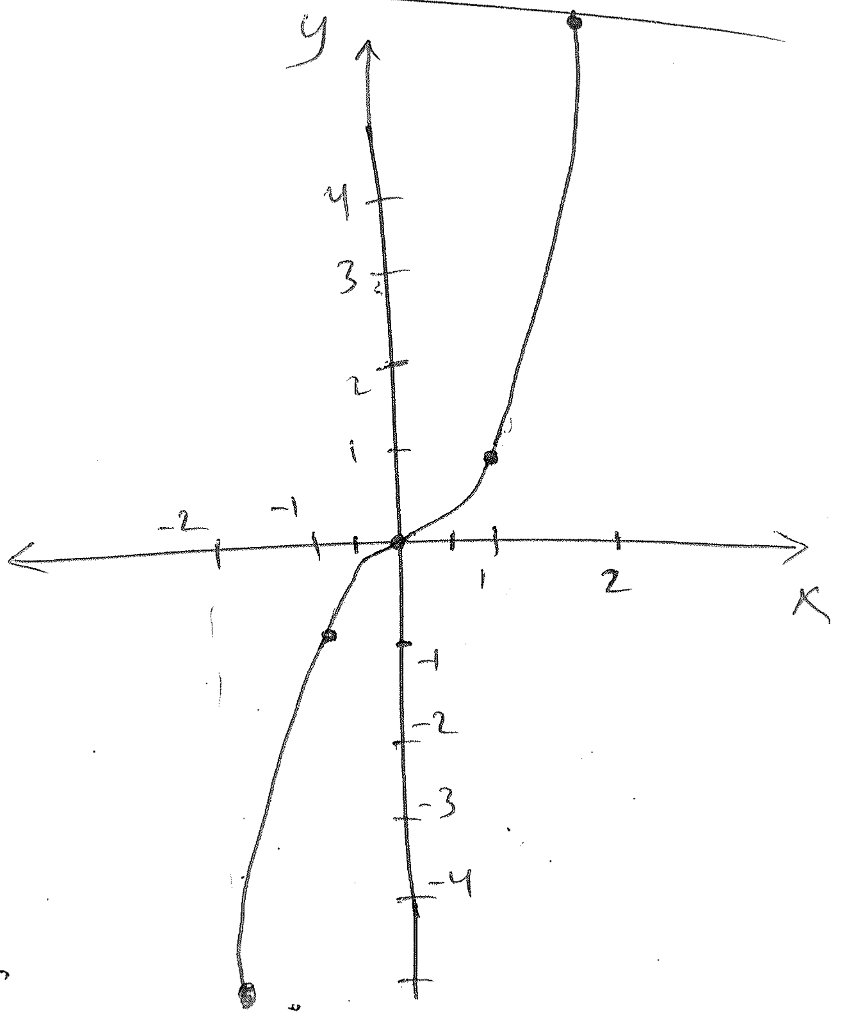
② $y = \frac{1}{x}$

x	y
-2	-0.5
-1	-1
-0.5	-2
→ 0	undefind
0.5	2
1	1
2	0.5



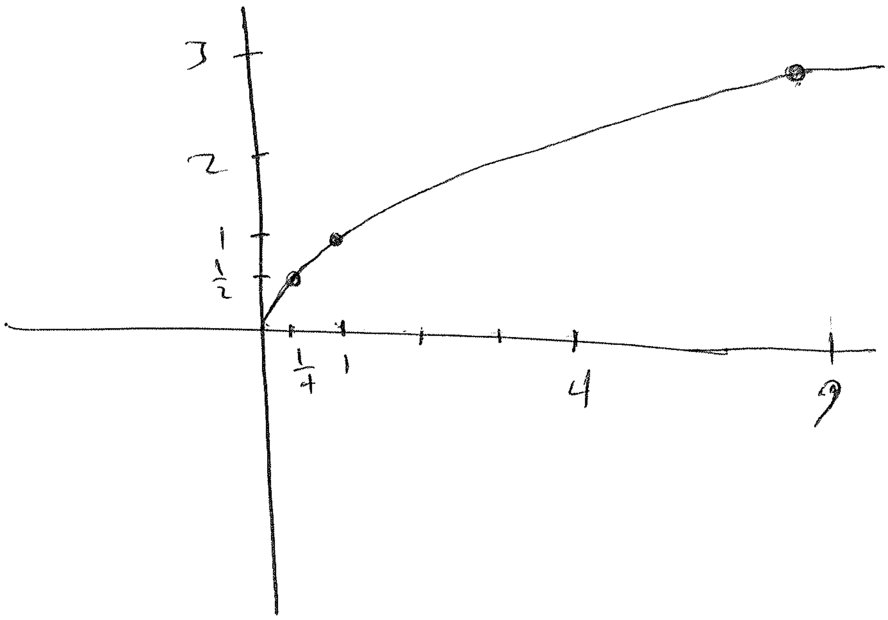
③ $y = x^3$

x	y
-2	-8
-1	-1
-0.5	-0.125
→ 0	0
0.5	0.125
1	1
2	8



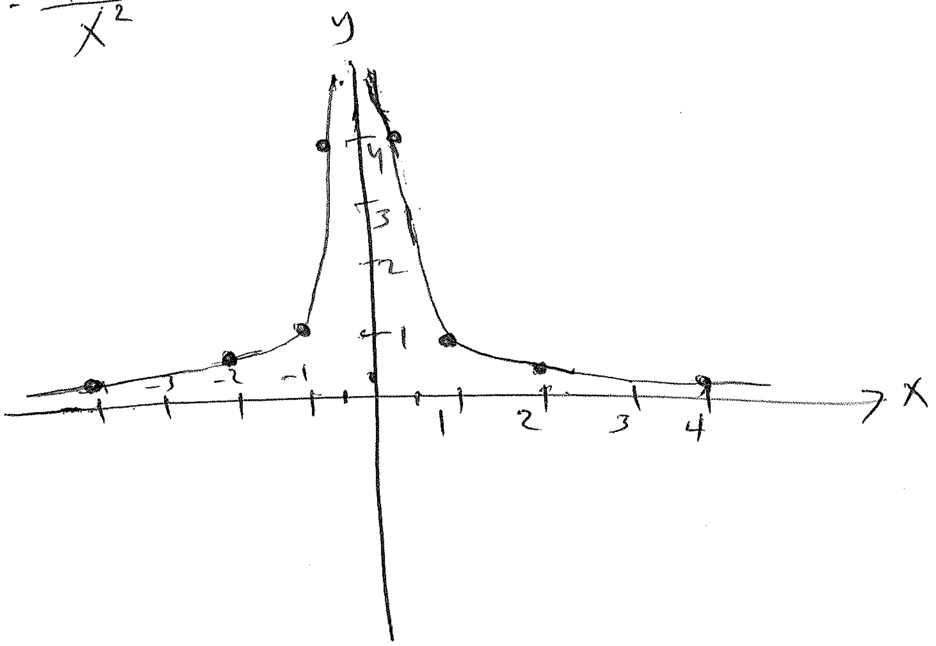
