Generating FM & PM Waves

Basically, there are two ways of generating FM waves: ***indirect*** and ***direct.*** We first describe the narrowband FM generator that is utilized in the ***indirect FM generation*** of wideband angle modulation signals.

NBFM & NBPM Generation

For NBFM and NBPM signals, we have shown earlier that because $|k_f a(t)| \ll 1$ And $|k_p m(t)| \ll 1$, respectively, the modulated signals can be approximated by

$$\varphi_{NBFM}(t) = A[\cos(w_c t) - k_f a(t)\sin(w_c t)]$$

$$\varphi_{NBPM}(t) = A[\cos(w_c t) - k_p m(t)\sin(w_c t)]$$

Both approximations are linear and are similar to the expression of the AM wave.

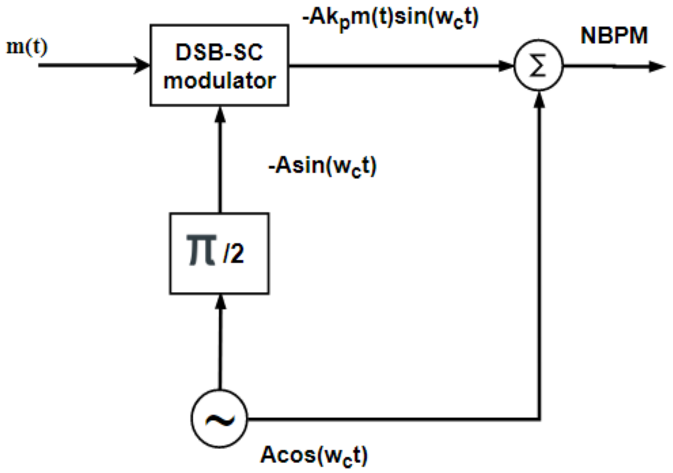


Figure: Generation NBPM

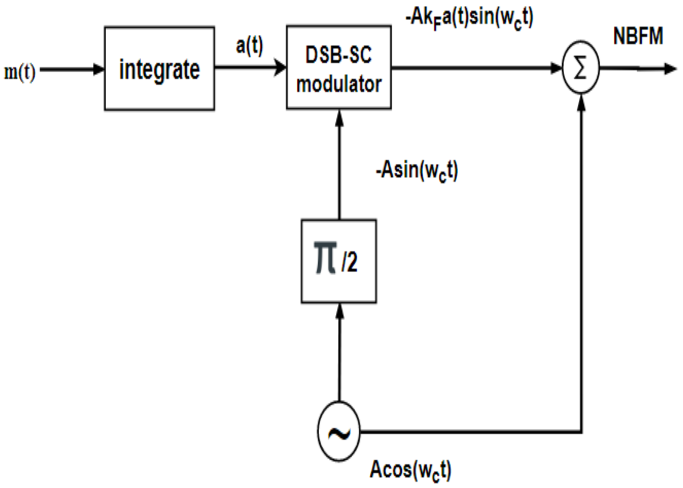
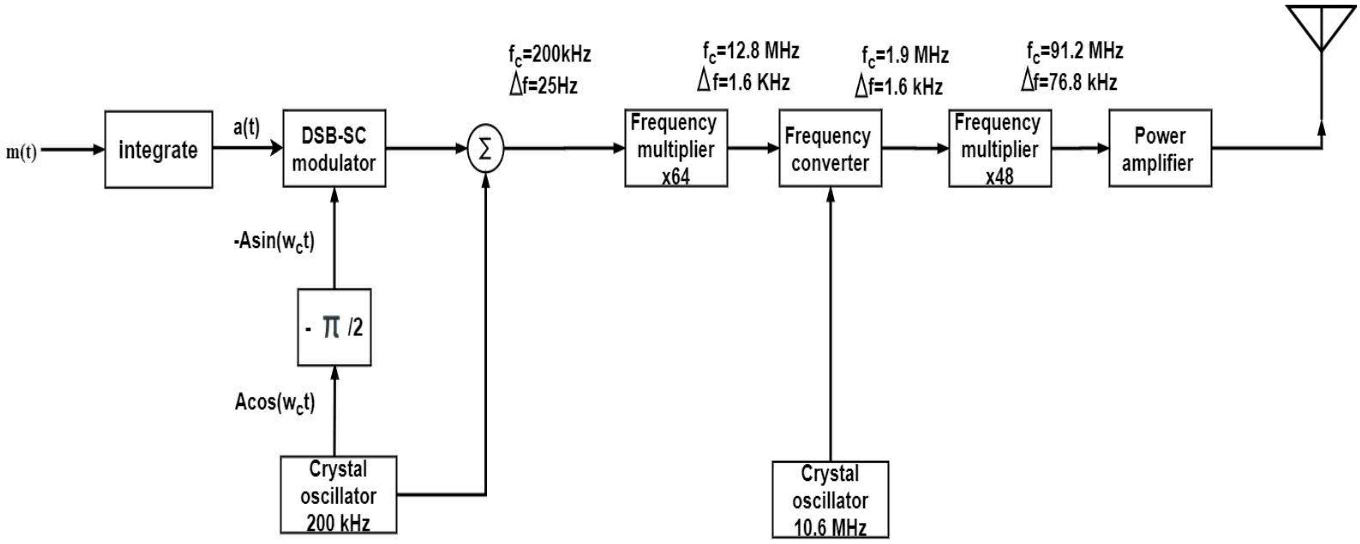


