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(V) Exponential & Logarithmic Functions

* the exponential Function is the function in the form : $f(x) = e^x$ or $y = e^x$ ← natural exponential function

where $e = 2.71828\ldots$

<u>H.W.</u> Why $e = 2.71828\ldots$?

* More generally an exponential function is a function of the form:

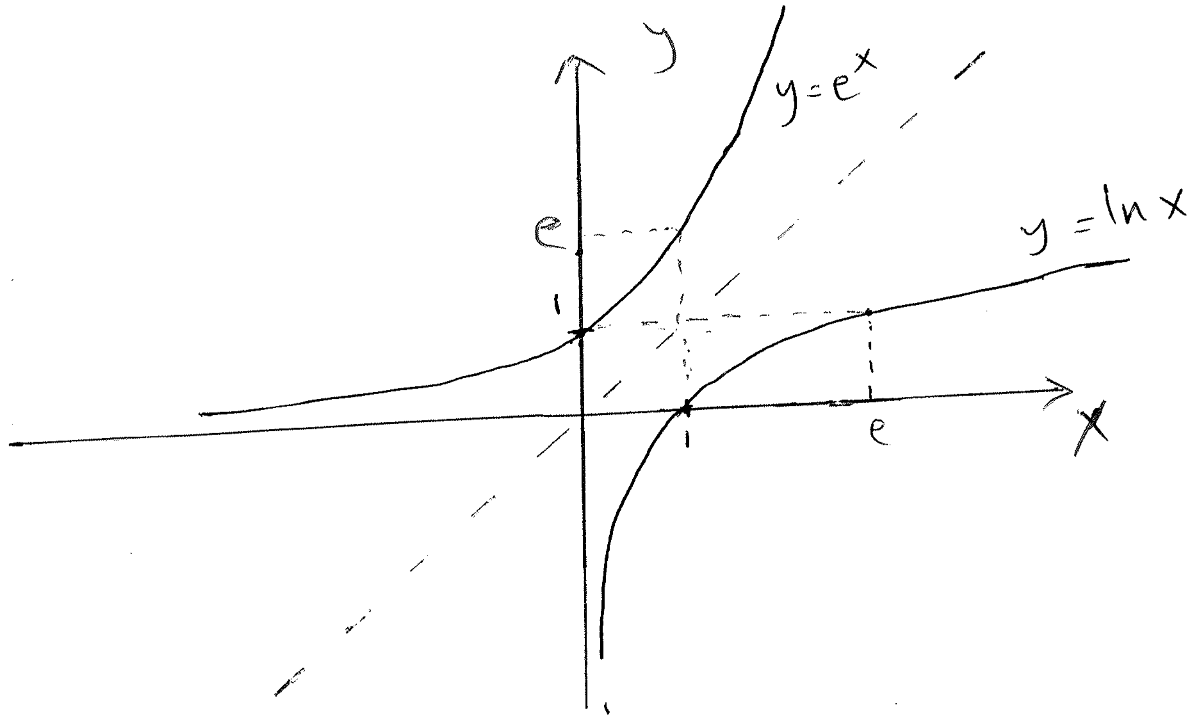
$$f(x) = a\,b^x$$

where b is the base $\in$ Real Number ($\mathbb{R}$). And x is the exponent.

* Logarithmic Functions are the inverses functions of exponential functions.

$$y = a^x \xleftrightarrow{\text{inverse}} x = a^y$$

$$y = e^x \xleftrightarrow{\text{inverse}} x = e^y$$



* Note: $y = \ln x$ has the same graph of $x = e^y$

$b^p \cdot b^q = b^{p+q}$, $\frac{b^p}{b^q} = b^{p-q}$, $(b^p)^q = b^{pq}$,

$b^0 = 1$, $b^1 = b$

* The natural logarithim function:

$$\ln x = \frac{\ln x}{\ln e} = \log_e x$$

where $\ln e = 1$

* $\log_b a = \frac{\ln a}{\ln b} = \frac{\log_{10} a}{\log_{10} b}$

* $\log_a a = 1$, $\log_a 1 = 0$

→

*If $b > 0$, $b \neq 1$, $a > 0$, $c > 0$, and r is any real number, then:

$\log_b(ac) = \log_b a + \log_b c$ product property

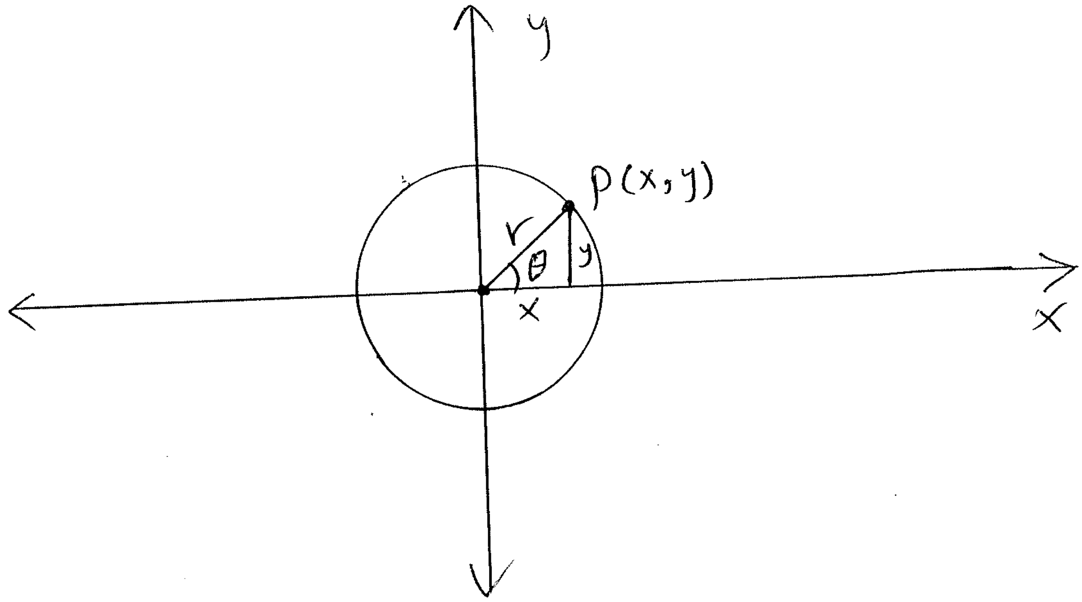
$\log_b(a/c) = \log_b a - \log_b c$ Quetient property

$\log_b(a)^r = r \log_b(a)$ Power Property

$\log_b(1/c) = -\log_b c$ Reciprocal property

(VI) Trigonometric Functions

If $p(x,y)$ is a point on the circle ~~when~~ and the angle θ measure the angular displacement of the radius (r), as shown in fugure below.



Then θ can be defined by the following equation:

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$$

* Observe that $\tan\theta$ and $\sec\theta$ are not defined when $x=0$ or $\theta = \left(\frac{\pi}{2}, \frac{3\pi}{2}, \ldots, \frac{-\pi}{2}, \frac{-3\pi}{2}, \ldots\right)$ and $\cot\theta$, $\csc\theta$ are not defined when $y=0$ or $\theta = (0, \pi, 2\pi, \ldots, -\pi, -2\pi, -3\pi)$

* From Phythagorian theorem:

$x^2 + y^2 = r^2$ devide by r^2

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

In the same way we can find:-

$$\left.\begin{aligned} 1 + \tan^2\theta &= \sec^2\theta \\ 1 + \cot^2\theta &= \csc^2\theta \end{aligned}\right\} \text{Prove that?}$$

* The relation between Trigonometric functions (Identities):

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad \ldots\ldots (1)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B \quad \ldots\ldots (2)$$