④ Example: $y = \sqrt{x}$

x	y
~~-4~~	
0	0
$+\frac{1}{4}$	$\frac{1}{2}$
9	3



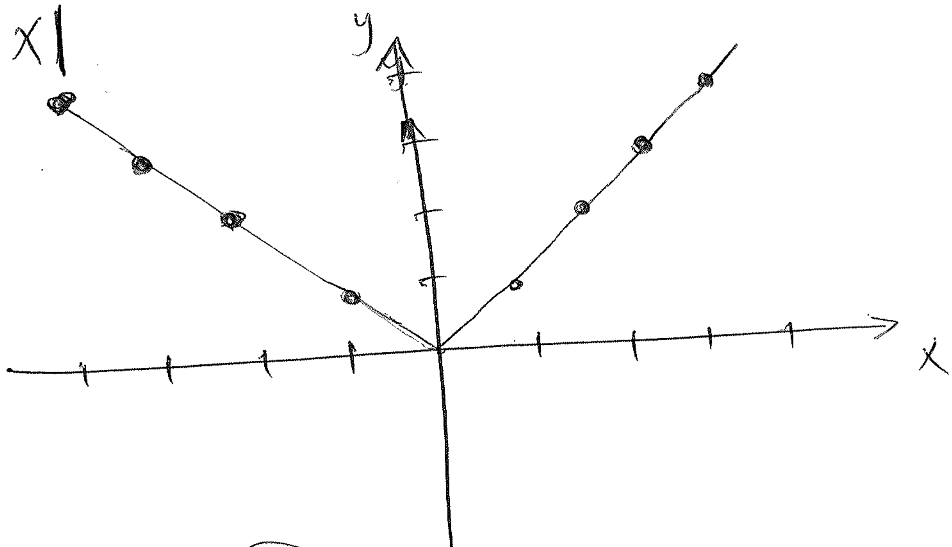
⑤ Example: $y = \frac{1}{x^2}$

x	y
-4	$\frac{1}{16}$
-2	$\frac{1}{4}$
-1	1
$-\frac{1}{2}$	4
0	0
1	1
2	$\frac{1}{4}$
4	$\frac{1}{16}$



