PROBABILITY, SIGNALS & SYSTEMS

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THE GENERAL MULTIPLICATION RULE

The probability that events A and B both occur can be found using the general multiplication rule

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

where $P(B \mid A)$ is the conditional probability that event B occurs given that event A has already occurred.

In words, this rule says that for both of two events to occur, first one must occur, and then given that the first event has occurred, the second must occur.

EXAMPLE

- Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

Sol:

Let D1 (resp. to the defective fuse in the first test), and D2 (resp.to the second defective fuse in the second test). We want to compute

$$P(D_1 \cap D_2) = P(D_1)P(D_2|D_1) = \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{21}$$

TREE DIAGRAMS

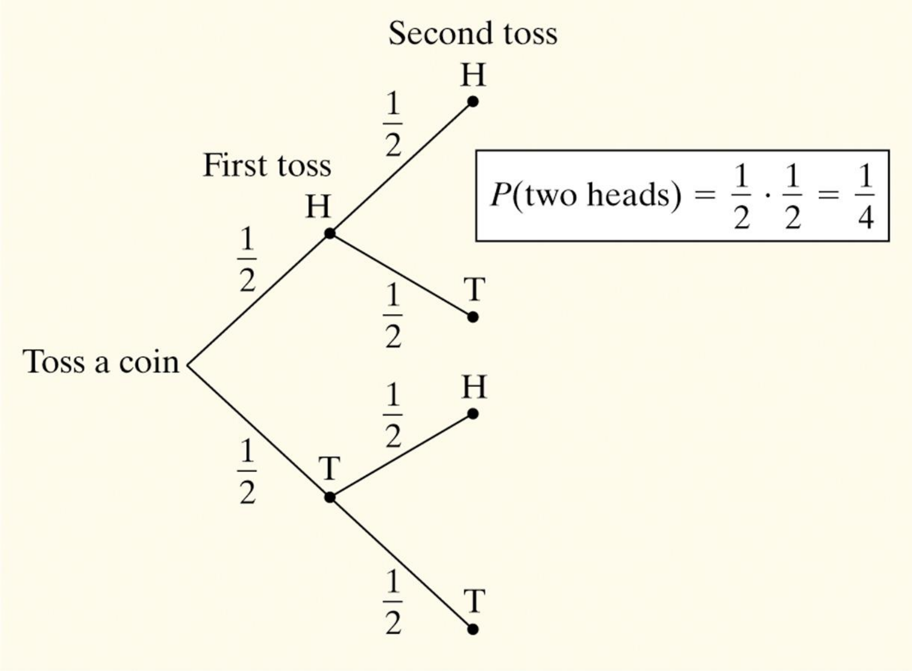
- The general multiplication rule is especially useful when a chance process involves a sequence of outcomes. In such cases, we can use a tree diagram to display the sample space.

Consider flipping a coin twice.

What is the probability of getting two heads?

<u>Sample Space:</u>
HH HT TH TT

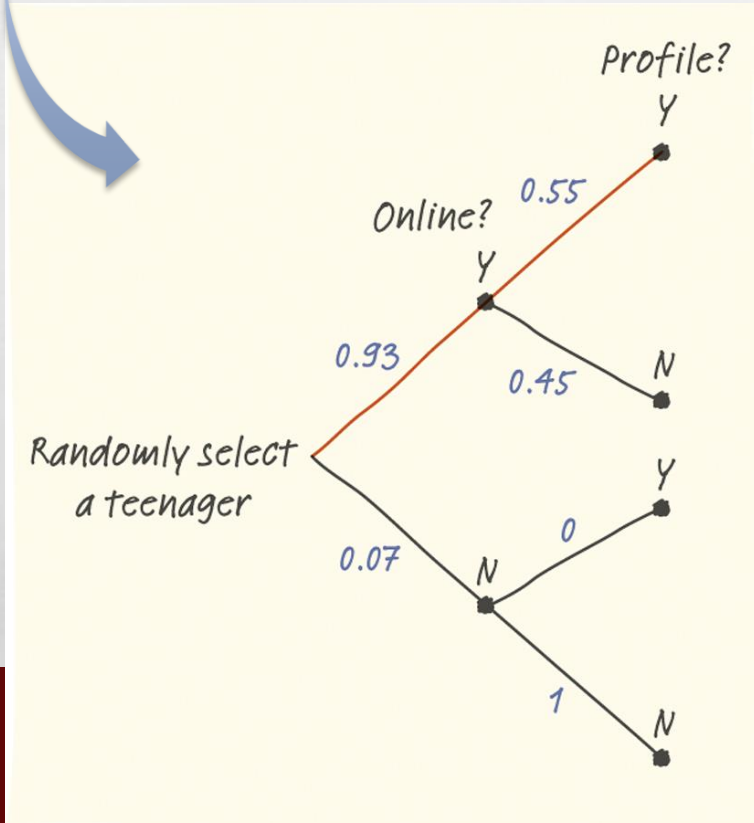
So, *P*(two heads) = *P*(HH) = 1/4



EXAMPLE: TREE DIAGRAMS

The Pew Internet and American Life Project finds that 93% of teenagers (ages 12 to 17) use the Internet, and that 55% of online teens have posted a profile on a social-networking site.