Figure: Generation NBFM

Indirect Method of Armstrong (WBFM & WBPM Generation)

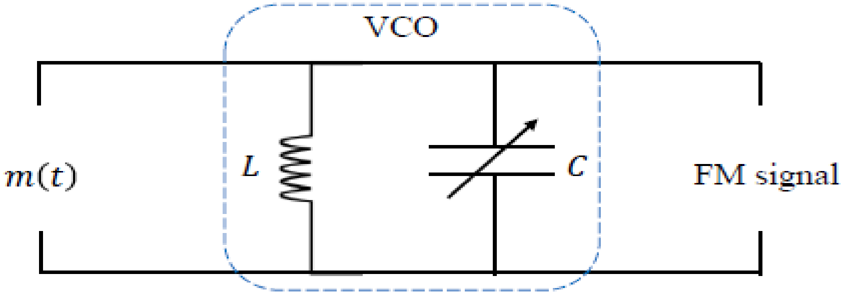
In Armstrong's indirect method, NBFM is generated as shown in Figure below. The NBFM is then converted to WBFM by using additional **frequency multipliers**. This forms the basis of the **Armstrong indirect frequency modulator**. First, generate an NBFM approximately. Then multiply the NBFM frequency and limit its amplitude variation. Generally, we require to increase Δf by a very large factor n. This increases the carrier frequency also by n. Such a large increase in the carrier frequency may not be needed. In this case we can apply **frequency mixing** to shift down the carrier frequency to the desired value.



This scheme has an **advantage** of **frequency stability**, but it **suffers** from inherent **noise** caused by excessive multiplication and **distortion** at lower modulating frequencies, where $\frac{\Delta f}{B}$ is not small enough.

Direct Generation

In a voltage-controlled oscillator (VCO), the frequency is controlled by an external voltage. The oscillation frequency varies linearly with the control voltage. We can generate an FM wave by using the modulating signal $m(t)$ as a control signal. One can construct a VCO using an operational amplifier and a hysteretic comparator (such as a Schmitt trigger circuit). Another way of accomplishing the same goal is to vary one of the reactive parameters (C or L) of the resonant circuit of an oscillator. A ***reverse-biased semiconductor diode*** acts as a capacitor whose capacitance varies with the bias voltage. The capacitance of these diodes, known under several trade names (e.g., Varicap, Varactor, Voltacap), can be approximated as a linear function of the bias voltage *m(t)* over a limited range.



DEMODULATION OF FM SIGNALS

To demodulate angle modulated signals, we need a device whose output is a function of its input frequency. This device is called the frequency discriminator.

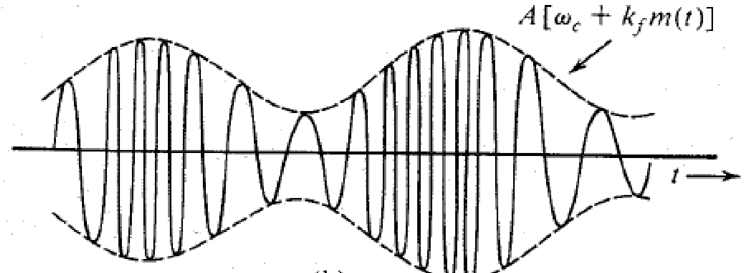
$$\boldsymbol{\varphi_{FM}(t) = A\cos(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto)}$$

$$\boldsymbol{\dot{\varphi}_{FM}(t) = \frac{d}{dt}[\varphi_{FM}(t)]}$$

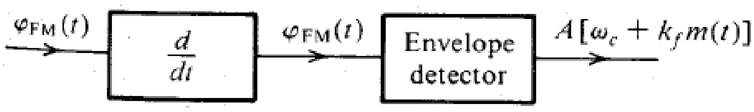
$$\boldsymbol{\dot{\varphi}_{FM}(t) = \frac{d}{dt}\left[A\cos(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto)\right]}$$

$$\boldsymbol{\dot{\varphi}_{FM}(t) = A(w_c + k_f m(t))\sin(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto) - \pi)}$$

Both the amplitude and the frequency of the signal $\boldsymbol{\dot{\varphi}_{FM}(t)}$ are modulated, the envelope being $\boldsymbol{A(w_c + k_f m(t))}$. Because $\boldsymbol{\Delta w = k_f m_p < w_c}$,
we have $\boldsymbol{w_c + k_f m(t) > 0}$ for all $\boldsymbol{t}$, and $m(t)$ can be obtained by **envelope detection** of $\boldsymbol{\dot{\varphi}_{FM}(t)}$.