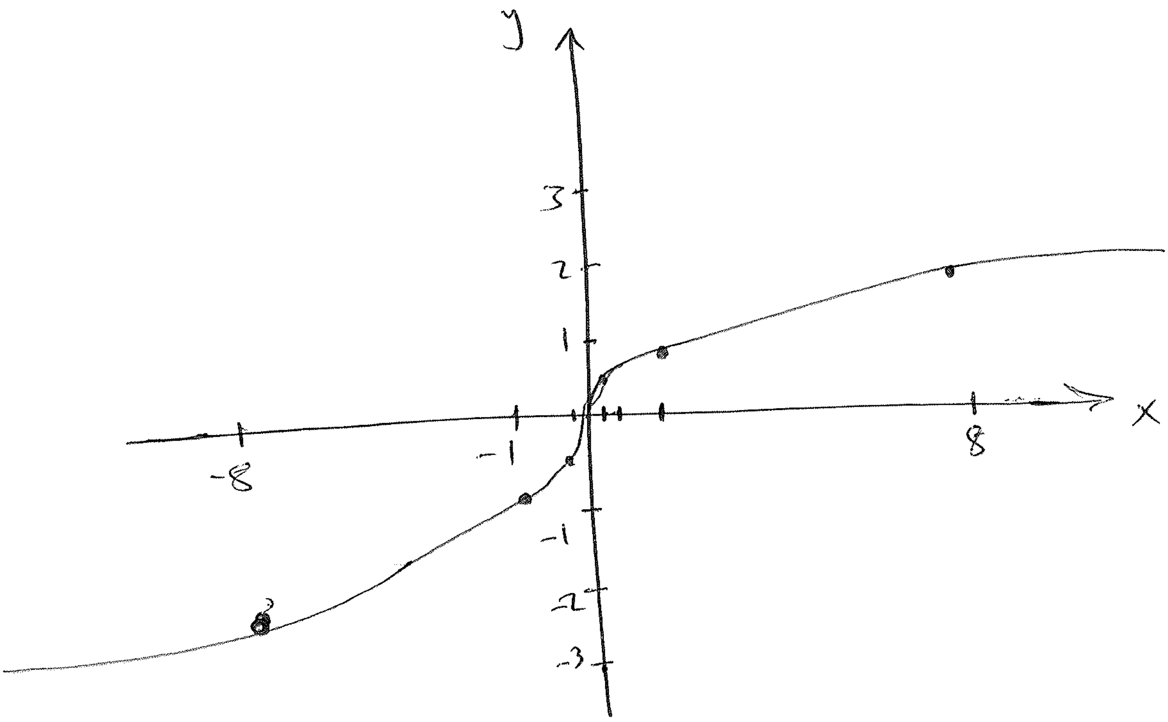
⑥ Example: $y = |x|$

x	y
-4	4
-2	2
-1	1
$-1/2$	$1/2$
0	0
$1/2$	$1/2$
2	2
4	4

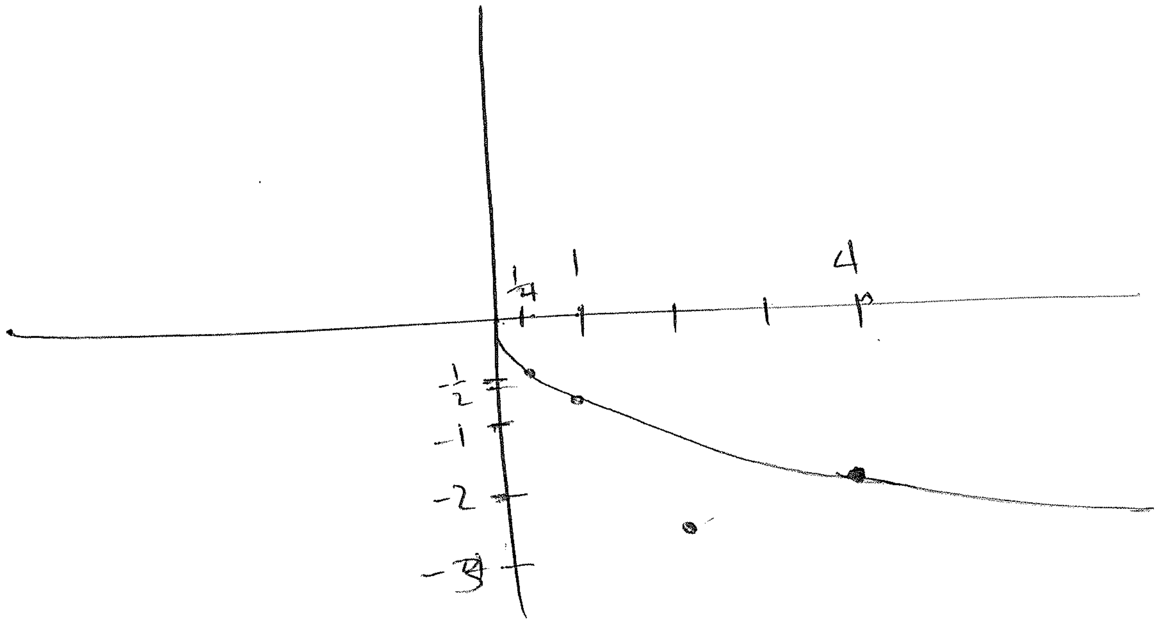


Example: $y = \sqrt[3]{x}$

x	y
-27	-3
-8	-2
-1	-1
$-\frac{1}{8}$	$\frac{1}{2}$
→ 0	0
$+\frac{1}{8}$	$\frac{1}{2}$
8	2
27	3



Example: $y = -\sqrt{x}$

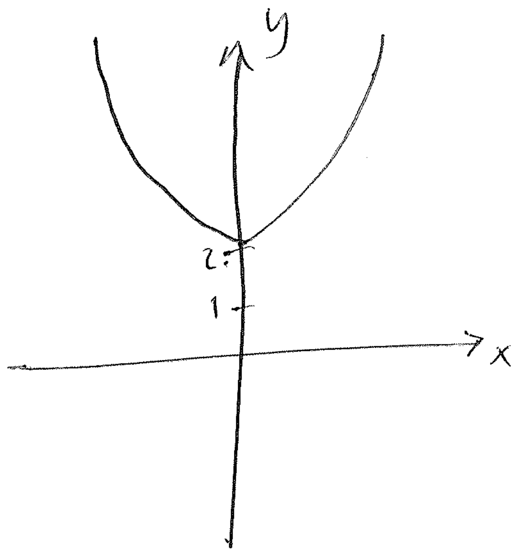


* Translation of Functions (Shifting the Graph)

