AL FURAT AL AWSAT TECHNICAL UNIVERSITY
NAJAF COLLEGE OF TECHNOLOGY
DEPARTMENT OF AVIONICS ENGINEERING

DIGITAL SIGNAL PROCESSING

3rd YEAR

By
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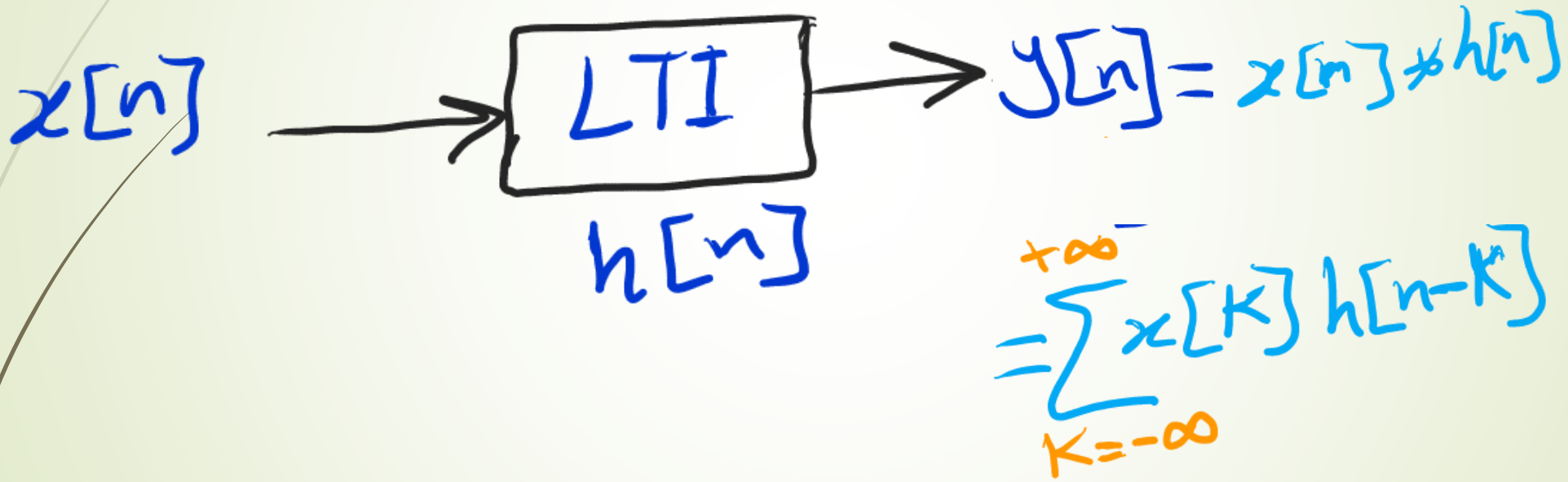
Discrete Convolution(convolution sum)

- Convolution is a mathematical way of combining two signals to form a third signal.



The steps of sum convolution:-

① Change (n) to (k).

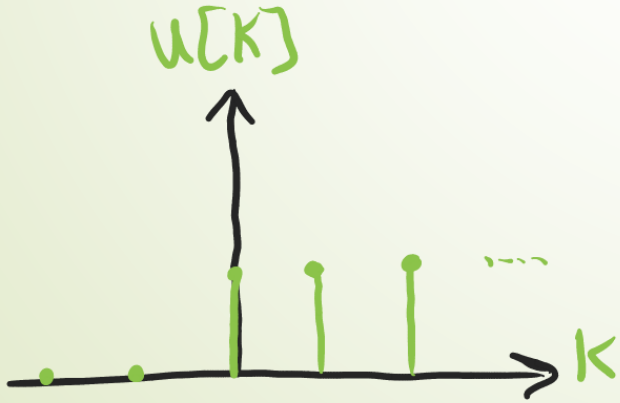
② Fold h(k) to be h(-k).

