AL FURAT AL AWSAT TECHNICAL UNIVERSITY
TECHNICAL ENGINEERING COLLEGE / AL-NAJAF
DEPARTMENT OF AVIONICS ENGINEERING

DIGITAL SIGNAL PROCESSING

3rd YEAR

BY

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OBJECTIVES OF COURSE

- Learning the student to understanding the mathematical analysis for signals and how can processing it.

SYLLABUS

The week	Details
1-3	Continuous and discrete signals and systems.
4-5	Linear time-invariant system: introduction (impulse response, unit step response)- properties of DSP system (linearity, time invariance, causality).
6-8	Discrete convolution: Linear convolution- properties of convolution- circular convolution.
9-11	Discrete correlation: cross- correlation and auto- correlation sequence- properties of cross- correlation and auto- correlation sequence.
12-16	Z-Transform: Definition of the Z-transform (Region of Convergence(ROC))- properties of the Z- transform- stability- evaluation of the inverse Z- transform – long division method – partial fraction expansion.
17-18	Solution of the linear difference equations.
19-20	Frequency analysis of signals and systems.
21-24	Discrete Fourier Transform (DFT)- Fast Fourier Transform (FFT).
25	Feedback system.
26-28	Implementation of discrete time system: structure of FIR system(direct form structure, cascade form structure)- structure for IIR system (direct form structure, cascade form structure, parallel form structure)
29-30	Introduction to programmable DSPs- Architecture of TMS320C5X.

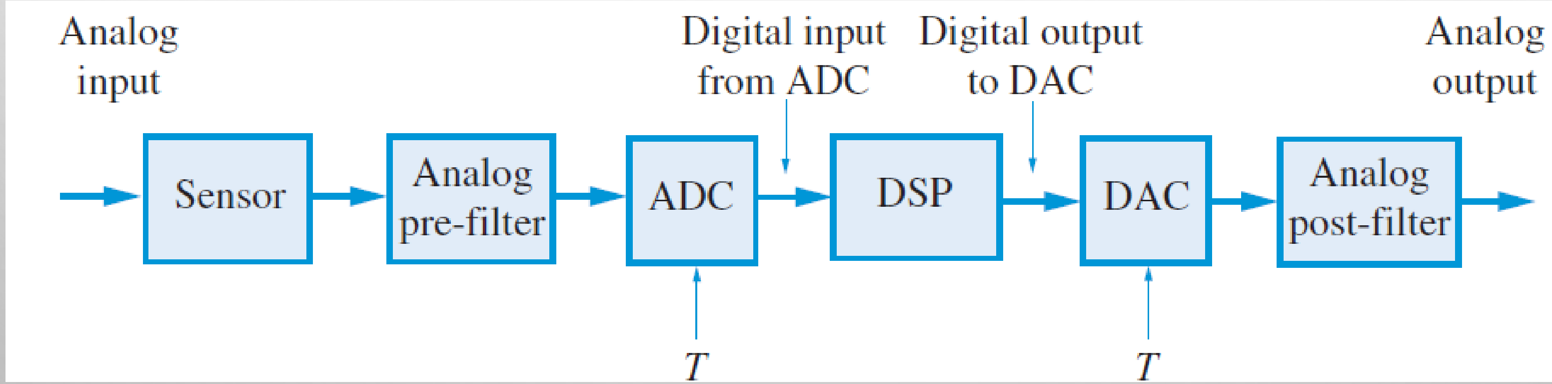
REFERENCES

- S Salivahanan and C Gnanapriya, "DIGITAL SIGNAL PROCESSING", Second Edition.
- Richard G. Lyons, “ UNDERSTANDING DIGITAL SIGNAL PROCESSING“, Second Edition.
- D. Williamson, “DISCRETE TIME SIGNAL PROCESSING“.

WHAT DOES "DIGITAL SIGNAL PROCESSING" MEAN?

- **Signal:** Physical quantity that varies with time or any other independent variable.
- Mathematically, we describe a signal as a dependent variable or function of one or more independent variables. For example, the function **s(t)= 5t, g(x1,x2)=x1+3x2**
- **Digital signal processing:** is concerned with the representation of analog signals by sequences of numbers, the processing of these sequences by numerical computation techniques, and the conversion of such sequences into analog signals.

•In practice, due to inherent real-world limitations, a typical system for the digital processing of analog signals includes the following parts (see fig. below)



A digital signal processing scheme.

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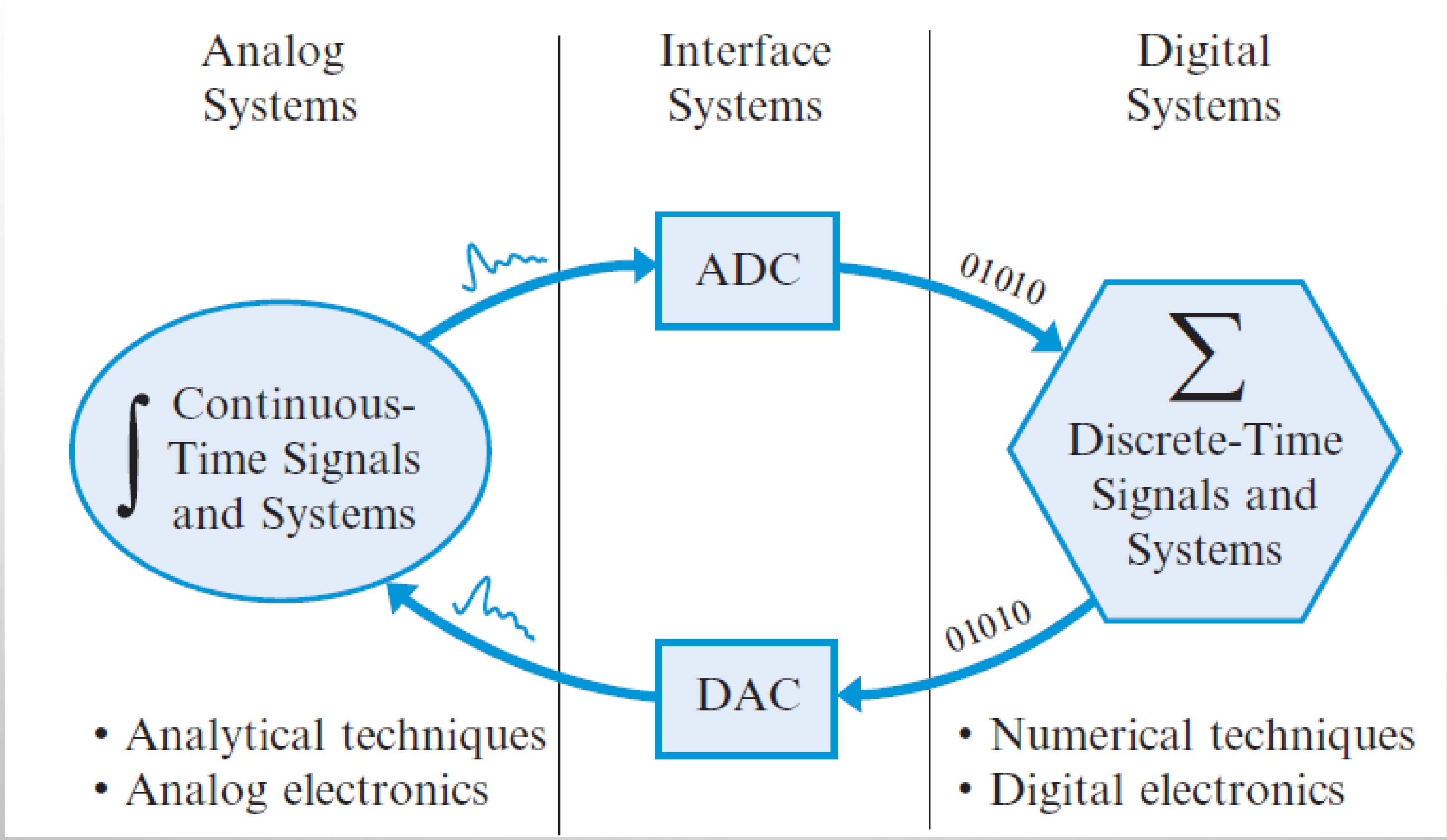