① Shifting to upward

$y = f(x) + c$

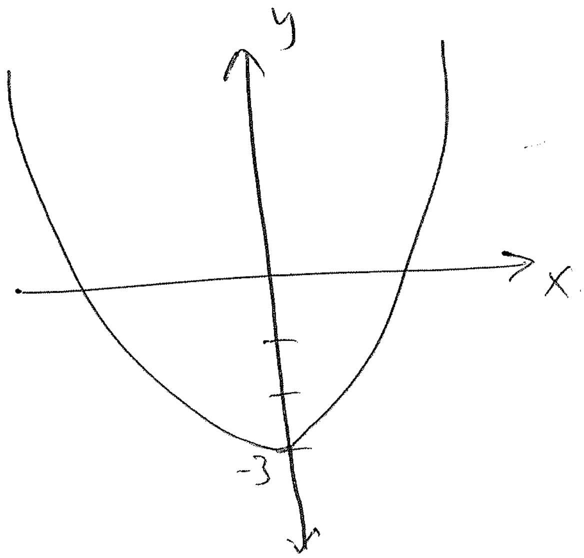
Example : $y = x^2 + 2$



② Shifting to downward

$y = f(x) - c$

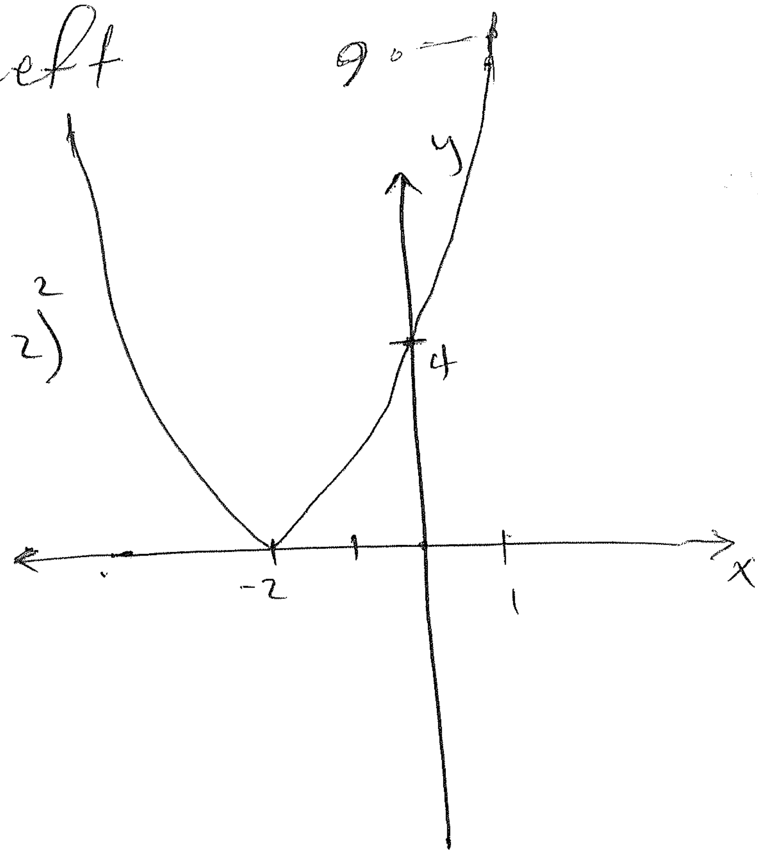
Example : $y = x^2 - 3$



③ Shifting to the left

$y = f(x+c)$

Example: $y = (x+2)^2$

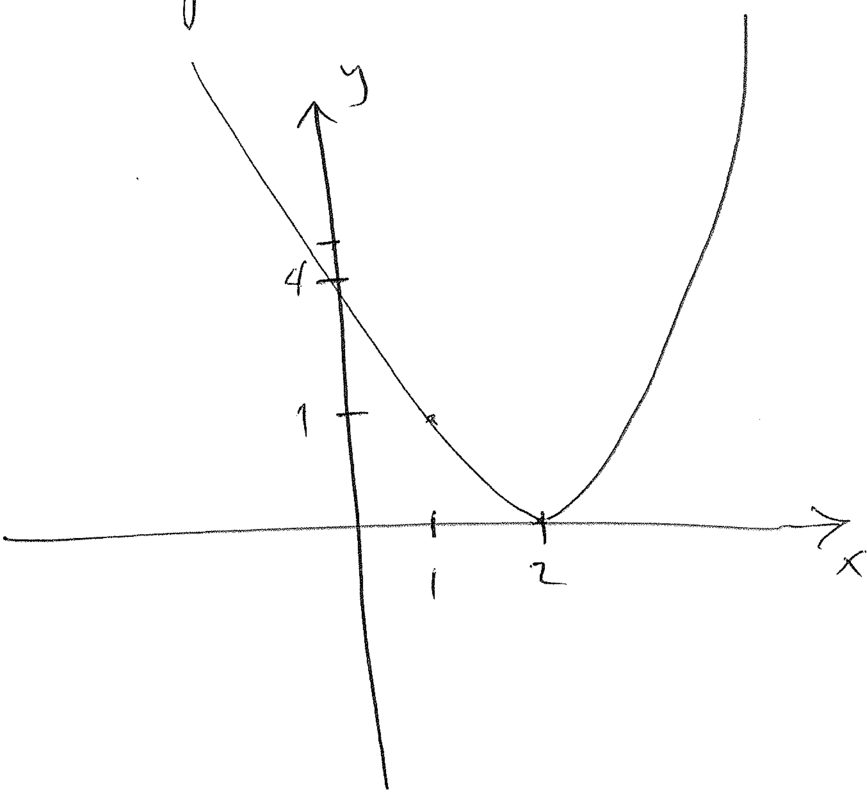


④ Shifting to the right

$y = f(x-c)$

Example:

$y = (x-2)^2$



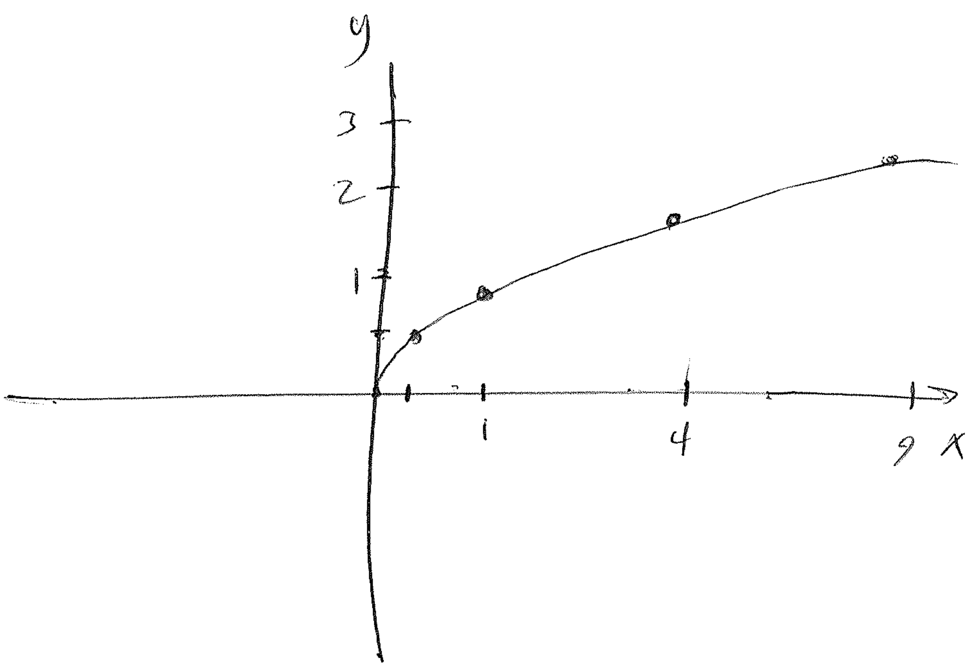
Example: Sketch the graph of

a.) $y = \sqrt{x-3}$, b.) $y = \sqrt{x+3}$, c.) $y = \sqrt{x} + 1$

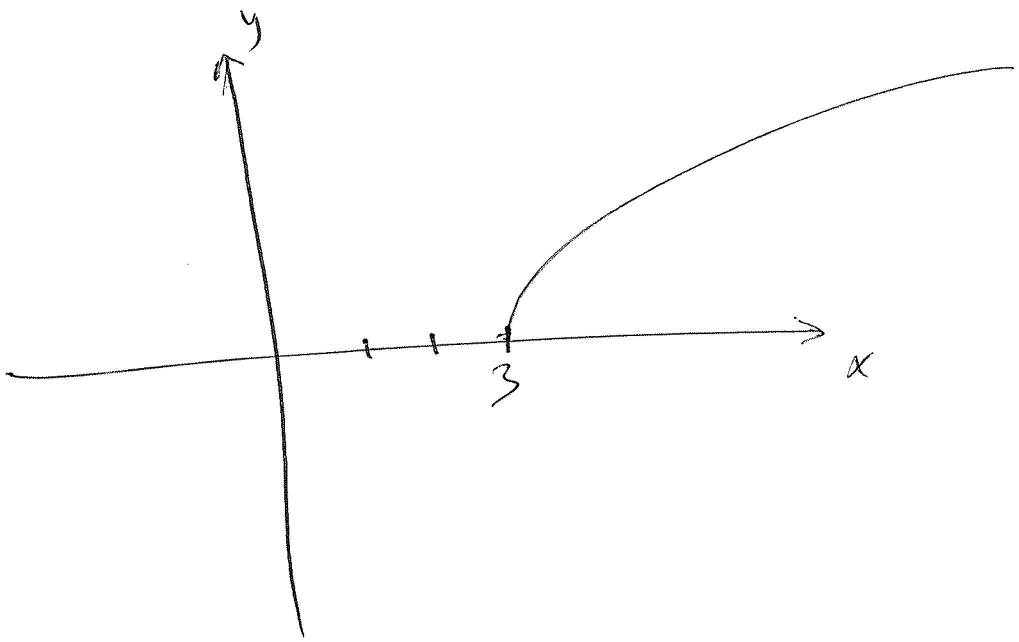
Solution

we know $y = \sqrt{x}$