(b)



(c)

The amplitude A of the incoming FM carrier must be constant. **If** the amplitude A were not constant, but a function of time, there would be an additional term containing $\frac{dA}{dt}$ on the right-hand side of equation above. Even if this term were neglected, the envelope of $\varphi_{FM}(t)$ would be $A(t)[w_c + k_f m(t)]$, and the envelope-detector output would be proportional to $m(t)A(t)$, still leading to distortions. Hence, it is essential to maintain A constant. Several factors, such as **channel noise and fading, cause A to vary**. This variation in A should be suppressed via the **bandpass limiter** before the signal is applied to the FM detector.

$$\varphi_{FM}(t) = A(t)\cos(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto)$$

$$\dot{\varphi_{FM}}(t) = \frac{d}{dt}[\varphi_{FM}(t)]$$

$$\dot{\varphi_{FM}}(t) = \frac{d}{dt}\left[A(t)\cos(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto)\right]$$

neglected

$$\dot{\varphi_{FM}}(t) = A(t)(w_c + k_f m(t))\sin(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto) - \pi) + \frac{dA(t)}{dt}\cos(w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto)$$

Bandpass Limiter

The amplitude variations of an angle-modulated carrier can be eliminated by what is known as a **bandpass limiter,** which consists of a hard limiter followed by a bandpass filter. The input-output characteristic of a hard limiter is shown in figure below. Observe that the bandpass limiter output to a sinusoid will be a square wave of unit amplitude regardless of the incoming sinusoidal amplitude. Thus an angle-modulated sinusoidal input $v_i(t) = A(t)\cos\theta(t)$ results in a constant amplitude, angle modulated square wave $v_o(t)$.

$$v_i(t) = A(t)\cos\theta(t)$$

$$\theta(t) = w_c t + k_f \int_{-\infty}^{t} m(\propto) d\propto$$

The output $v_o(t)$ of the hard limiter is $+1$ *or* -1, depending on whether $v_i(t) = A(t)\cos\theta(t)$ is positive or negative. Because $A(t) \geq 0$, $v_o(t)$ can be expressed as a function of θ :

$$v_o(t) = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

Hence, v_o as a function of θ is a periodic square wave function with period 2π figure below, which can be expanded by a Fourier series .

$$v_o(\theta) = \frac{4}{\pi}\left(\cos\theta(t) - \frac{1}{3}\cos 3\theta(t) + \frac{1}{5}\cos 5\theta(t) + \cdots\right)$$

At any instant t, $\boldsymbol{\theta(t) = w_c t + k_f \int_{-\infty}^{t} m(\propto) d \propto}$. Hence, the output $\boldsymbol{v_o}$ as a function of time is given by

$$\boldsymbol{v_o}\big(\theta(t)\big) = \boldsymbol{v_o}\left(\boldsymbol{w_c t + k_f \int_{-\infty}^{t} m(\propto) d \propto}\right)$$

$$\boldsymbol{v_o}(\theta) = \frac{\mathbf{4}}{\boldsymbol{\pi}}\left(\boldsymbol{cos}\left[\boldsymbol{w_c t + k_f \int_{-\infty}^{t} m(\propto) d \propto}\right] - \frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{cos}\,\mathbf{3}\left[\boldsymbol{w_c t + k_f \int_{-\infty}^{t} m(\propto) d \propto}\right] + \frac{\mathbf{1}}{\mathbf{5}}\boldsymbol{cos}\,\mathbf{5}\left[\boldsymbol{w_c t + k_f \int_{-\infty}^{t} m(\propto) d \propto}\right] + \cdots\right)$$

The output, therefore, has the original FM wave plus frequency-multiplied FM waves with multiplication factors of 3, 5, 7, ... We can pass the output of the hard limiter through a bandpass filter with a center frequency $\boldsymbol{w_c}$ and a bandwidth B_{FM}. The filter output $\boldsymbol{e_o(t)}$ is the desired **angle-modulated carrier with a constant amplitude.**

$$\boldsymbol{e_o(t)} = \frac{\mathbf{4}}{\boldsymbol{\pi}}\boldsymbol{cos}\left[\boldsymbol{w_c t + k_f \int_{-\infty}^{t} m(\propto) d \propto}\right]$$