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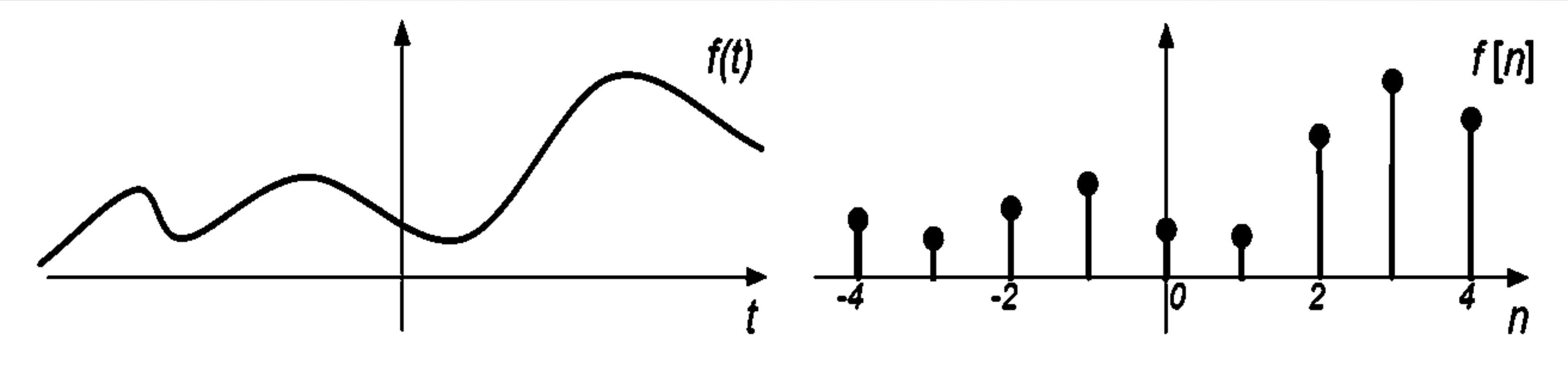
CLASSIFICATION OF SIGNALS

Signals can be classified in terms of the continuity of the independent and dependent variables as follows:

- **analog signals** or ***Continuous-time signals*** **:** are defined for every value of time.
- **Discrete-time signals :** are defined only at certain specific values of time.
- **Digital Signals:** is defined as a function of an integer independent variable and its values are taken from a finite set of possible values, which are represented by a string of 0's and l's .



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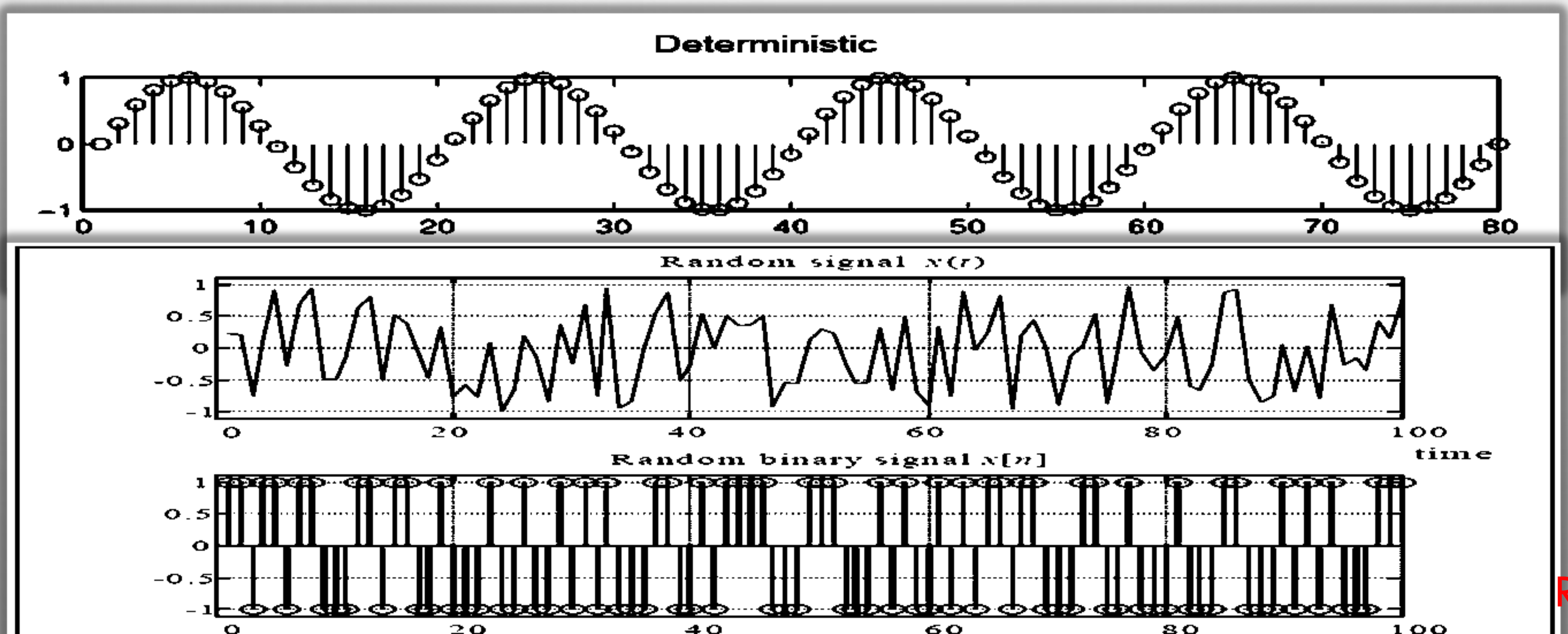
CLASSIFICATION OF SIGNALS

Signals can also be classified in terms of the predictability of the dependent variables with respect to the independent variable as follows:

- **Deterministic signals:** if the dependent variable is predictable at any instance of the independent variable time.
- **Non deterministic(random) signal:** unpredictable dependent variable at any instance of the independent variable time.



