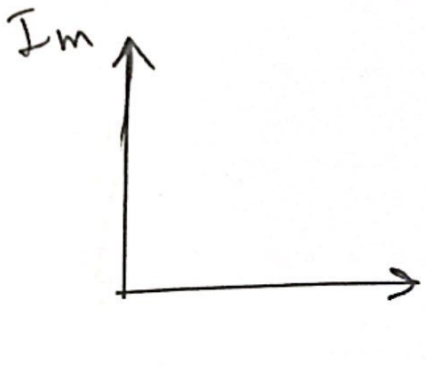
Complex Numbers : المحاضرة التاسعة
الأعداد المركبة

Any number which consists of Real and Imaginary part with an index ($j = \sqrt{-1}$) is called complex number.



$$z = Re + j\,Im \Rightarrow \text{Cartesian Form}$$

→ Complex addition :

$$z_1 = a_1 + j\,b_1$$

$$z_2 = a_2 + j\,b_2$$

$$z_1 + z_2 = a_1 + a_2 + j(b_1 + b_2)$$

$$z_1 - z_2 = a_1 - a_2 + j(b_1 - b_2)$$

→

→ Multiplication:

$$z_1 = a_1 + jb_1$$
$$z_2 = a_2 + jb_2$$

$$z_1 * z_2 = a_1a_2 + ja_1b_2 + ja_2b_1 + j^2 b_1b_2$$
$$= (a_1a_2 - b_1b_2) + j(a_1b_2 + a_2b_1)$$

→ Division: نضرب بمرافق المقام

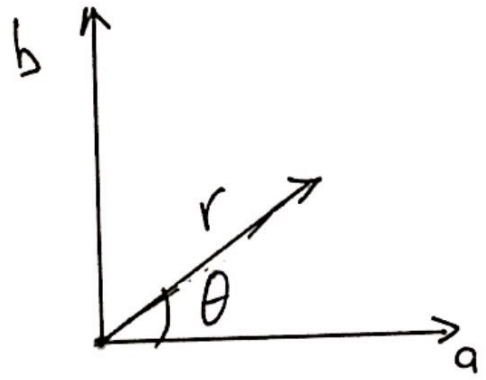
$$\frac{z_1}{z_2} = \frac{a_1 + jb_1}{a_2 + jb_2} * \frac{a_2 - jb_2}{a_2 - jb_2}$$

$$= \frac{a_1a_2 - ja_1b_2 + ja_2b_1 + b_1b_2}{a_2^2 - ja_2b_2 + ja_2b_2 + b_2^2}$$

$$= \frac{a_1a_2 + b_1b_2 + j(a_2b_1 - a_1b_2)}{a_2^2 + b_2^2}$$

*Polar Form:

$$z = a + jb \Rightarrow \text{Cartesian Form}$$



$$\cos\theta = \frac{a}{r} \Rightarrow a = r\cos\theta$$

$$\sin\theta = \frac{b}{r} \Rightarrow b = r\sin\theta$$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\therefore z = r\cos\theta + jr\sin\theta \quad \text{(Polar Form)}$$

by using Euler's formula

$$z = re^{j\theta}$$

In Angle notation

$$z = r\angle\theta$$

Example ①: $(2+j)^2 = 4 + 4j - 1$

$= 3 + 4j$

Example ②: $(2+j)^8$

يصعب حلها لذلك يتم تحويلها الى Polar (Phasor) $(re^{j\theta})$

we have:

$r = \sqrt{4+1} = \sqrt{5}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta = 0.464$

$\therefore z = (2+j)^8$

$2 + j = 5^{\frac{1}{2}} e^{j(0.464)}$

$\therefore z = \left(5^{\frac{1}{2}} \cdot e^{j(0.464)}\right)^8$

$z = 5^4 e^{3.712j}$

$z = 5^4 (\cos(3.712) + j\sin(3.712))$

$z = -526.051 - j337.484$ (cartesian)

Example ③ : Find the Value of x & y

if $(x+jy)(2+j) = 3-j$

such that $x \in R$, $y \in R$

solution :

$$2x + jx + j2y - y = 3 - j$$

$$2x - y + j(x+2y) = 3 - j$$

$$\therefore \quad 2x - y = 3 \quad \text{---- ①}$$

$$x + 2y = -1 \quad \text{---- ②}$$

solve eq.① + $-2 *$ eq.② :

$$-y - 4y = 3 + 2$$

$$-5y = 5$$

$$\boxed{y = -1}$$

sub. in ② :

$$x - 2 = -1$$

$$\boxed{x = 1}$$

* Note ①:

$$\text{cis}\,\theta = \cos\theta + j\sin\theta$$

$$\text{cis}^n\theta = \cos(n\theta) + j\sin(n\theta)$$

Note ②: Complex numbers are handled in section 4.5 in Calculas (P: 1539)

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