* if we replace B by A in equation (1)

or $(A = B)$,

$$\cos(2A) = \cos^2 A - \sin^2 A \quad \ldots\ldots (3a)$$

$$= 1 - \sin^2 A - \sin^2 A$$

$$= 1 - 2\sin^2 A \quad \ldots\ldots (3b)$$

Half Angle Formula

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or

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$= \cos^2 A - (1 - \cos^2 A)$$

$$= 2\cos^2 A - 1 \quad \ldots\ldots (3c)$$

$\Rightarrow$

* If we solve (3c :

$$\cos(2A) = 2\cos^2 A - 1$$

$$2\cos^2 A = \cos(2A) + 1$$

$$\cos^2 A = \frac{\cos 2A + 1}{2}$$

The formula of double Angle ---- (4a

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* If we solve (3b

$$\cos(2A) = 1 - 2\sin^2 A$$

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$ ---- (4b

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$\sin(A+B) = \sin A \cos B + \cos A \sin B$ ---- (5)

$\sin(A-B) = \sin A \cos B - \cos A \sin B$ ---- (6)

* if we replace B by A in (5):

$\sin(2A) = 2 \sin A \cos A$ ---- (7)

* by combining (5) and (6) by addition and subtraction we find that :-

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$ ---- (8)

$\sin B \cos A = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$ ---- (9)

* Similarly combining (1) and (2) by addition and subtraction we find that :

$\cos A . \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$ ---- (10)

$\sin A . \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ ---- (10.b)

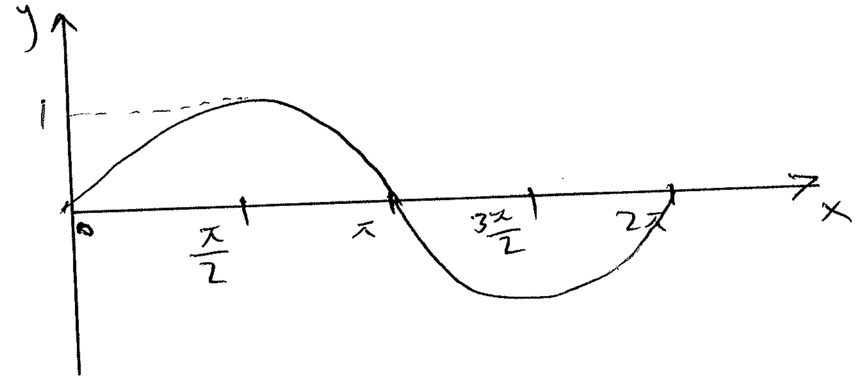
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B} \quad \text{------ (11)}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B} \quad \text{---- (12)}$$