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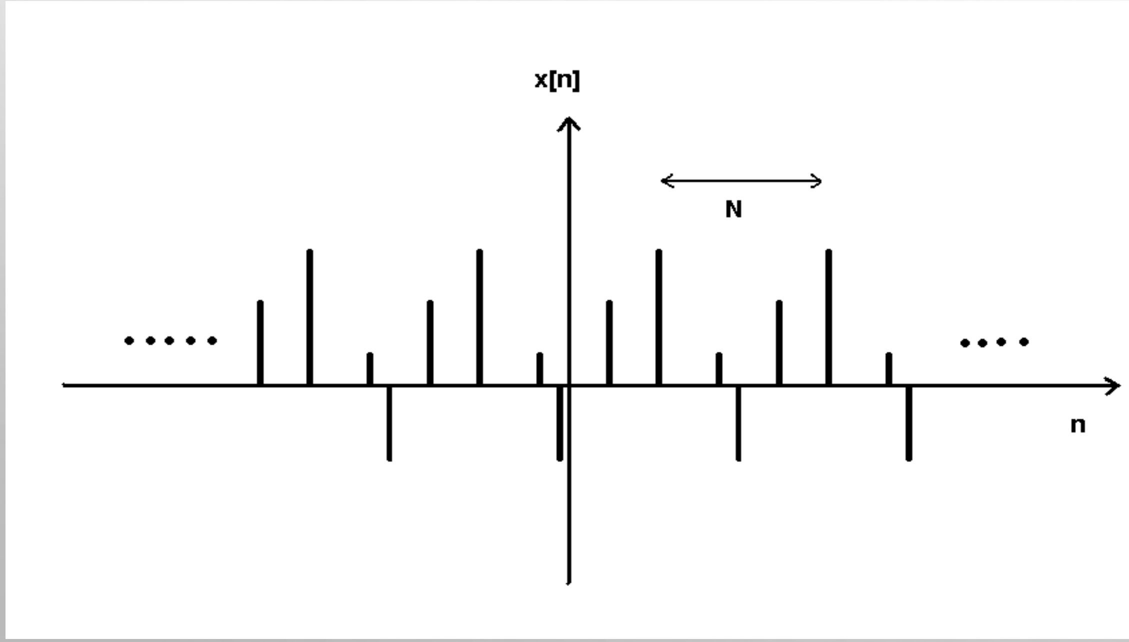
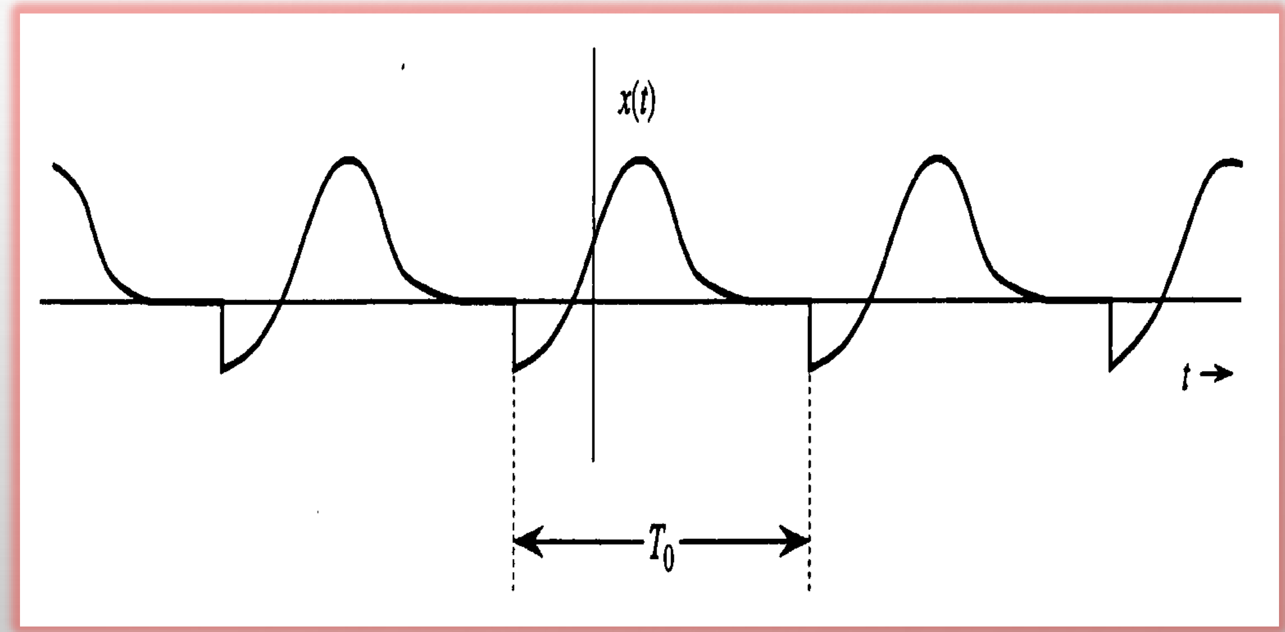
CLASSIFICATION OF SIGNALS

- **Periodic and non-periodic signals**

For a continuous signal, if we can find a number **T** such that **$x(t + kT) = x(t)$** for all **t**, then the signal is **periodic**. In the case of a discrete signal we have **$x[n + kN] = x[n]$** for all **n**. The quantity T and N is referred to as the **period** of the signal.

* If there are no such numbers **T** or **N** the signal is **non-periodic**.





PROPERTIES OF PERIODIC SIGNALS

- For continuous time signals the two quantities closely related to the period are the **frequency** and **angular frequency**, denoted as f and w, respectively. These quantities are defined as:

$$f = \frac{1}{T} \quad \text{and} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

Theorem (**Sum of periodic functions**). Suppose that $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental periods T_1 and T_2, respectively. Then, the sum $y(t) = x_1(t) + x_2(t)$ is a periodic signal **if and only if** the ratio T_1 / T_2 is a **rational number** (i.e., the quotient of two integers). Suppose that $T_1 / T_2 = q/r$ where q and r are integers and coprime (i.e., have no common factors), then the fundamental period of y(t) is $r\,T_1$ (or equivalently, $q\,T_2$, since $r\,T_1 = q\,T_2$). (Note that $r\,T_1$ is simply the least common multiple of T_1 and T_2.)

EXAMPLES

1. Is x[n] = $\cos(\pi/_8\, n)$ periodic? If so, what is the period?

SOL:-

x[n] = $\cos(\pi/_8\, n) \equiv \cos(wn)$

$\omega = \pi/_8$

$\omega = 2\pi f$ ➡ $f = \pi/_{16\pi}$ ➡ $f = 1/_{16}$ =rational $= k/_N$ ➡ periodic ➡ N=16

2. Determine the fundamental period of the following sequence:

$$x[n] = \cos(1.1\pi n) + \sin(0.7\pi n)$$

Sol:- $\omega_1 = 1.1\,\pi = 11\pi/_{10}$, $\omega_2 = 0.7\,\pi = 7\pi/_{10}$

$f_1 = 11\pi/_{20\pi} = 11/_{20} = k/_{N1}$ ➡ periodic ➡ N_1 =20

$f_2 = 7\pi/_{20\pi} = 7/_{20} = k/_{N2}$ ➡ periodic ➡ N_2 =20

N =LCM(N1,N2)

EXAMPLES

1. Let $x1(t)$ = sinπt and $x2(t)$ = sin$t$. Determine whether the signal $y(t) = x1(t)+x2(t)$ is periodic.

Sol: Denote the fundamental periods of $x1(t)$ and $x2(t)$ as $T1$ and $T2$, respectively. We then have $T_1 = \frac{2\pi}{f}$

$T_1 = \frac{2\pi}{\pi} = 2$, $T_2 = \frac{2\pi}{1} = 2\pi$, $\frac{T_1}{T_2} = \frac{2}{2\pi} = \frac{1}{\pi}$

Since π is an **irrational number**, this quotient is not rational. Therefore, $y(t)$ is not periodic.

2. Let $x1(t) = \cos(6\pi t)$ and $x2(t) = \sin(30\pi t)$. Determine whether the signal $y(t) = x1(t)+x2(t)$ is periodic.

Sol:

Let $T1$ and $T2$ denote the fundamental periods of $x1(t)$ and $x2(t)$, respectively. Thus, we have:

$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3}$, $T_2 = \frac{2\pi}{30\pi} = \frac{1}{15}$, $\frac{T_1}{T_2} = \frac{15}{3} = \frac{5}{1}$

Therefore, $y(t)$ is periodic.