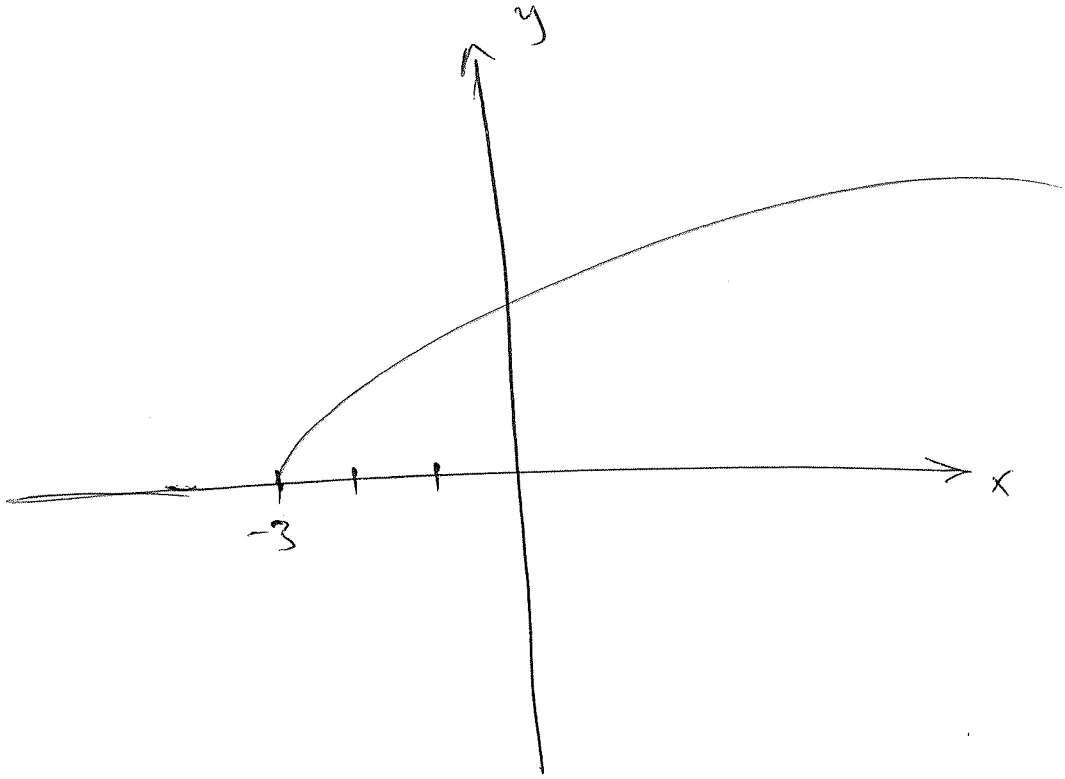
x	y
0	0
1/4	1/2
1	1
4	2
9	3



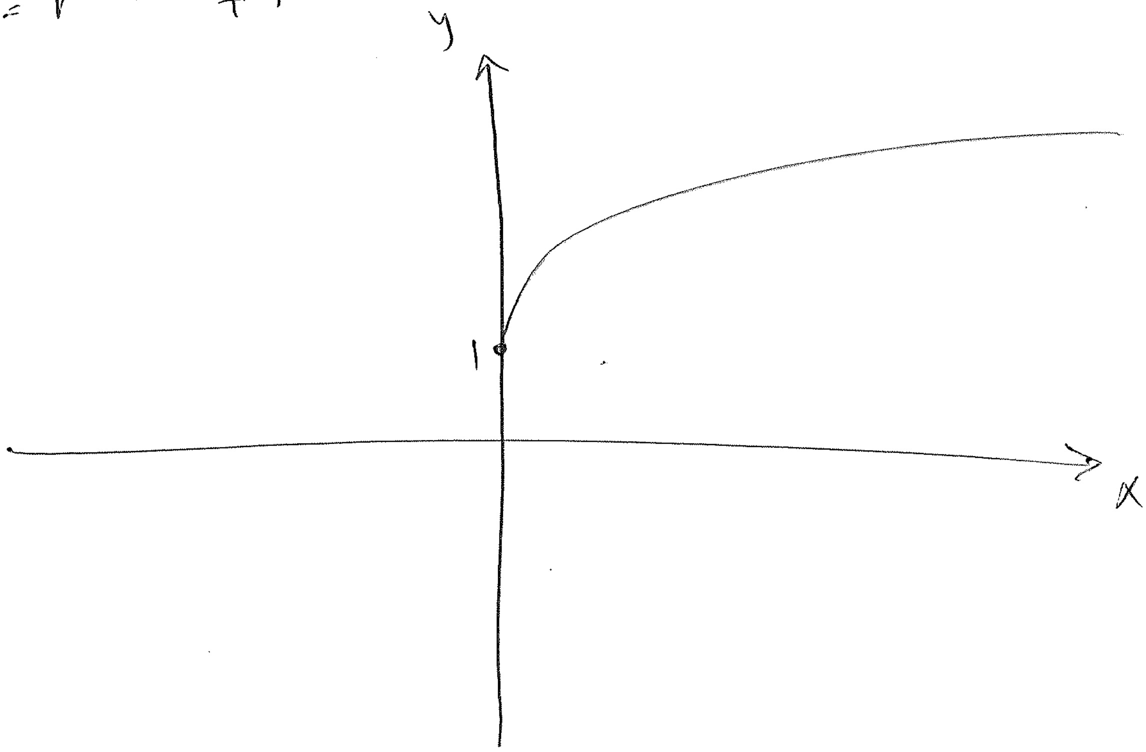
a.) $y = \sqrt{x-3}$



b.) $y = \sqrt{x+3}$



c.) $y = \sqrt{x} + 1$

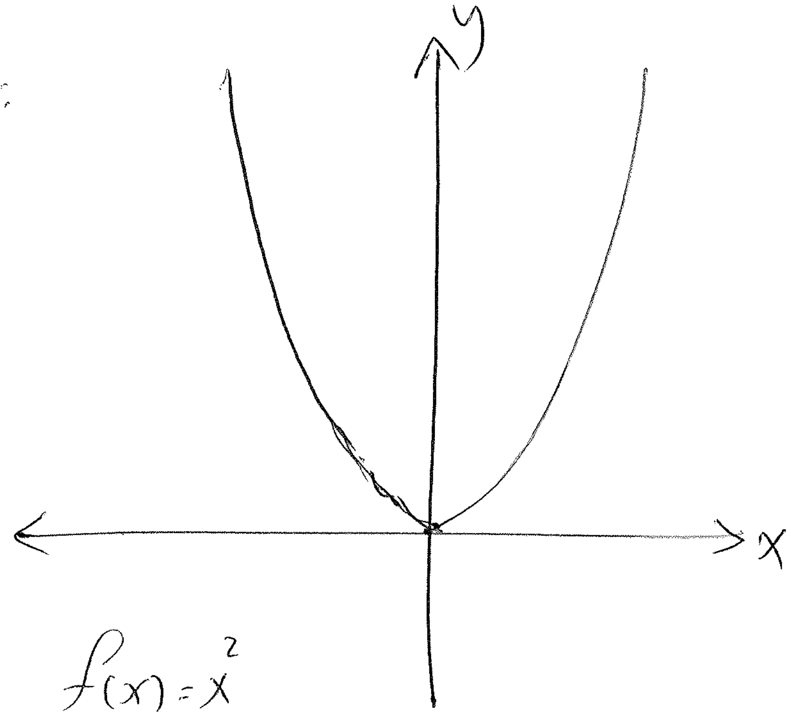