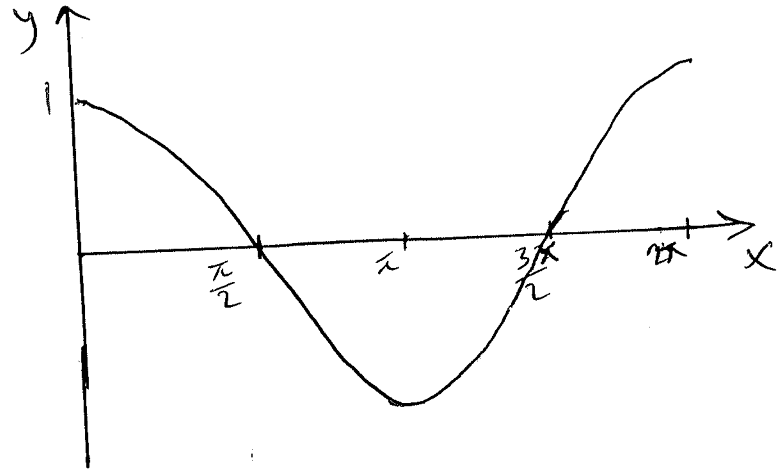
9

Graph of the Trigonometric Function

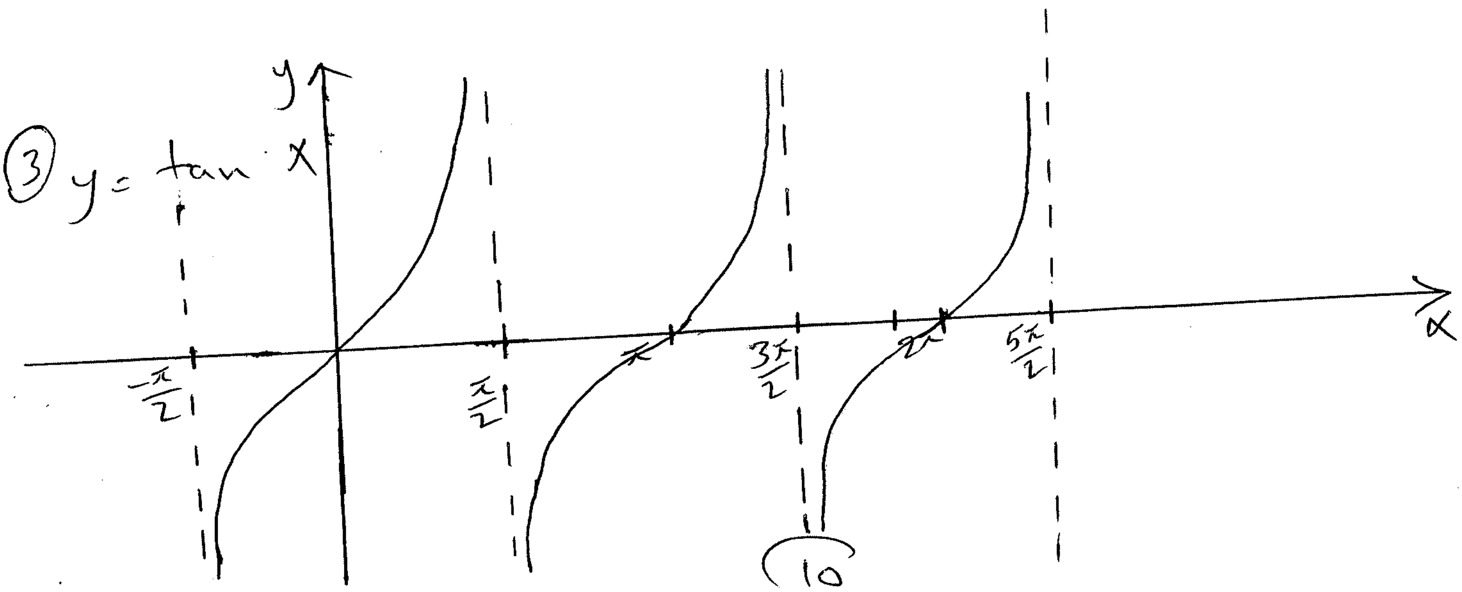
① $y = \sin x$



② $y = \cos x$



③ $y = \tan x$



Some Properties of the trigonometrix functions

1. $\sin(180-\alpha)=\sin\alpha$
2. $\cos(180-\alpha)=-\cos\alpha$
3. $\tan(180-\alpha)=-\tan\alpha$
4. $\sin(180+\alpha)=-\sin\alpha$
5. $\cos(180+\alpha)=-\cos\alpha$
6. $\tan(180+\alpha)=\tan\alpha$
7. $\sin(-\alpha)=-\sin\alpha$
8. $\cos(-\alpha)=\cos\alpha$
9. $\tan(-\alpha)=-\tan\alpha$
10. $\sin(90-\alpha)=\cos\alpha$
11. $\cos(90-\alpha)=\sin\alpha$
12. $\sin(90+\alpha)=\cos\alpha$
13. $\cos(90+\alpha)=-\sin\alpha$
14. $\sin(270-\alpha)=-\cos\alpha$
15. $\cos(270-\alpha)=-\sin\alpha$
16. $\sin(270+\alpha)=-\cos\alpha$
17. $\cos(270+\alpha)=\sin\alpha$