③ Time shifting of h(-k) to be h(n-k)

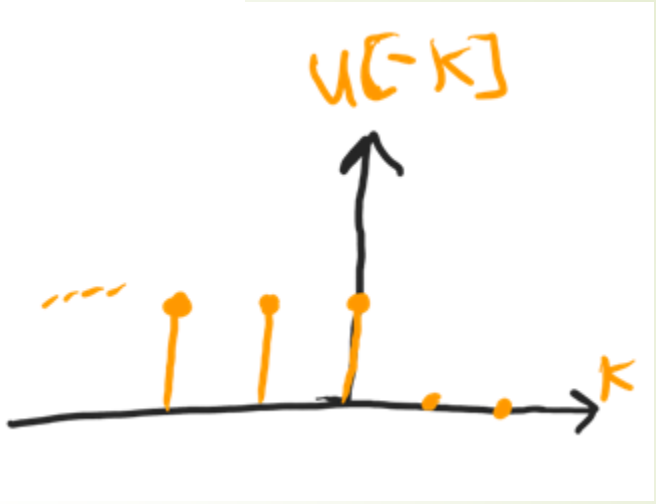
④ multiply $x[k] * h[n-k]$

Ex/ Calculate y[n] in the system shown below.

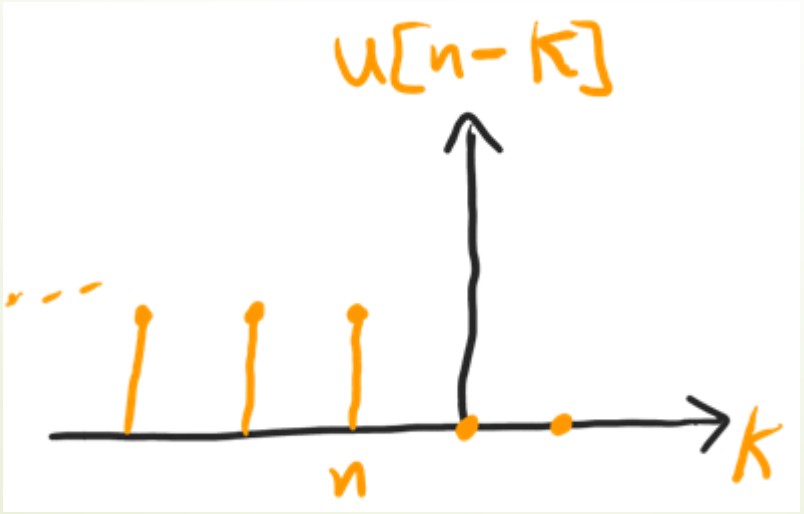
① Change (n) to (k)