What percent of teens are online and have posted a profile?



$$P(\text{online}) = 0.93$$

$$P(\text{profile} \mid \text{online}) = 0.55$$

$$\begin{aligned} P(\text{online and have profile}) &= P(\text{online}) \times P(\text{profile} \mid \text{online}) \\ &= (0.93)(0.55) \\ &= 0.5115 \end{aligned}$$

51.15% of teens are online *and* have posted a profile.

EXAMPLE

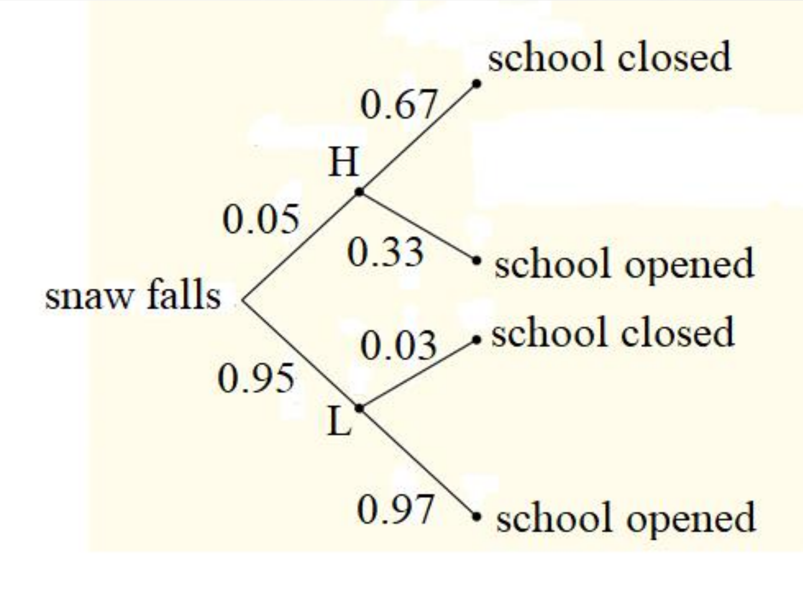
Of all snowfalls , 5% are heavy. After a heavy snowfall, schools are closed 67% of the time. After a light snowfall, schools are closed 3% of the time.

- Find the probability that the snowfall is light and the schools are closed.

  = 0.95*0.03 = 0.0285

- Find p(schools open, heavy snow)

  = 0.33*0.05 = 0.0165



BAYES' RULE

- For any two events A and B, where P(A) ≠ 0, we have

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}.$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B)P(B).$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

BAYES' RULE

If $B_1, B_2, B_3, \cdots$ is a partition of the sample space S, and A is any event with $P(A) > 0$, we have

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_i P(A|B_i)P(B_i)}.$$

CONDITIONAL INDEPENDENCE

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B), \quad \text{or equivalently, } P(A|B) = P(A).$$

Two events A and B are conditionally independent given an event C if and only if

$$P(A \cap B|C) = P(A|C)P(B|C).$$

CHAIN RULE FOR CONDITIONAL PROBABILITY

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B \cap A) = P(A)P(B|A)$$

We can extend this to 3 or more events:

$$P(B \cap A \cap C) = P(A)P(B|A)P(C|A \cap B).$$

EXAMPLE

• Consider a manufacturing firm that receives shipment of parts from two suppliers. It get 65 percent of it's parts from supplier 1 and 35 percent from supplier 2. The Quality levels differ between suppliers are shown in the table below. Find the following probabilities:

1. the probability of selecting a part from supplier 1 that is good.
2. A bad part broke one of it's machines. What is the probability the part came from suppler 1.
3. The probability of selecting a bad part.

