PROBABILITY, SIGNALS & SYSTEMS

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MEAN, VARIANCE, AND STANDARD DEVIATION

Mean:

$$\mu = \Sigma\ X\ .\ P(X)$$

Variance:

$$\sigma^2 = \Sigma\ [X^2\ .\ P(X)] - \mu^2$$

Standard Deviation:

$$\sigma = \sqrt{\Sigma\ [X^2\ .\ P(X)] - \mu^2}$$

EXPECTATION OF FUNCTIONS OF RANDOM VARIABLES

- The expected value, or expectation, of a discrete random variable of a probability distribution is the theoretical average of the variable.

- The expected is by definition, the mean of the probability distribution.

$$E(X) = \mu = \Sigma X . P(X)$$

EXPECTATION OF FUNCTIONS OF RANDOM VARIABLES

X is **discrete**

$$E\left[g(X)\right] = \sum_{x} g(x)\,p(x) = \sum_{i} g(x_i)\,p(x_i)$$

X is **continuous**

$$E\left[g(X)\right] = \int_{-\infty}^{\infty} g(x)\,f(x)\,dx$$

EXAMPLE

Find the mean of the number of spots that appear when a die is tossed.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Solution:

$\mu = \Sigma X . P(X)$

$= 1 . 1/6 + 2 . 1/6 + 3 . 1/6 + 4 . 1/6 + 5 . 1/6 + 6 . 1/6$

$= 21/6 = 3.5$

EXAMPLE

Compute the Variance and standard deviation for the probability distribution.

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Solution:

$\sigma^2 = \Sigma\,[X^2 \,.\, P(X)] - \mu^2$

$= 1^2 \,.\, 1/6 + 2^2 \,.\, 1/6 + 3^2 \,.\, 1/6 + 4^2 \,.\, 1/6 + 5^2 \,.\, 1/6 + 6^2 \,.\, 1/6 - (3.5)^2$

$\sigma^2 = 2.9,$ $\sigma = 1.7$

VARIANCE

- The average of the squared deviations about the mean is called the variance.
- a variance can of two types which are:

1- Variance of a population

2- Variance of a sample

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N}$$ **For population variance**

the mean square deviation (POPULATION VARIANCE)

$$s^2 = \frac{\sum(x-\bar{X})^2}{n-1}$$ **For sample variance**

The variance of a population is denoted by σ^2 and the variance of a sample by S^2.

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POPULATION VARIANCE

- The population variance is the mean squared deviation from the population mean:

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x-\mu)^2}{N}$$

- Where σ^2 stands for the population variance
- μ is the population mean
- N is the total number of values in the population
- x is the value of the i-th observation.
- $\sum$ represents a summation

SAMPLE VARIANCE

- The sample variance is defined as follows:

$$s^2 = \frac{\sum_{i=1}^{N}(x - \bar{X})^2}{n - 1}$$

- Where s^2 stands for the sample variance
- $\bar{x}$ is the sample mean
- n is the total number of values in the sample
- x_i is the value of the i-th observation.
- Σ represents a summation

STANDARD DEVIATION

standard deviation - is the positive square root of the variance

- **population standard deviation:** $\sigma = \sqrt{\sigma^2}$ $\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}}$

the root mean square deviation (POPULATION STANDARD DEVIATION)

- **sample standard deviation:** $s = \sqrt{s^2}$ $s = \sqrt{\frac{\Sigma(x-\bar{X})^2}{n-1}}$

EXAMPLE

e.g. Find the msd of the following data:

x	7	9	14

Mean, $\bar{x} = \dfrac{\sum x}{n} \Rightarrow \bar{x} = \dfrac{30}{3} = 10$

(i) $\sigma^2 = \dfrac{\sum (x-\bar{x})^2}{n} = \dfrac{(7-10)^2 + (9-10)^2 + (14-10)^2}{3}$