② Time reversal or folding

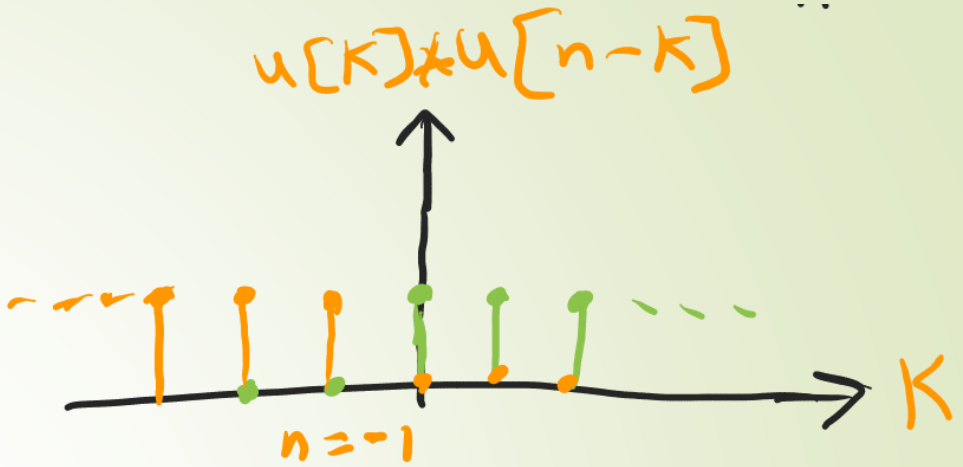


③ time shifting



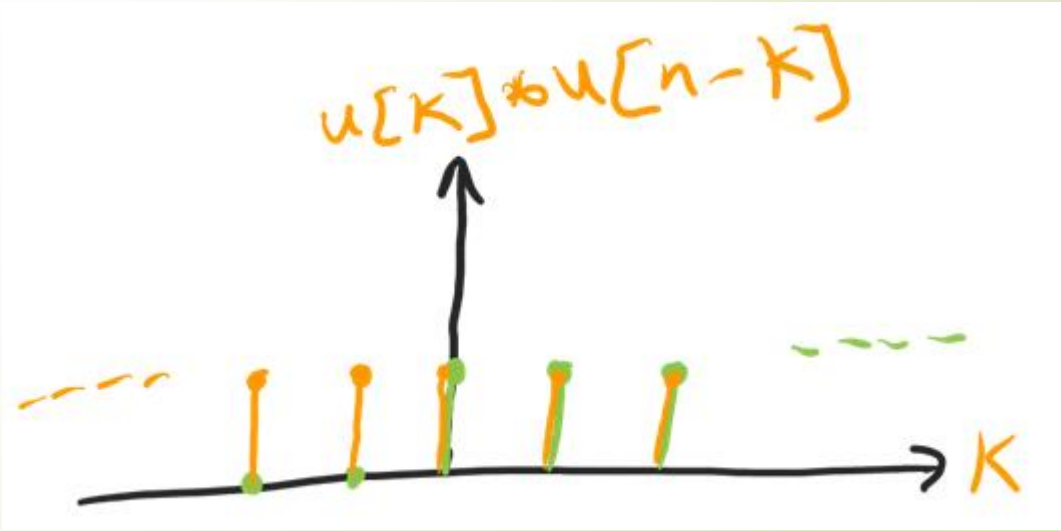
Case 1 when $n<0$

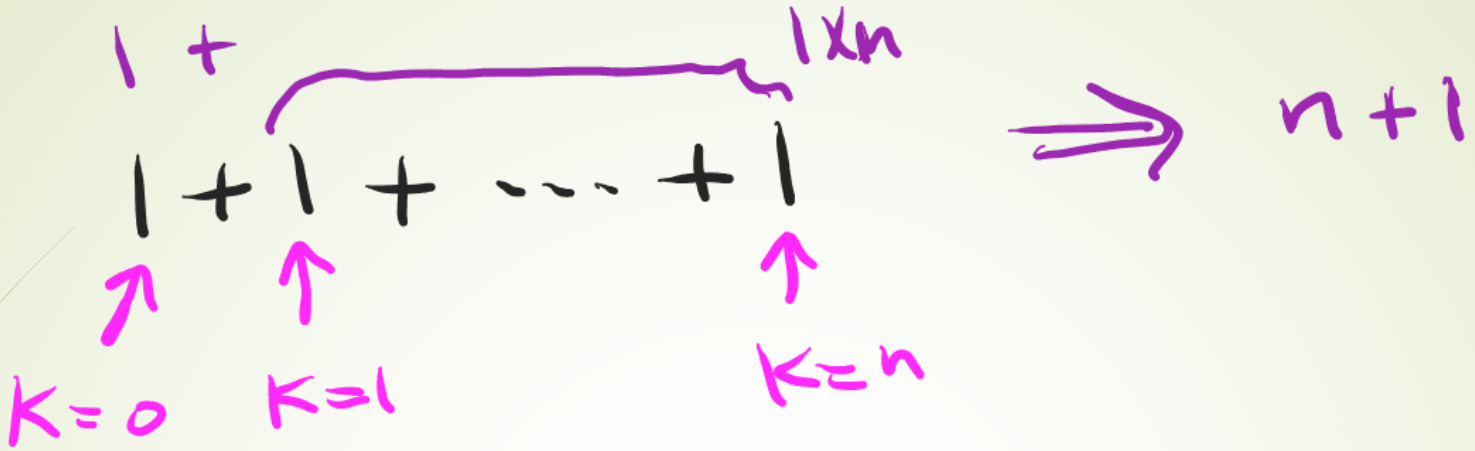
$$y[n]=\sum_{k=-\infty}^{0}=0$$