	Percentage Good Parts	Percentage Bad Parts
Supplier 1	98	2
Supplier 2	95	5

DATA

Let A_1 denote the event that a part is received from supplier 1; A_2 is the event the part is received from supplier 2

P(A1) = .65   and P(A2) = .35

Let G denote that a part is good and B denote the event that a part is bad. Thus we have the following conditional probabilities:

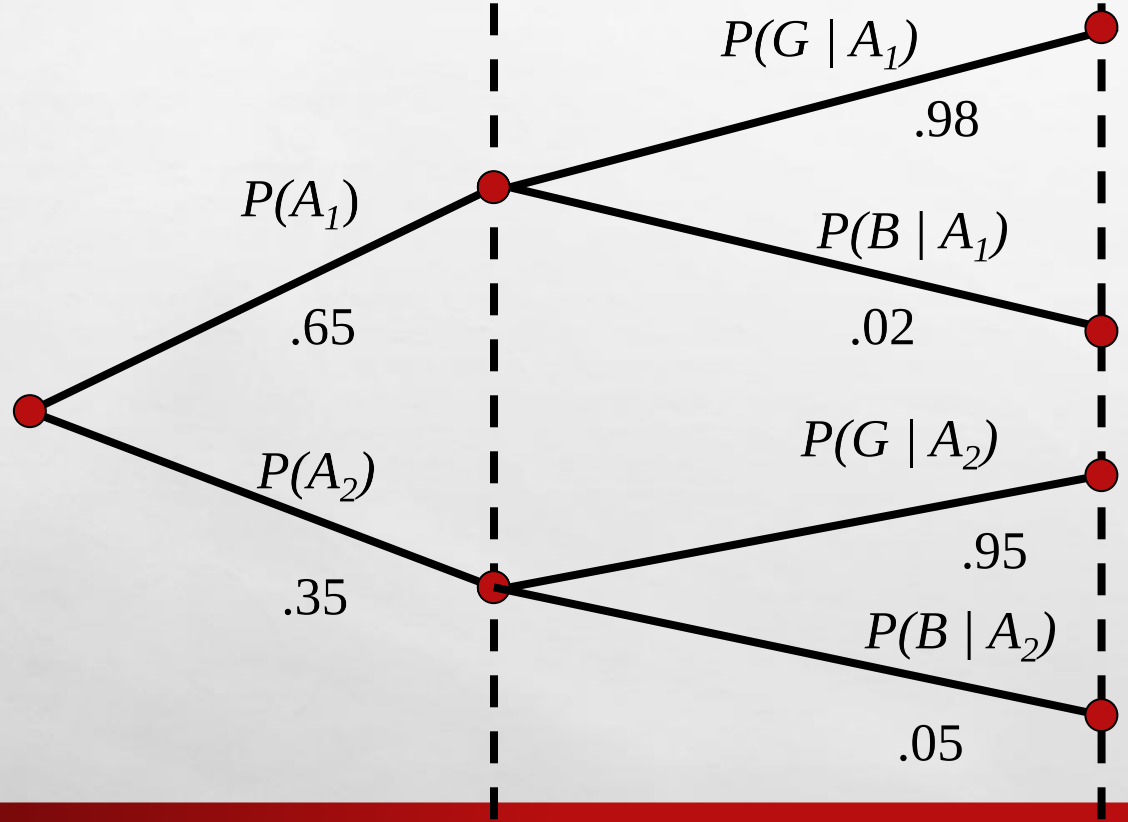
$$P(G \mid A_1) = .98 \text{ and } P(B \mid A_1) = .02$$

$$P(G \mid A_2) = .95 \text{ and } P(B \mid A_2) = .05$$

SOLUTION

1-

Step 1 Supplier | Step 2 Condition | Probability of Outcome



$P(A_1 \cap G) = P(A_1)P(G \mid A_1) = .6370$

$P(A_1 \cap B) = P(A_1)P(B \mid A_1) = .0130$

$P(A_2 \cap G) = P(A_2)P(G \mid A_2) = .3325$

$P(A_2 \cap B) = P(A_2)P(B \mid A_2) = .0175$

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SOLUTION

2-

$$P(A_1 | B) = \frac{P(A_1)P(B | A_1)}{P(A_1)P(B | A_1) + P(A_2)P(B | A_2)}$$

$$= \frac{(.65)(.02)}{(.65)(.02) + (.35)(.05)} = \frac{.0130}{.0305} = .4262$$

SOLUTION

3- The probability of selecting a bad part is found by adding together the probability of selecting a bad part from supplier 1 and the probability of selecting bad part from supplier 2.

That is:

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B)$$
$$= P(A_1)P(B|A_1) + P(A_2)P(B/A_2)$$

TABULAR APPROACH TO BAYES' THEOREM 2-SUPPLIER PROBLEM

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B \mid A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5) Posterior Probabilities $P(A_i \mid B)$
A_1	.65	.02	.0130	.0130/.0305 =.4262
A_2	.35	.05	.0175	.0175/.0305 =.5738
	1.00		P(B)=.0305	**1.0000**

H.W

A school system compiled the following information from a survey it sent to people who were juniors 10 years earlier. 85% of the students graduated from high school of the students who graduated, 90% are happy with their present job. of the students who did not graduate, 60% are happy with their present job.

Find the probability that the person graduated and is happy.
Find p(did not graduate and is happy).
Make a tree diagram.