EXAMPLES

The angular frequency of the discrete time periodic sequences is given by,

$\omega = \frac{2\pi}{N}$

Therefore, the time period of the sequence is,

$$N = \frac{2\pi}{\omega}$$

Example:-

x(n)= sin 10πn

Sol:-

$\omega = 10\ \pi$

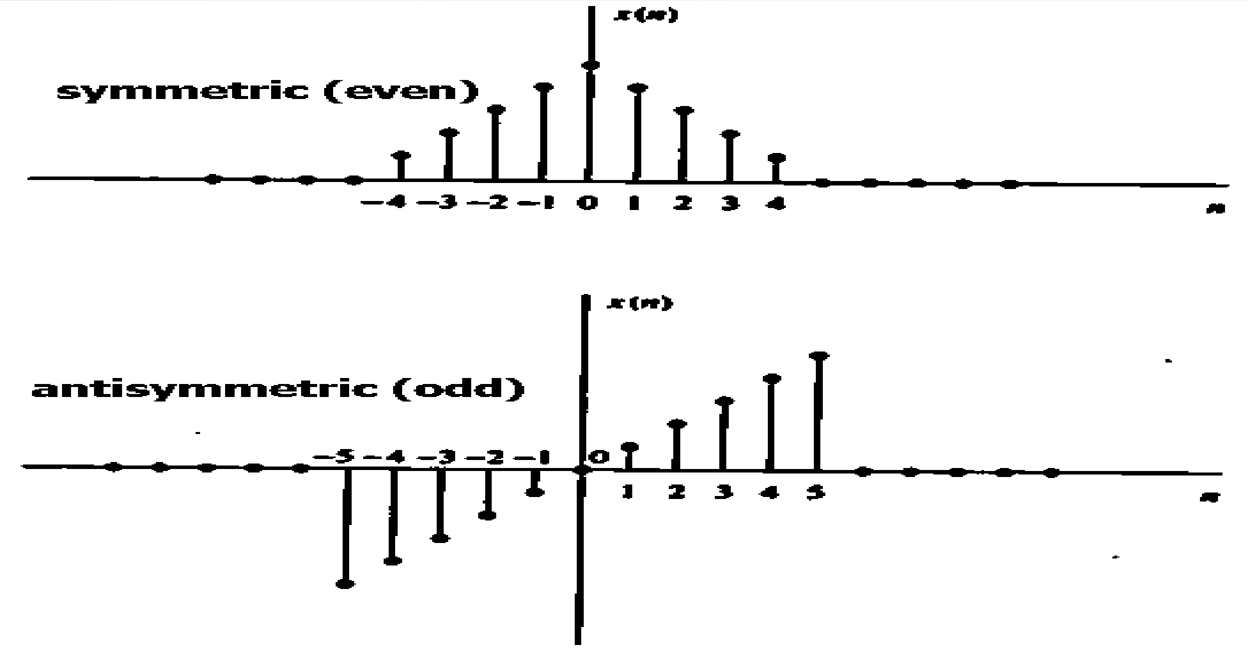
N= $\frac{2\pi}{\omega} = \frac{2\pi}{10\ \pi} = \frac{1}{5} = 0.2$

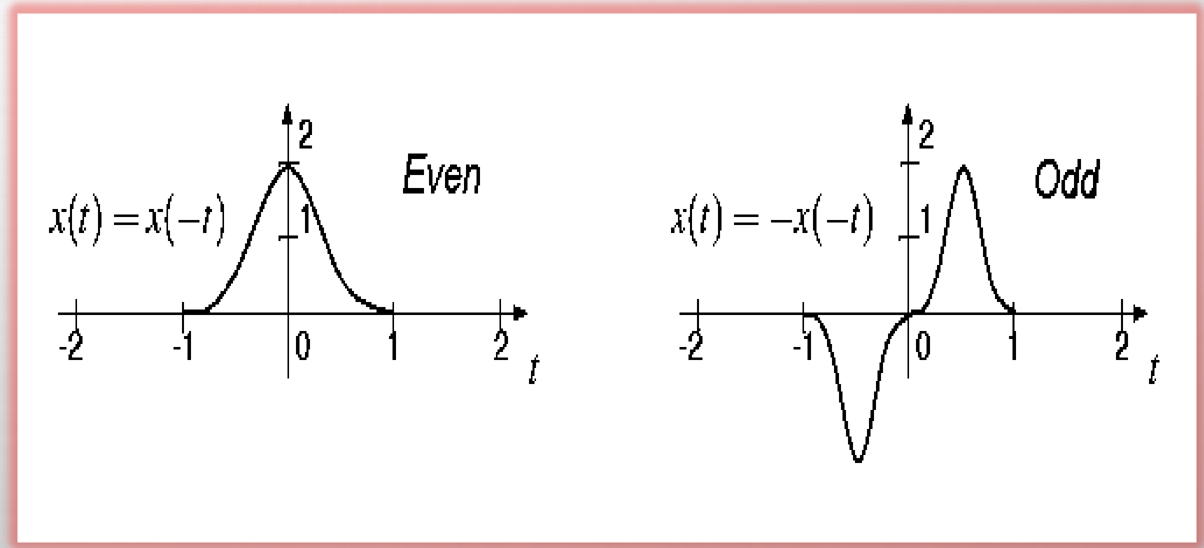
CLASSIFICATION OF SIGNALS

- **Symmetric (Even) and antisymmetric (odd) signals**

For a continuous signal, if **x(-t) = x(t)** for all **t**, then the signal is **even**. In the case of a discrete signal we have **x[-n] = x[n]** for all **n**.

* For a continuous signal, if **x(-t) = -x(t)** for all **t**, then the signal is **odd**. In the case of a discrete signal we have **x[-n] = -x[n]** for all **n** . An odd signal is 0 at t = 0 or n = 0, since x(0) = -x(0) ⇒ x(0) = 0 .





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PROPERTIES OF EVEN AND ODD SIGNALS

- Any signal $x(t)$ can be represented as the sum of the form:

$$x(t) = x_e(t) + x_o(t)$$

where x_e (t) and x_o (t) are even and odd, respectively, and given by:

$$x_e(t) = \frac{1}{2}(x(t)+x(-t)) \quad \text{and} \quad x_o(t) = \frac{1}{2}(x(t) - x(-t))$$

- we can easily confirm that $x_e(t) + x_o(t) = x(t)$ as follows:

$$x_e(t) + x_o(t) = 1/2\,(x(t)+x(-t)) + 1/2\,(x(t)-x(-t))$$

$$= \tfrac{1}{2}x(t) + \tfrac{1}{2}\cancel{x(-t)} + \tfrac{1}{2}x(t) - \tfrac{1}{2}\cancel{x(-t)}$$

$$= x(t)$$

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PROPERTIES OF EVEN AND ODD SIGNALS

- we can easily verify that $x_e\ (t)$ is even and $x_o\ (t)$ is odd.

x_e(-t) = $\frac{1}{2}$ (x(-t)+x(- (-t)))

= $\frac{1}{2}$ (x(t)+x(−t))

= x_e (t)

Thus, $x_e\ (t)$ is even.

x_o(-t) = $\frac{1}{2}$ (x(-t) - x(-(-t)))

= $\frac{1}{2}$ (x(-t) - x(t))

= - x_o (t)

Thus, $x_o\ (t)$ is odd.

PROPERTIES OF EVEN AND ODD SIGNALS

1. The sum of two even signals is even signal.
2. The sum of two odd signals is odd signal.
3. The sum of an even signal and an odd signal is neither even nor odd signal.
4. The product of two even signals is even signal.
5. The product of two odd signals is even signal.
6. The product of an even signal and an odd signal is odd signal.

EXAMPLES

- cos(wn) is an even signal because cos(- Ɵ) = cos (Ɵ).
- sin(wn) is an odd signal because sin(- Ɵ) = - sin (Ɵ).
- cos(wn) . sin(wn) is an *odd* signal (even times odd).
- sin(w1n) . sin(w2n) is an *even* signal (odd times odd).

EXAMPLE

- Find the even and odd components of the following signal:

$x(t)=\cos(t)+\sin(t)+\cos(t)\sin(t)$

Sol:- There are two methods for solution:

1- using time reversal method

$X(-t)=\cos(-t)+\sin(-t)+\cos(-t)\sin(-t)$

$\cos(-t)=\cos(t)$

$\sin(-t)= - \sin(t)$

$X(-t)=\cos(t)-\sin(t)-\cos(t)\sin(t)$

$x_e(t) = \frac{1}{2}(x(t)+x(-t))$

$= \frac{1}{2}(\cos(t)+\sin(t)+\cos(t)\sin(t)+\cos(t)-\sin(t)-\cos(t)\sin(t))$

$= \cos(t)$

$x_o(t) = \frac{1}{2}(x(t)-x(-t))$

$= \frac{1}{2}(\cos(t)+\sin(t)+\cos(t)\sin(t)-\cos(t)+\sin(t)+\cos(t)\sin(t))$

$= \sin(t)+\cos(t)\sin(t)$

EXAMPLE

- Find the even and odd components of the following signal:

$x(t)=\cos(t)+\sin(t)+\cos(t)\sin(t)$

Sol:-

2- using even and odd signals properties

$\cos(t)=$ even

$\sin(t)=$ odd

$\cos(t)\sin(t)=$ odd X even= odd

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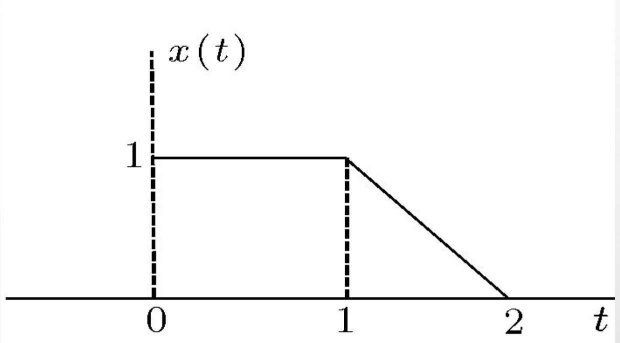
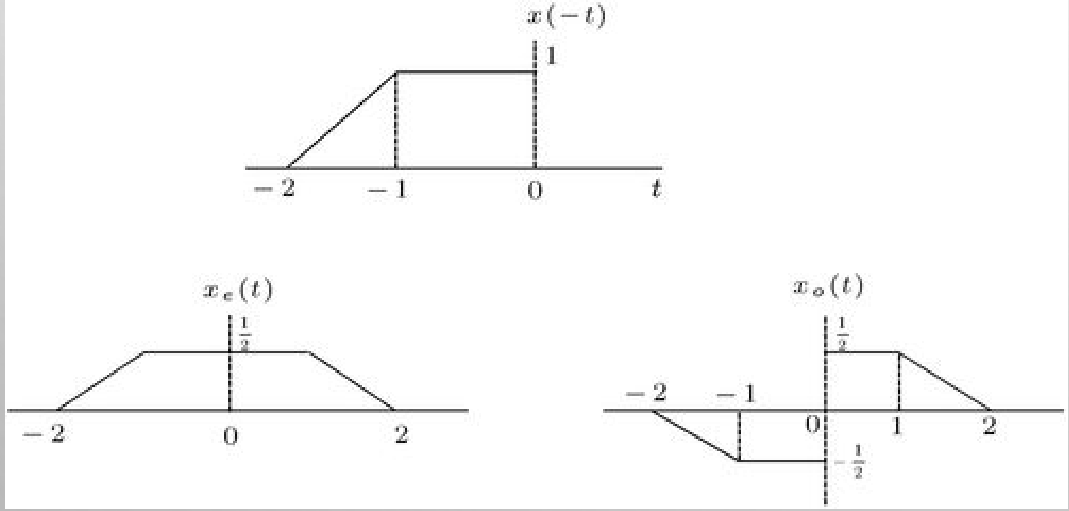
EXAMPLE

- Find the even and odd components of the following signal:

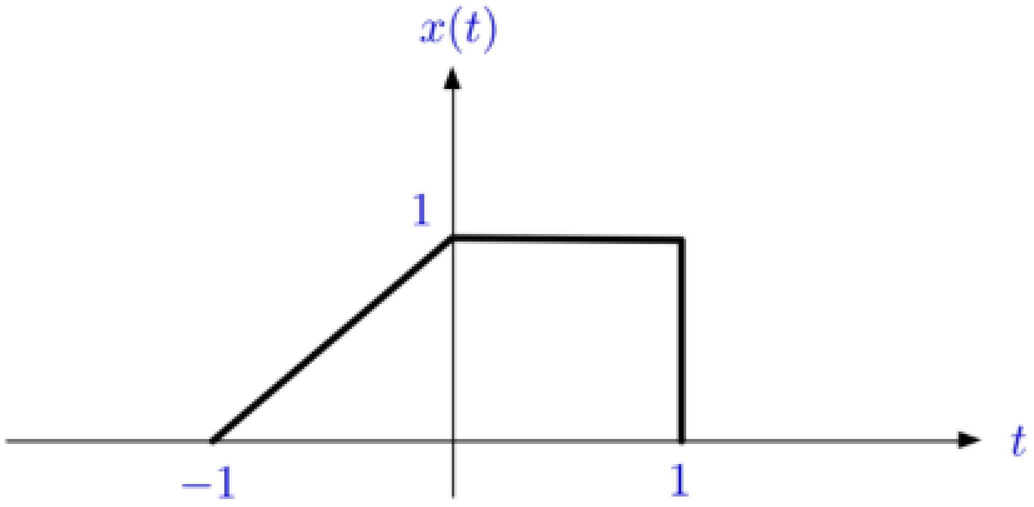


Sol:-



HOMEWORK

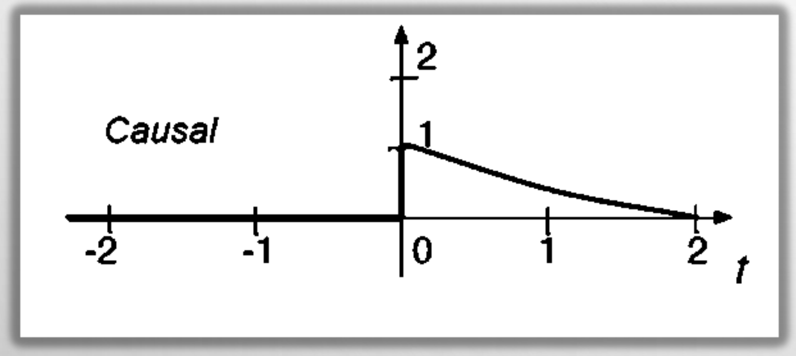
Plot the even and odd parts $x_e(t)$ and $x_o(t)$ for a CT signal $x(t)$ as shown below:

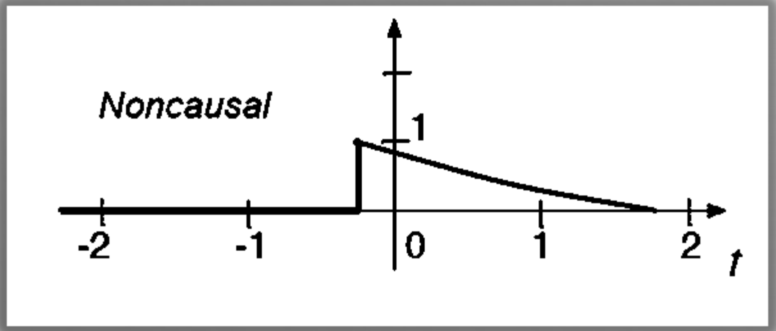


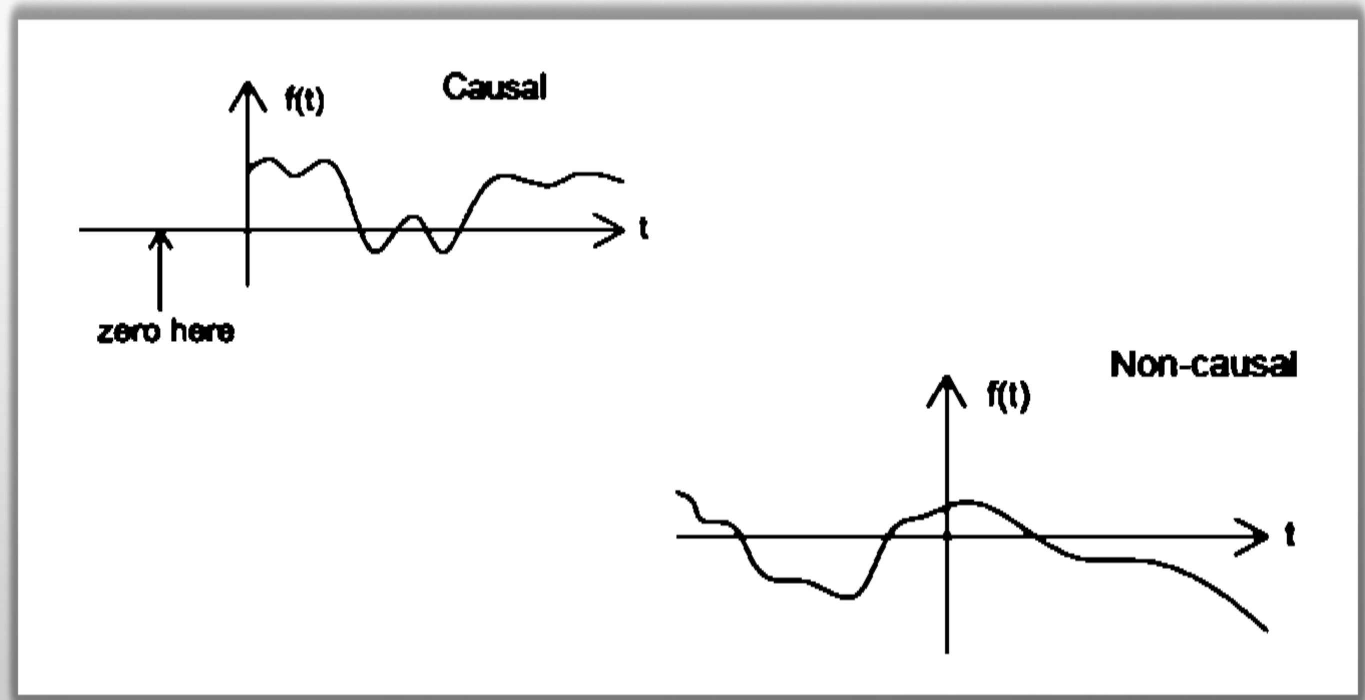
CLASSIFICATION OF SIGNALS

- **Causal and Non-Causal Signals**

If $x(t) = 0$, for $t < t0$ or for discrete signals $x[n] = 0$, for $n < n0$ then we have *causal* signals. The starting point t0 or n0 is very often taken to be the origin.







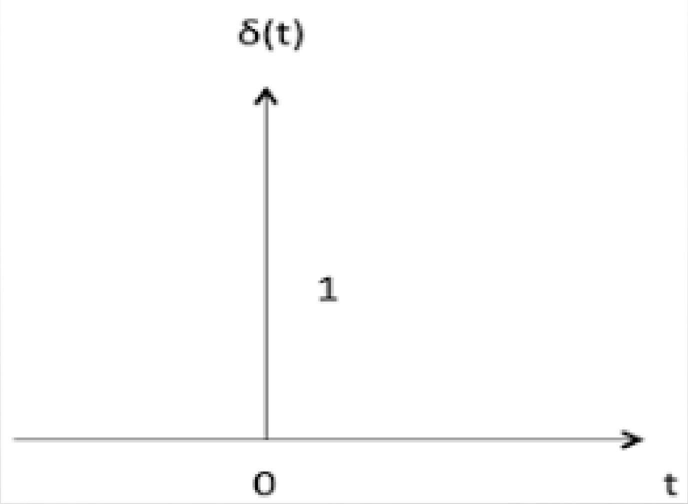
SOME USEFUL FUNCTIONS

1. Unit impulse function:

- The unit impulse function, also known as the dirac delta function, δ(t), for continuous signals is defined by:

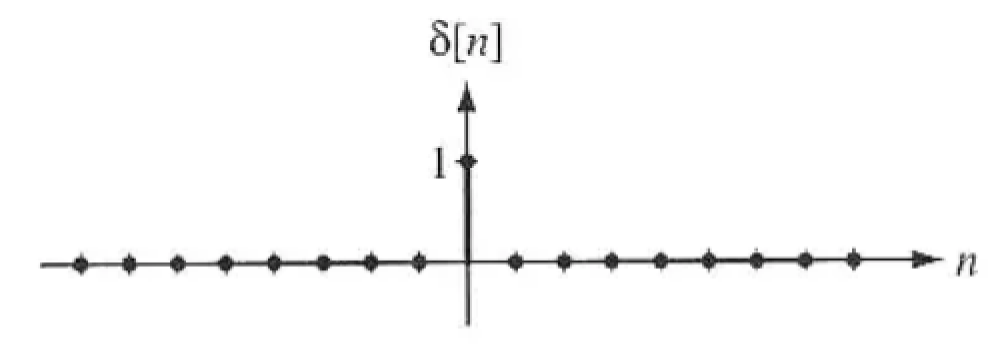
$$\int_{-\infty}^{\infty} \delta(t)dt = 1.........and$$

$$\delta(t) = \begin{cases} \infty,......t = 0 \\ 0,........t \neq 0 \end{cases}$$



- The unit impulse function δ(n) for discrete signals is :

$$\delta[n] = \begin{cases} 1\ , n = 0 \\ 0\ , n \neq 0 \end{cases}$$

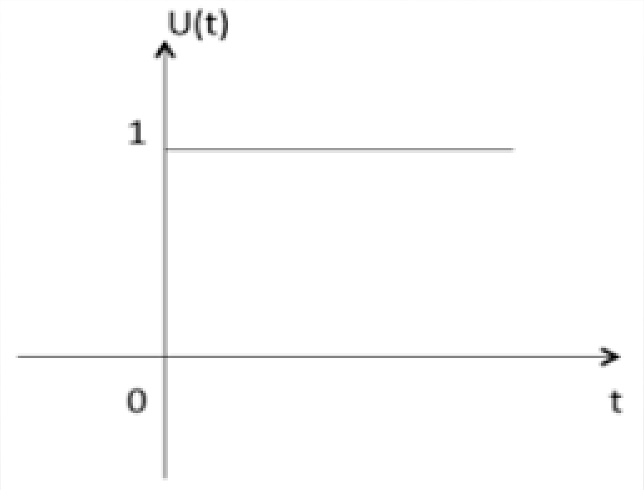


SOME USEFUL FUNCTIONS

2. Unit step function:

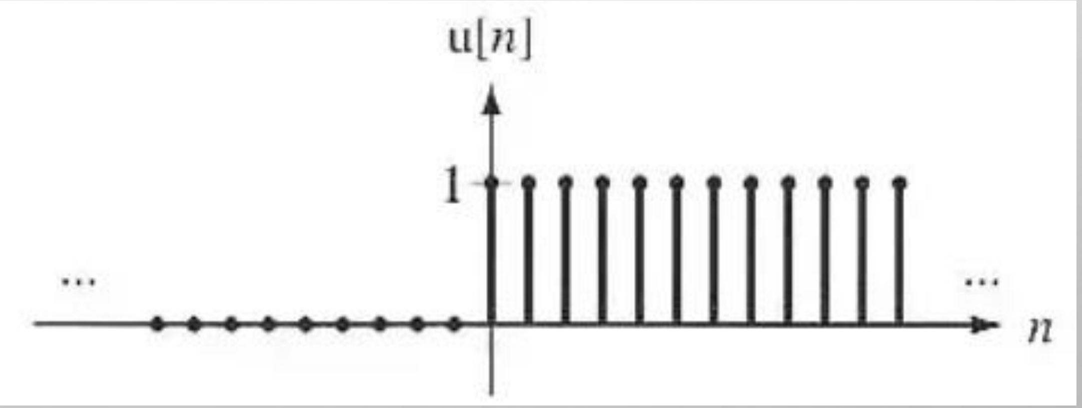
- The unit step function u(t) for continuous signals is :

$$u(t)=\begin{cases} 0 & \text{for} \quad t<0 \\ 1 & \text{for} \quad t\geq 0 \end{cases}$$