Case 2 when $n\geq 0$

$$y[n]=\sum_{K=0}^{n}1 = n+1$$



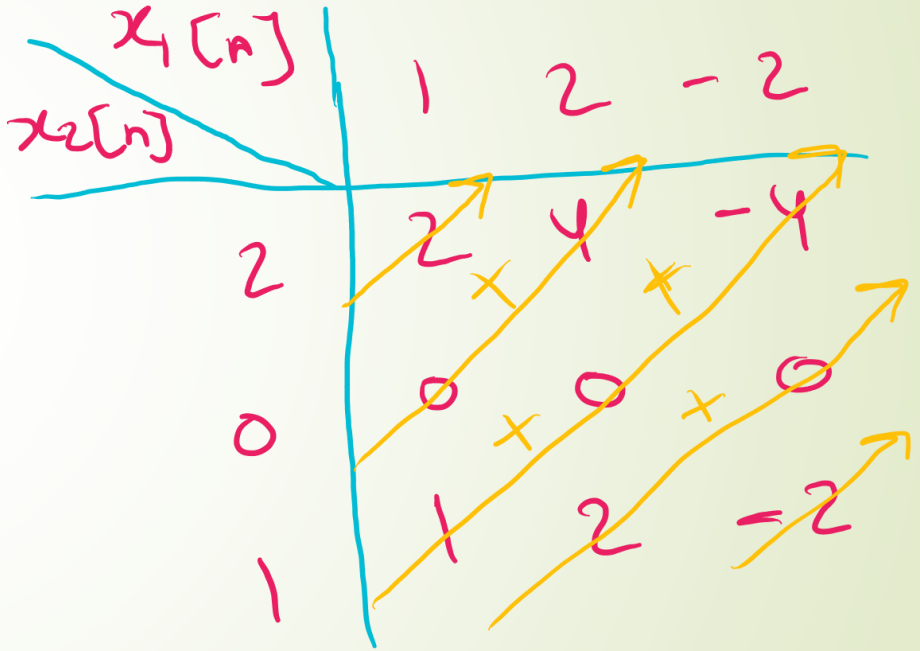


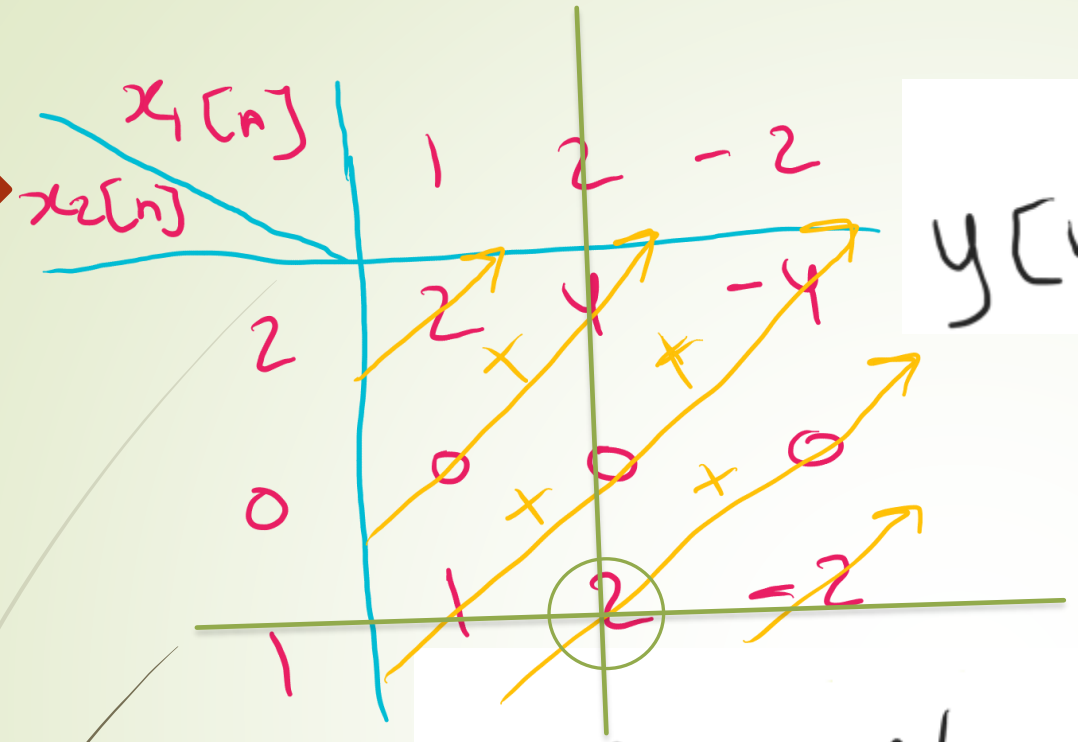
$$\therefore y[n] = \begin{cases} 0 , & n < 0 \\ n+1 , & n \geq 0 \end{cases}$$