* Even & odd Functions :

① → Even Functions :

$$f(x) = f(-x)$$

$$f(x) - f(-x) = 0$$



Y-axis التناظر حول

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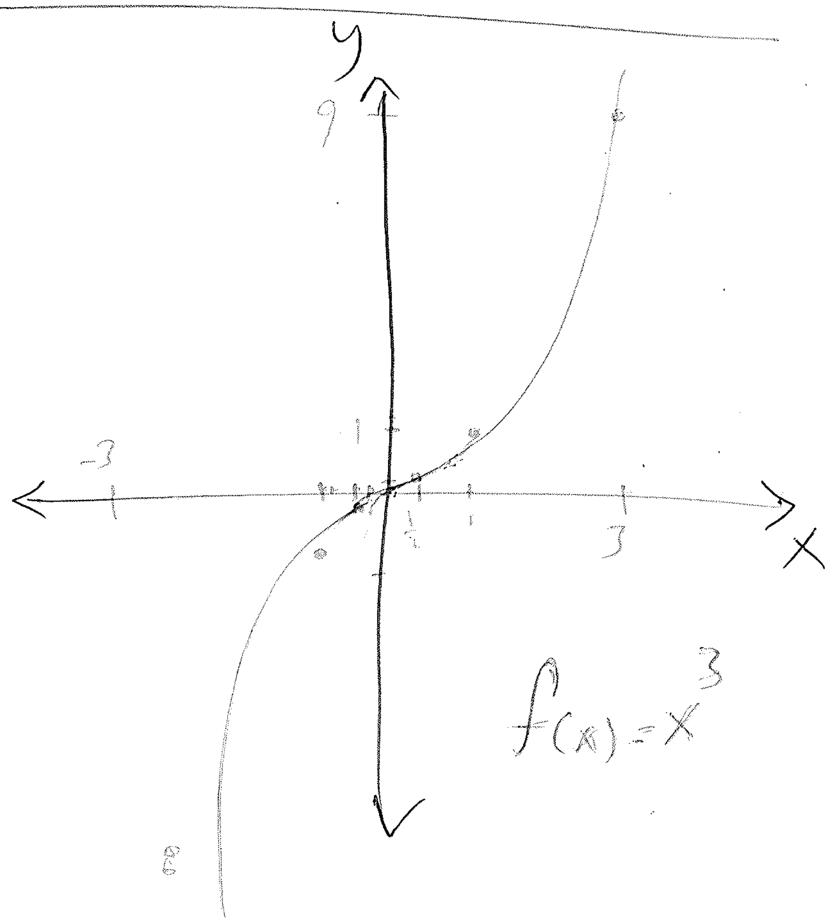
② Odd Functions

