

# **PROBABILITY, SIGNALS & SYSTEMS**

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# OBJECTIVE OF COURSE

- To develop understanding of fundamentals of probability including various probability distributions and laws of statistics and elementary statistical techniques to effectively analyze scientific data.

# SYLLABUS

<b>Weeks</b>	<b>Contents</b>
1-2	Introduction: Set Theory - Basic concepts of probability
3-4	Probability types: Conditional probability - Independent events
5	Baye's formula
6-7	Discrete and continuous random variables - Distributions and density functions
8-9	Probability distributions (binomial, Poisson, Hyper geometric, Normal, Uniform and exponential)
10-15	Mean - Variance - Standard deviations - Moments and generation functions - Linear regression and curve fitting - Limits theorems - Stochastic processes - First and second order characteristics - Applications
16-23	Signals, spectrum, and filters Singularity functions - Periodic signals and Fourier series - Non periodic signals and Fourier transform - Convolution and impulses system response and filters - Correlation and spectral density - Parseval's theorem for energy signals. Laplace Transform - Z-Transform - Analysis of signals and System.
24-27	Ideal & practical filters: LPF(RC & RL) - HPF(RC &RL) - BPF - BSF.
28-30	Noise Band limited white noise - Thermal noise - Noise figure.

# REFERENCES

- Introduction to Statistics by Walpole
- Modern Elementary Statistics by John E. Freund.
- Probability and its engineering uses by T.C.Fry.
- Elementary Statistics by P. A. Games & G. R. Klaro.
- Probability and Statistics by Nestollor, Rourke and Thomas.
- Introduction to Signals and Systems by Oppeheim.
- Signals and Systems- An Introduction by Leslie Balme.

# INTRODUCTION TO SET THEORY

- A **Set** is any well defined collection of “objects.”
- The **elements** of a set are the objects in a set.
- Usually we denote sets with upper-case letters, elements with lower-case letters. The following notation is used to show set membership
- $x \in A$  means that  $x$  is a member of the set  $A$
- $x \notin A$  means that  $x$  is not a member of the set  $A$ .



# WAYS OF DESCRIBING SETS

- List the elements

$$A = \{1, 2, 3, 4, 5, 6\}$$

- Give a verbal description

“A is the set of all integers from 1 to 6, inclusive”

- Give a mathematical inclusion rule

$$A = \{ \text{Integers } x \mid 1 \leq x \leq 6 \}$$

# SOME SPECIAL SETS

- The **Null Set** or **Empty Set**. This is a set with no elements, often symbolized by



- The **Universal Set** or **Sample set**. This is the set of all elements currently under consideration and is often symbolized by

$\Omega$

OR

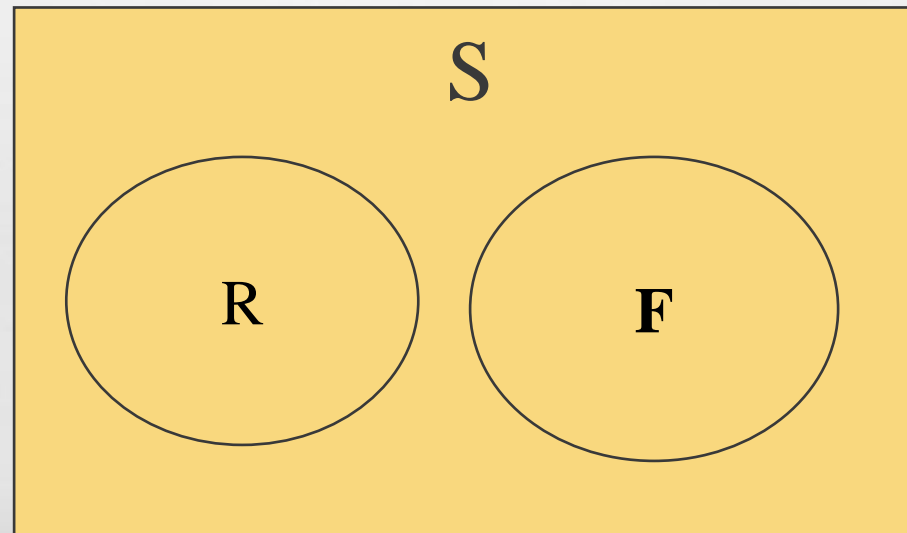
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# VENN DIAGRAMS

A **Venn Diagram** shows events as potentially intersecting circles or closed surfaces.