Ex/ if $x_1[n] = \{1, \underset{\uparrow}{2}, -2\}$ and $x_2[n] = \{2, 0, \underset{\uparrow}{1}\}$

find $y[n] = x_1[n] * x_2[n]$

$x_2[n]$ \ $x_1[n]$	1	2	-2
2	2	4	-4
0	0	0	0
1	1	2	-2





$$y[n] = \{2, 4, -3, 2, -2\}$$

no. of samples signal ① + no. of samples signal ② − 1

$$3 + 3 - 1 = 5$$

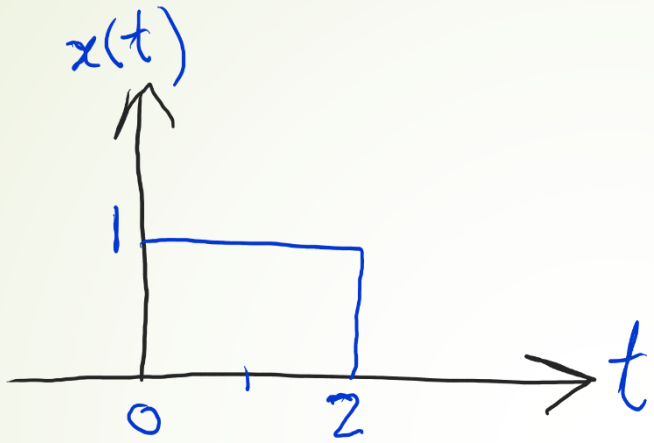
Continuous convolution(convolution integral)

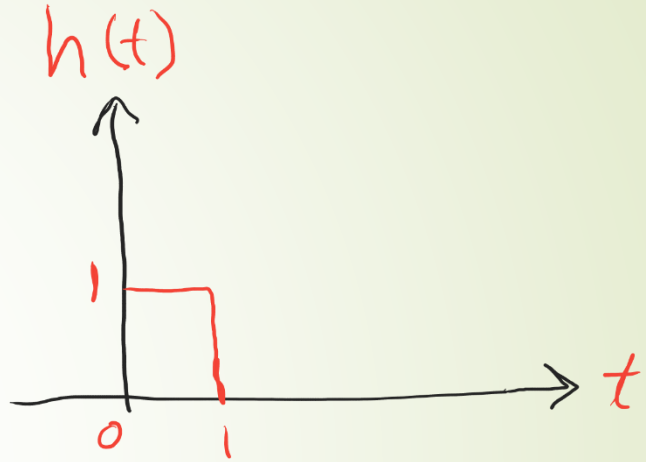
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{-\infty}^{+\infty} x(t-\tau)h(\tau)d\tau$$

Example

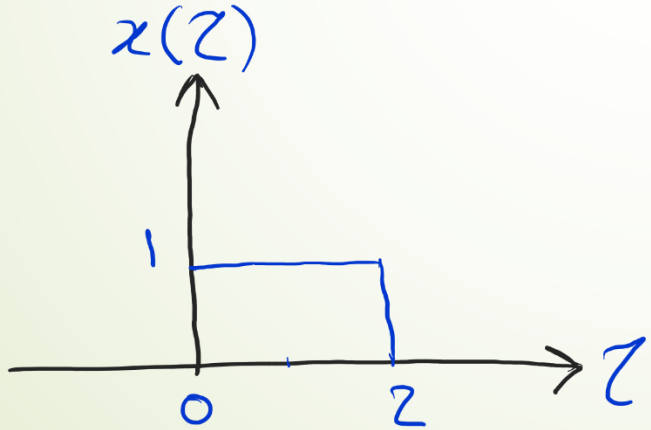
Using graphical direct method find y[n] for the signals shown below:



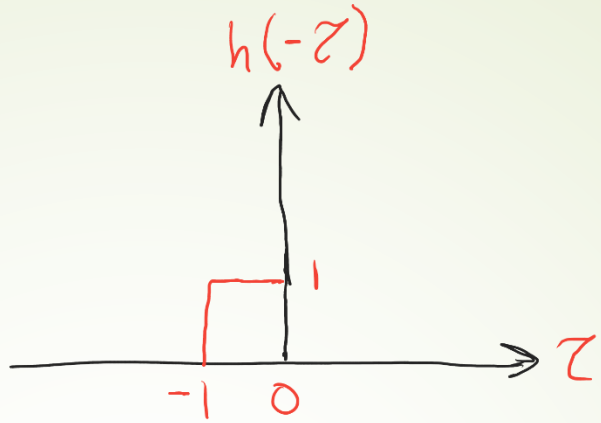


Solution:

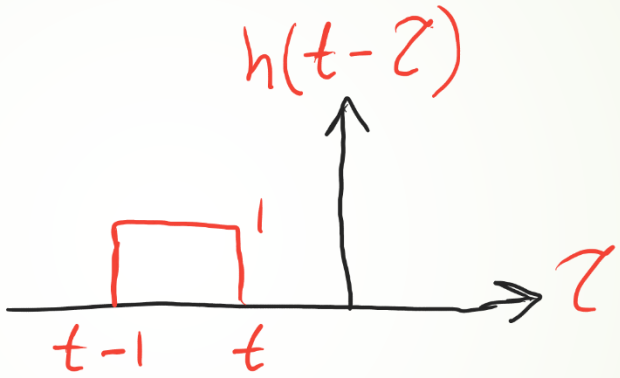
Step1:- convert (t) to (τ).



Step2:- time reverse h(τ).



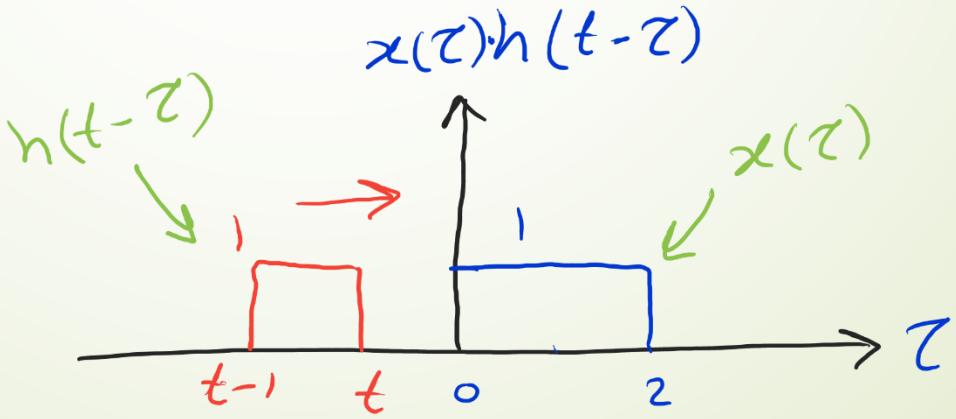
Step3:- time shift h(-τ).



Step4:- multiply x(τ) with h(t-τ)