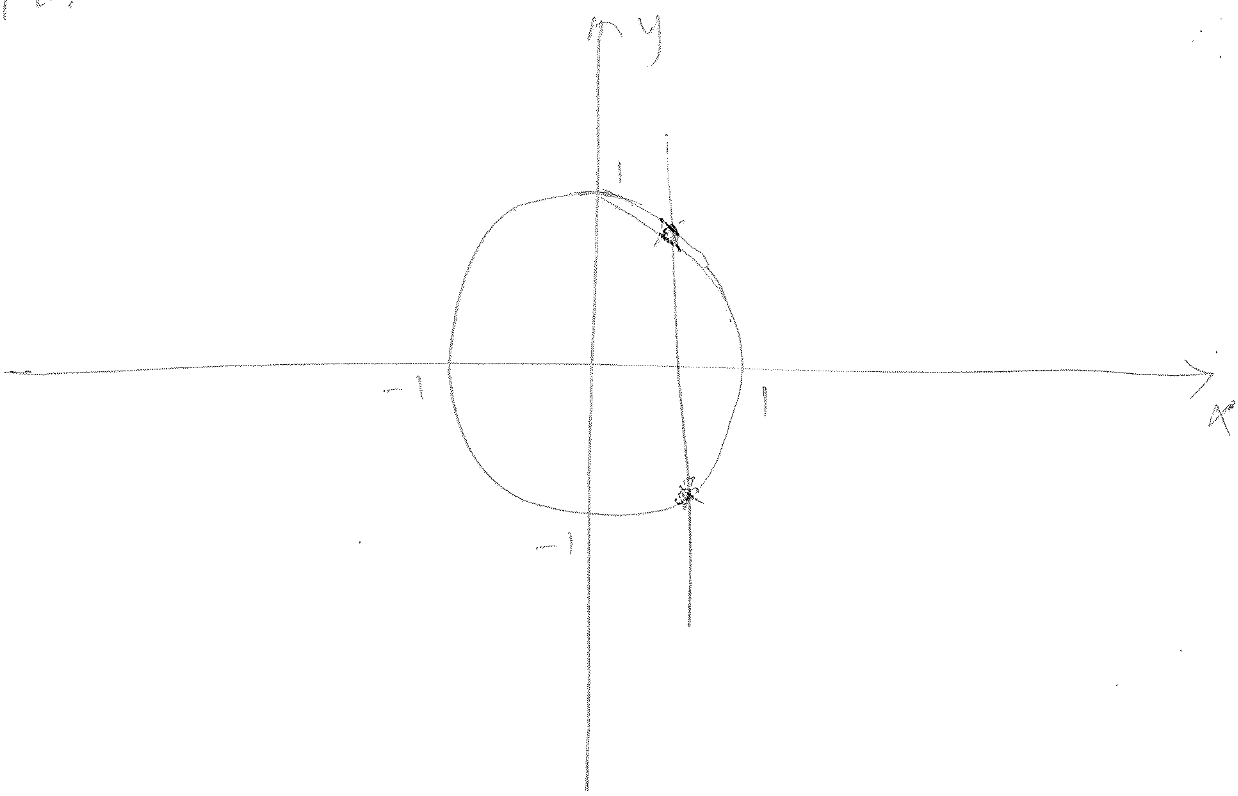
$$-f(x) = f(-x)$$

$$f(x) + f(-x) = 0$$



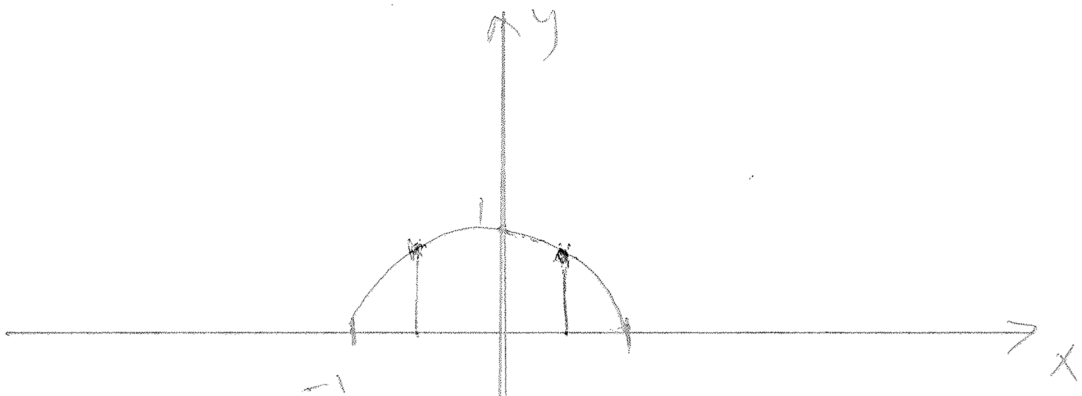
X-axis التناظر حول

Note:



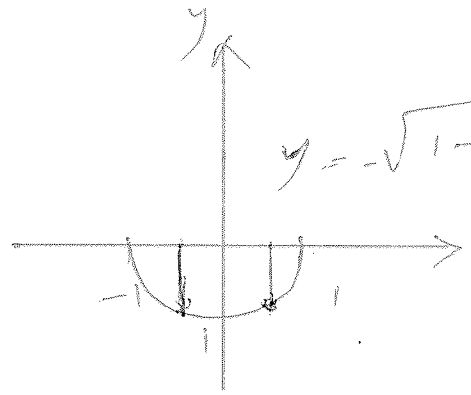
$x^2+y^2=1$

$\therefore$ The circle is not a graph for a function



$y=\sqrt{1-x^2}$

$\therefore$ The upper & lower semicircles are both considered a graph of a function --



$y=-\sqrt{1-x^2}$