- The unit step function u(n) for discrete signals is :

$$u[n]=\begin{cases} 0, & n<0 \\ 1, & n\geq 0 \end{cases}$$

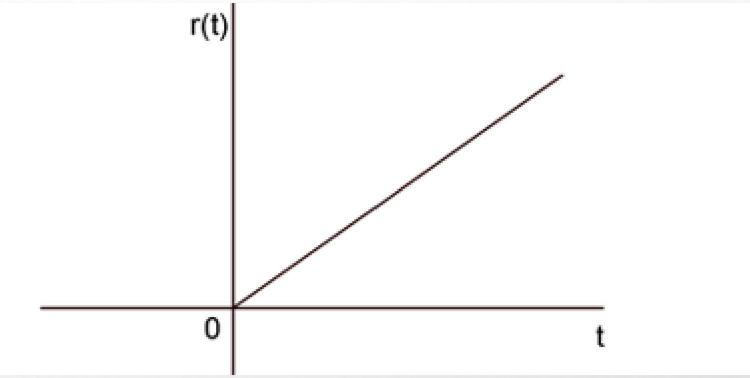


SOME USEFUL FUNCTIONS

3. Ramp function:

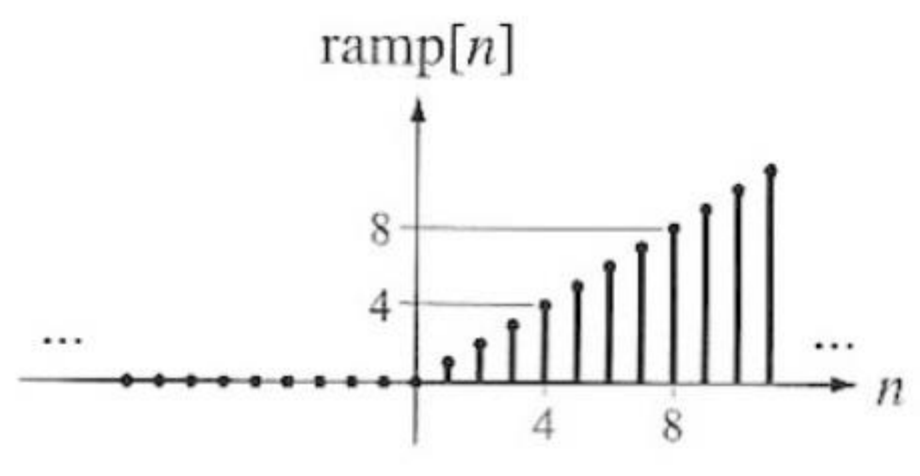
- The ramp function r(t) for continuous signals is :

$$r(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ t & \text{for} \quad t \geq 0 \end{cases}$$



- The ramp function r(n) for discrete signals is :

$$\text{ramp}[n] = \begin{Bmatrix} n\ , n \geq 0 \\ 0\ , n < 0 \end{Bmatrix} = n\text{u}[n]$$

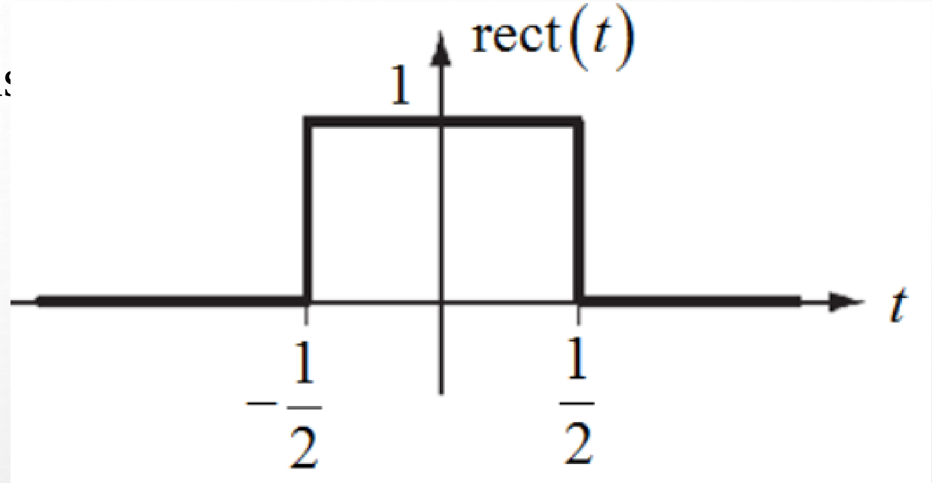


SOME USEFUL FUNCTIONS

4. rectangular function:

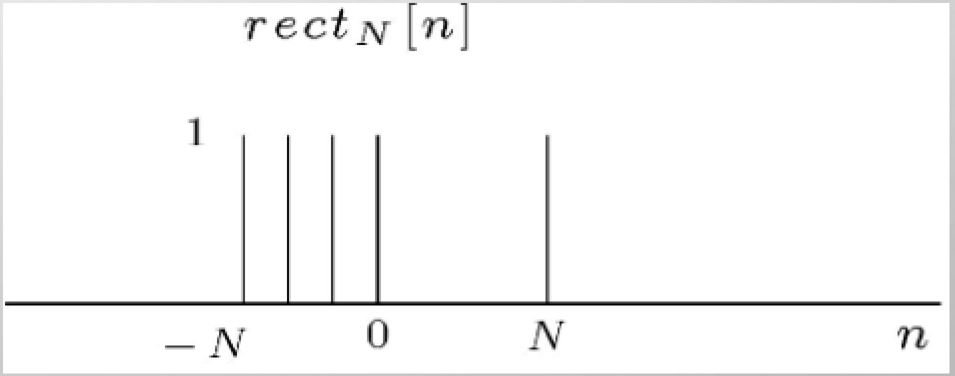
- The rectangular function Π(t) for continuous signals is

$$rect(t) = \begin{cases} 1 & \text{if } \frac{-1}{2} \geq t \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



- The rectangular function Π(n) for discrete signals is :

$$rect\left(\frac{n}{2N}\right) = \begin{cases} 1 \ldots\ldots if \;..|n| \leq N \\ 0 \ldots\ldots\ldots else \end{cases}$$

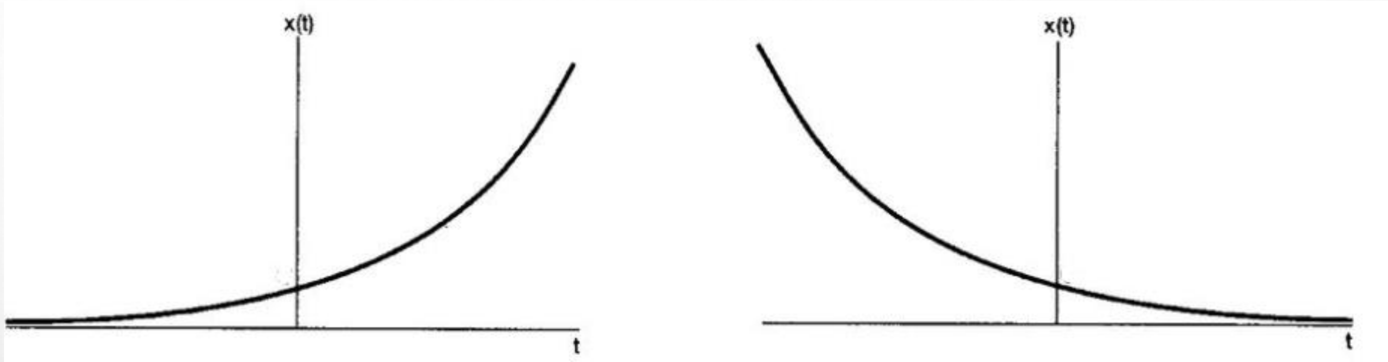


SOME USEFUL FUNCTIONS

5. exponential function:

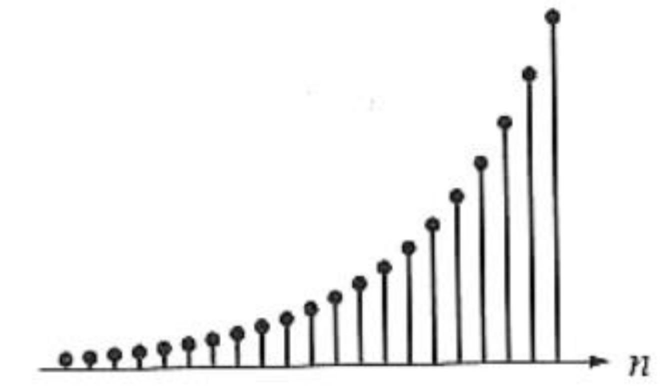
- The exponential function for continuous signals is :