Case 1

$t<0$



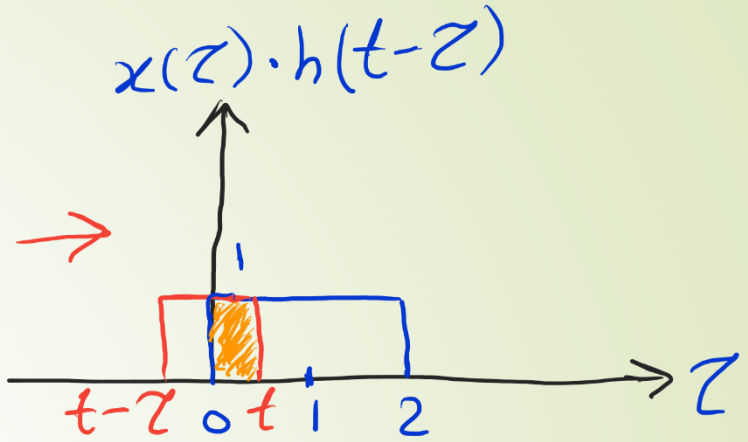
Case 2

$0 < t < 1$

$$y(t) = \int_0^t 1 \, d\tau$$

$$= [\tau]_0^t$$

$$= t$$



Case 3

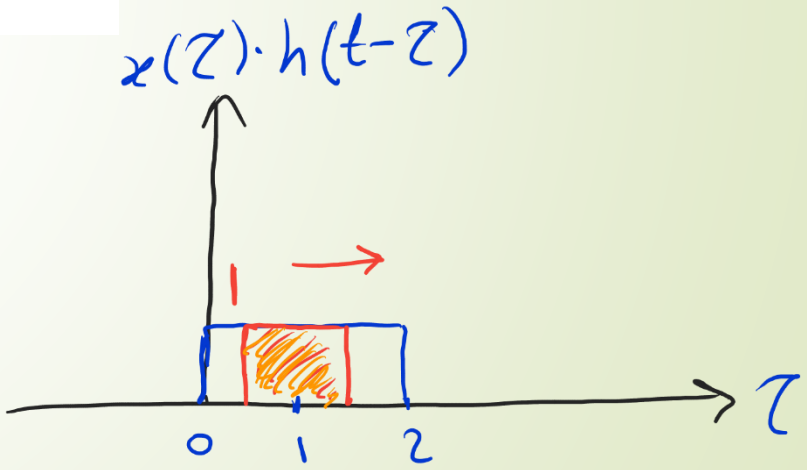
$1 < t < 2$

$$y(t) = \int_{t-1}^t 1 \, d\tau$$

$$= [\tau]_{t-1}^t$$

$$= t - t + 1$$

$$= 1$$



Case 4

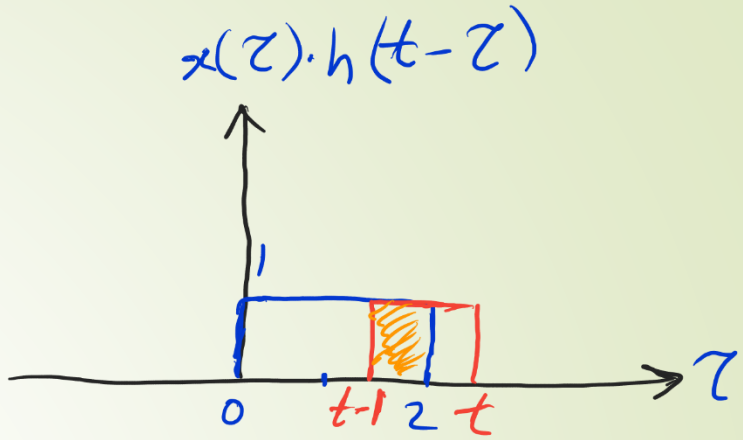
$2 < t < 3$

$$y(t) = \int_{t-1}^{2} 1 \, d\tau$$

$$= [\tau]_{t-1}^{2}$$

$$= 2 - t + 1$$

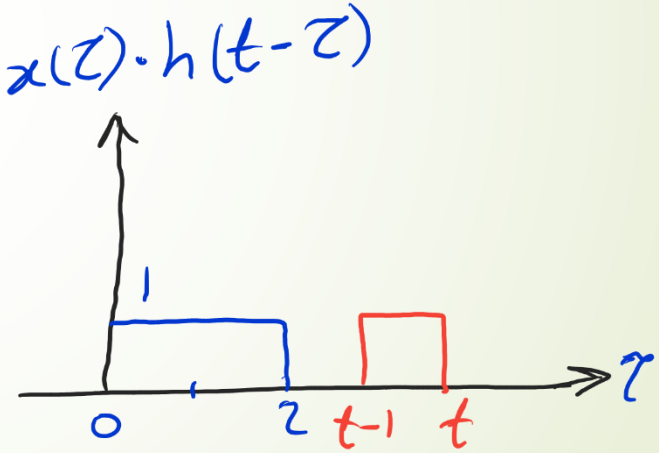
$$= 3 - t$$



Case 5

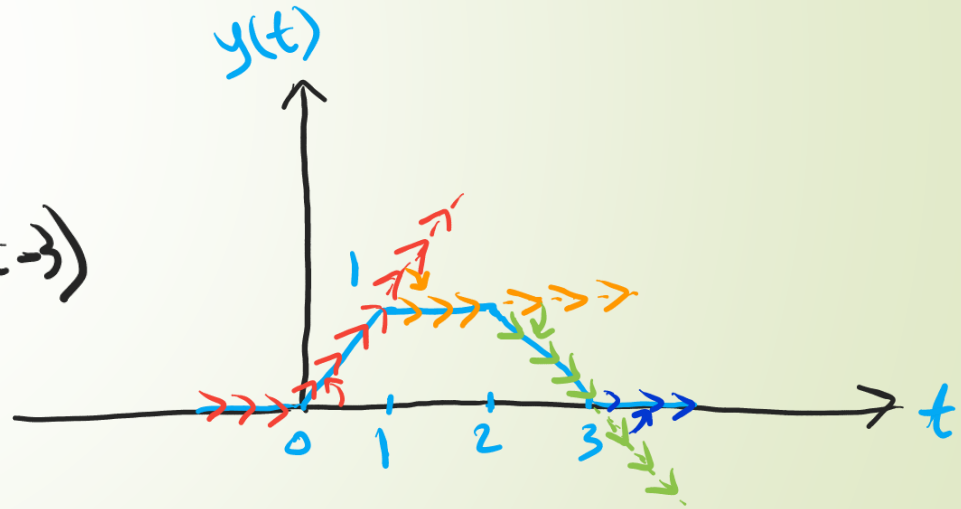
$t > 3$

$$y(t) = 0$$



$$\therefore y(t) = \begin{cases} 0, & t<0 \\ t, & 0<t<1 \\ 1, & 1<t<2 \\ 3-t, & 2<t<3 \\ 0, & t>3 \end{cases}$$

