المحاضرة الرابعة

* Continuity الإستمرارية

A function f is said to be continous at $x = c$ provided the following conditions are satisfied:

① $f(c)$ is defined

② $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = L$

③ $\lim_{x \to c} f(x) = f(c)$

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* Example: Determine whether the following functions are continuous at $x = 2$ or not?

① $f(x) = \dfrac{x^2 - 4}{x - 2}$

② $g(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$

③ $h(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$

$\longrightarrow$

$\Rightarrow$ solution $f(x) = \frac{x^2-4}{x-2}$

① f is not defined at $x=2$ so $f(x)$ is not continous at $x=2$.

②

$$g(x) = \begin{cases} \frac{x^2-4}{x-2} & , x \neq 2 \\ 3 & , x = 2 \end{cases}$$

- $g(x)$ is defined at $x=2$, $g(2)=3$

- $\lim_{x \to 2} g(x) = \lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$= 4$

- But $g(2) \neq \lim_{x \to 2} g(x)$, so the function $g(x)$ is not continous at $x=2$

$\rightarrow$

③

$$h(x) = \begin{cases} \dfrac{x^2-4}{x-2} & , x \neq 2 \\ 4 & , x = 2 \end{cases}$$

- $h(2)$ is defined at $x=2$, $h(2)=4$

- $\lim_{x\to 2} h(x) = \lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} x+2 = 4$

- $h(2) = \lim_{x\to 2} h(x)$, so the function $h(x)$ is continous at $x=2$

Ex: Find the value of the constant K, if possible, that will make the function continous every where.

① $f(x) = \begin{cases} 7x-2 & , x \leq 1 \\ Kx^2 & , x > 1 \end{cases}$

② $g(x) = \begin{cases} Kx^2 & , x \leq 2 \\ 2x+K & , x > 2 \end{cases}$

Solution:

① - $f(1) = 5$

- $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\lim_{x \to 1^+} (Kx^2) = \lim_{x \to 1^-} (7x-2)$

$\boxed{K = 5}$

- $\lim_{x \to 1} f(x) = f(1) = 5$