e.g., R=Republican

F=Female





# MEMBERSHIP RELATIONSHIPS

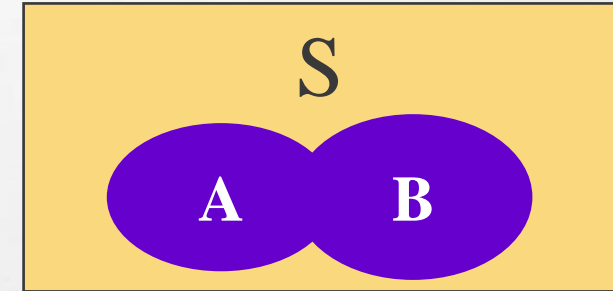
**Subset.**

$A \subset B$  “A is a subset of B”

- We say “A is a subset of B” if all the elements of A are also elements of B. The notation for subset is very similar to the notation for “less than or equal to,” and means, in terms of the sets, “included in or equal to.”

# COMBINING SETS – SET UNION

$$A \cup B$$



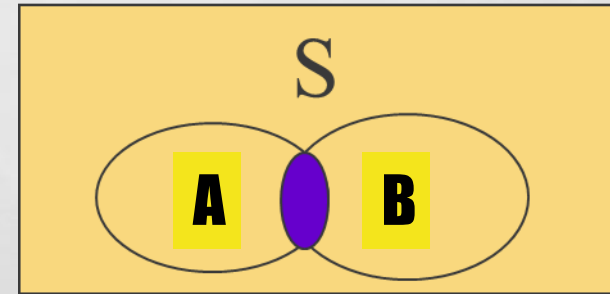
“A union B” is the set of all elements that are in A, or B, or both.

This is similar to the logical “**or**” operator.

# COMBINING SETS – SET INTERSECTION

$$A \cap B$$

“A intersect B” is the set of all elements that are in both A and B.  
This is similar to the logical “**and**”

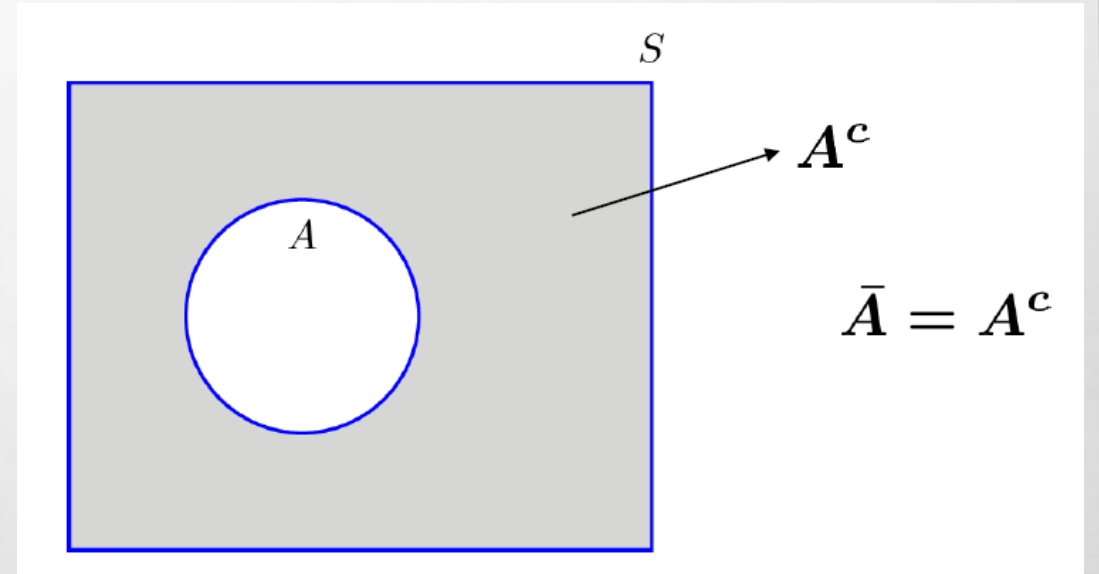


# SET COMPLEMENT

$$\bar{A} = A^c$$

- “A **complement**,” or “**not A**” is the set of all elements not in A.
- The complement operator is similar to the logical not, and is reflexive, that is,