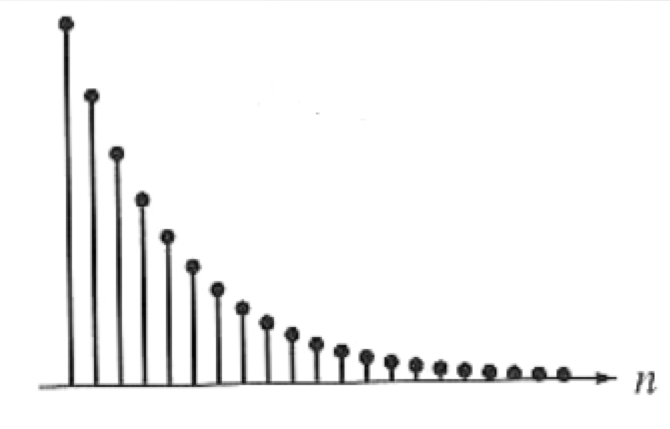
$$x(t) = e^{\alpha t}$$



- The exponential function for discrete signals is :

$$x(n)=a^{n} \text{ for all n}$$



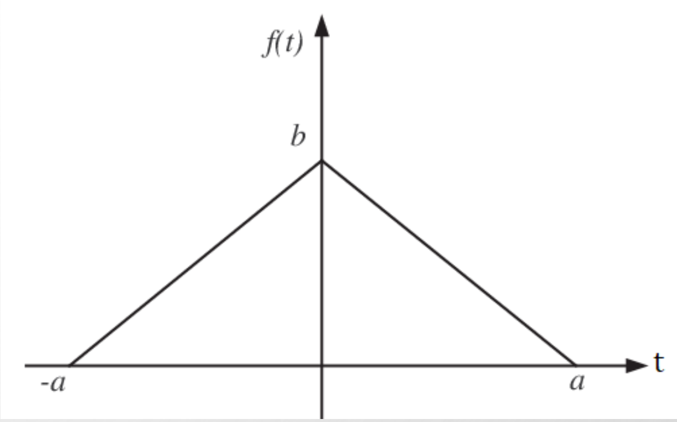


SOME USEFUL FUNCTIONS

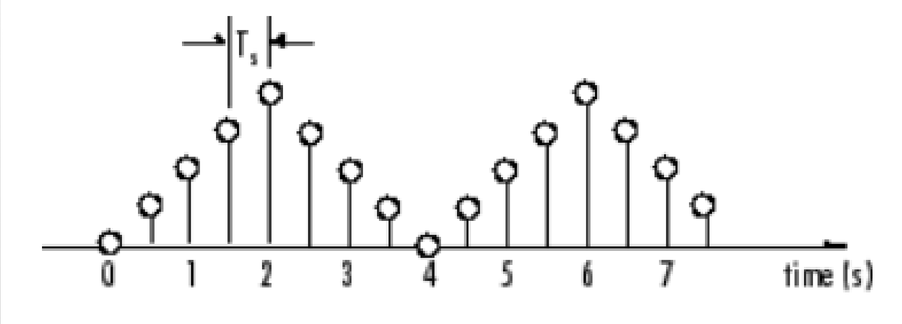
6. triangle function:

- The triangle function for continuous signals is :

$$tri(t) = \begin{cases} b(1+\frac{t}{a}) & for \quad -1 \le t \le 0 \\ b(1-\frac{t}{a}) & for \quad 0 \le t \le 1 \end{cases}$$



- The triangle function for discrete signals is :

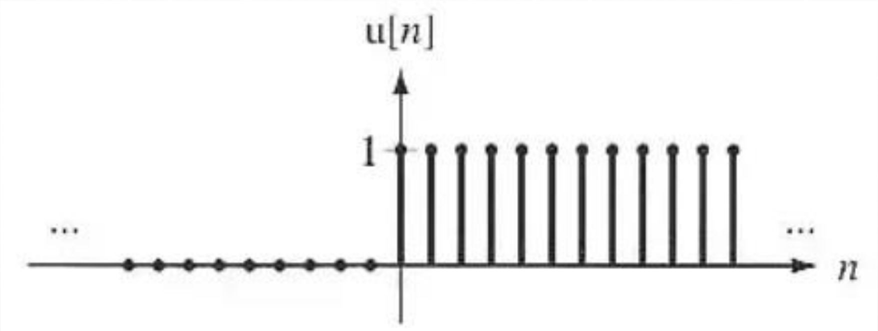


HOW DRAW THE SIGNAL AND WRITE ITS EQUATION

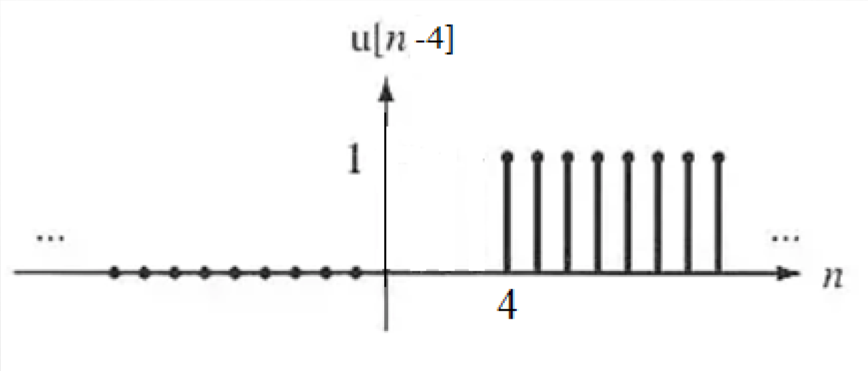
Example:- Draw the signal x[n]=u(n)-u(n-4)

Sol:-

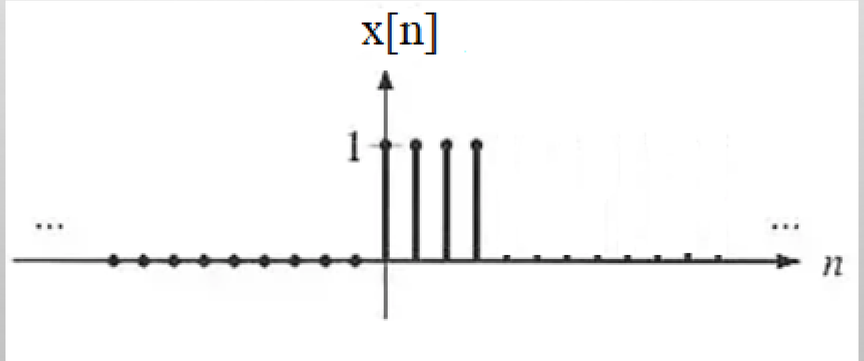
u(n)= $\begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$



u(n-4)= $\begin{cases} 0, & n<4 \\ 1, & n \geq 4 \end{cases}$

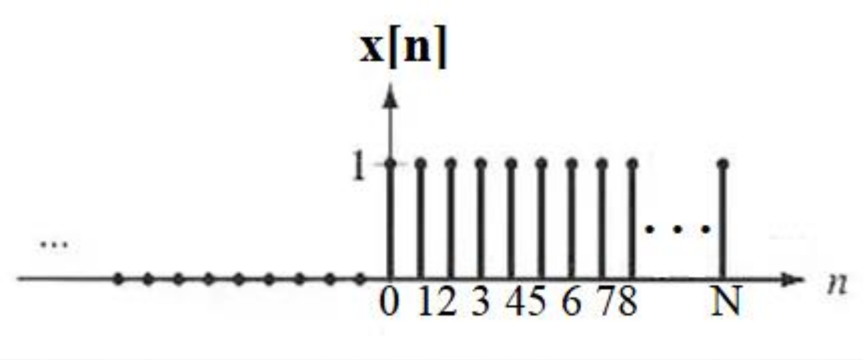


- x[n]= $\begin{cases} 1, & 0 \leq n \leq 3 \\ 0, & otherwise \end{cases}$
- Or we can write it as:
- x[n]={1,1,1,1}



HOW DRAW THE SIGNAL AND WRITE ITS EQUATION

Example:- write the corresponding equation of the following signal in term of unit step function



Sol:-

- x[n]=u[n]-u[n-(N+1)]