$= \dfrac{9+1+16}{3} = 8 \cdot 67$

(ii) $\sigma^2 = \dfrac{\sum x^2}{n} - \bar{x}^2 = \dfrac{49+81+196}{3} - 10^2 = \dfrac{326}{3} - 100 = 8 \cdot 67$

EXAMPLE

e.g. Find the sample mean and sample Variance of the following data:

x	7	9	14

Mean, $\bar{x} = \dfrac{\sum x}{n} \Rightarrow \bar{x} = \dfrac{30}{3} = 10$

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{(7-10)^2 + (9-10)^2 + (14-10)^2}{2}$$

$$= \frac{9+1+16}{2} = 13.0$$

SUMMARY

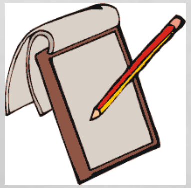
- The *msd* or variance, measure the spread or variability in the data.

- The sample standard deviation is the **larger** than the rmsd because we divide by (n-1)

- To find the *msd* or sample variance, we square the relevant quantity given by the calculator:

$$msd = (rmsd)^2 \qquad \text{sample variance} = s^2$$

Then, we divide by n for the *msd* or $(n - 1)$ for s^2.



SAMPLE MEAN FOR FREQUENCY DATA

$$mean\ \bar{x} = \frac{\sum x\,f}{\sum f} = \frac{\sum x\,f}{n}$$

Where,
f is the frequency
x is the data
n is the summation of the frequency

SAMPLE VARIANCE FOR FREQUENCY DATA

Sample variance $S^2 = \frac{\sum f.x^2 - \frac{(\sum x f)^2}{n}}{n-1}$

Where,
f is the frequency
x is the data
n is the summation of the frequency

SAMPLE STANDARD DEVIATION FOR FREQUENCY DATA

$$\text{Sample Standard deviation } S = \sqrt{\frac{\sum f \cdot x^2 - \frac{(\sum f \cdot x)^2}{n}}{n - 1}}$$

Where,
f is the frequency
x is the data
n is the summation of the frequency

EXAMPLE

Find the mean and sample standard deviation of the following data:

x	1	2	5	10
Frequency, f	3	5	8	4

Solution:

$$\overline{x} = \frac{\sum x f}{n} = \frac{1*3+2*5+5*8+10*4}{3+5+8+4} = \frac{93}{20} = 4 \cdot 65$$

$$S^2 = \frac{\sum f.x^2 - \frac{(\sum x f)^2}{n}}{n-1} = \frac{1^2\times3+\ldots+10^2\times4-\frac{(1\times3+\ldots+10\times4)^2}{20}}{19} = 10.029$$

$$S = \sqrt{S^2} = \sqrt{10.029} = 3.17$$

EXAMPLE

Find the sample standard deviation of the following lengths:

Solution: We need the class mid-values

Length (cm)	1-9	10-14	15-19	20-29
x	5	12	17	24·5
Frequency, f	2	7	12	9
xf	10	84	204	220.5
x^2	25	144	289	600.25
x^2f	50	1008	3468	5402.25

$$n = \sum f = 30$$

$$\bar{x} = \frac{\sum xf}{\sum f} = 17.283$$

$$S^2 = \frac{\sum f.x^2 - \frac{(\sum x f)^2}{n}}{n-1} = \frac{9928.25 - 8.961}{29} = 33.351$$

Standard deviation, $s = 5 \cdot 77$

EXAMPLE

Find the mean and sample variance of 20 values of x given the following:

$$\sum x = 82 \quad \text{and} \quad \sum x^2 = 370$$

Solution:

Since we only have summary data, we must use the formulae

sample mean, $\bar{x} = \frac{\sum x}{n}$ $\Rightarrow \bar{x} = \frac{82}{20} = 4 \cdot 1$

sample variance, $S^2 = \frac{\sum x^2 - \bar{x}^2}{n-1} = \frac{370 - 16.81}{19} = 1 \cdot 78$