$$\overline{\bar{A}} = A$$



# SET DIFFERENCE (SUBTRACTION)

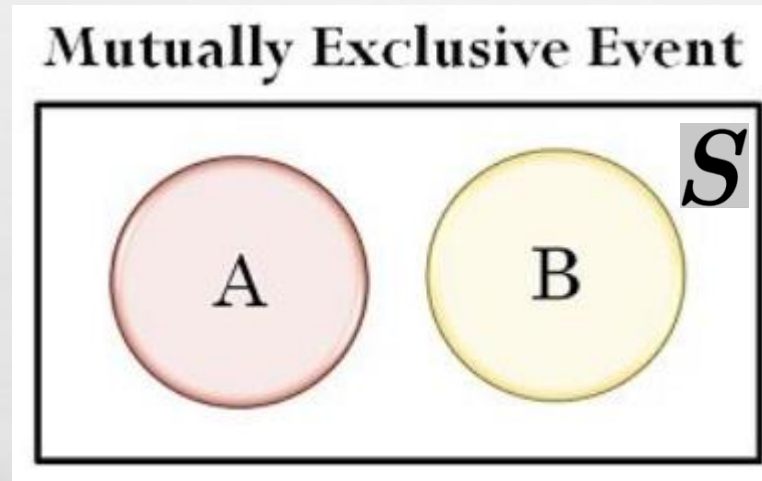
$$A - B$$

- The set **difference** “**A minus B**” is the set of all elements that are in A, and not in B, so

$$A - B = A \cap \bar{B}$$

# MUTUALLY EXCLUSIVE SET (DISJOINT)

- Two sets A and B are mutually exclusive (or disjoint) if  $A \cap B = \emptyset$



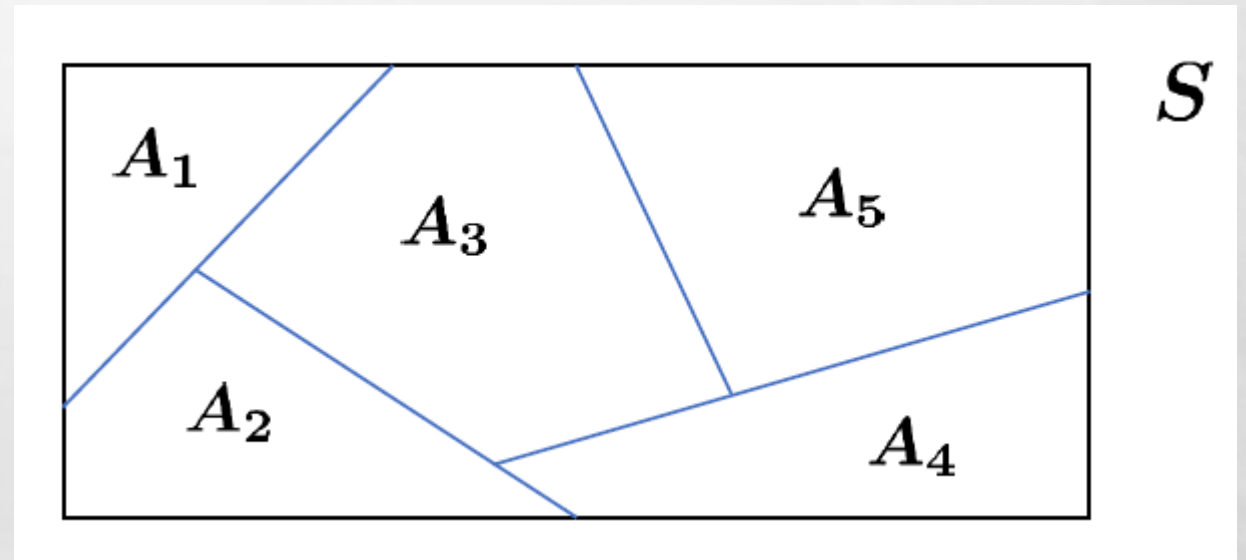


# PARTITION

• A collection of sets  $A_1, A_2, \dots, A_n$  is a Partition of  $S$  if

1. They are disjoint .

2.  $A_1 \cup A_2 \cup \dots \cup A_n = S$



# DE MORGAN'S LAW

- Theorem : De Morgan's law

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

# EXAMPLES

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5, 6\}$$

$$A \cap B = \{3\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$B - A = \{4, 5, 6\}$$

$$\bar{B} = \{1, 2\}$$

# DISTRIBUTIVE LAW

- Theorem : Distributive law

$$\bullet A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$\bullet A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

# SOME TEST QUESTIONS

$$A \cup \emptyset = ?$$

# SOME TEST QUESTIONS

$$A \cup \bar{A} = ?$$



# SOME TEST QUESTIONS

$$A \cap \emptyset = ?$$

# SOME TEST QUESTIONS

$$A - \bar{A} = ?$$

# SOME TEST QUESTIONS

$$A \cap \bar{A} = ?$$

# SOME TEST QUESTIONS

$$A \cup \Omega = ?$$

# SOME TEST QUESTIONS

$$A \cap \Omega = ?$$

# SOME TEST QUESTIONS

If  $A \subset B$  then

$$A \cap B = ?$$



## **SOME TEST QUESTIONS**

If  $A \subset B$  then

$$A \cup B = ?$$