$$y(t) = 0 + 1\cdot r(t-0) - 1\cdot r(t-1) - 1\cdot r(t-2) + 1\cdot r(t-3)$$

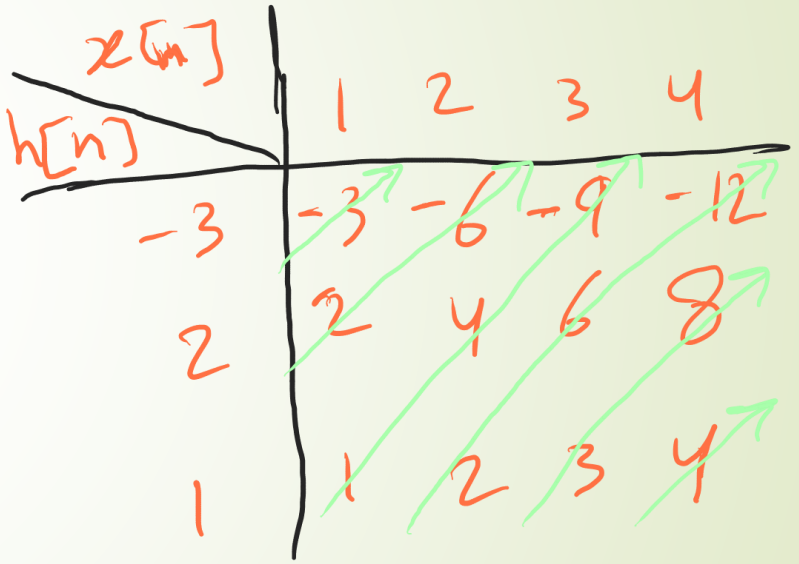


$$\therefore y(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$$

Circular or periodic convolution

Example:- Using circular convolution find y[n] if x[n]={1,2,3,4} and h[n]={-3,2,1}

h[n] \ x[n]	1	2	3	4
-3	-3	-6	-9	-12
2	2	4	6	8
1	1	2	3	4



$y[n]=\{-3,-4,-4,-4,11,4\}$

N=4+3-1=6

x[n]={1,2,3,4} h[n]={-3,2,1} N=4+3-1=6

$$x[n]=\{1,2,3,4,0,0\} \qquad h[n]=\{-3,2,1,0,0,0\}$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \\ -4 \\ -4 \\ 11 \\ 4 \end{bmatrix}$$

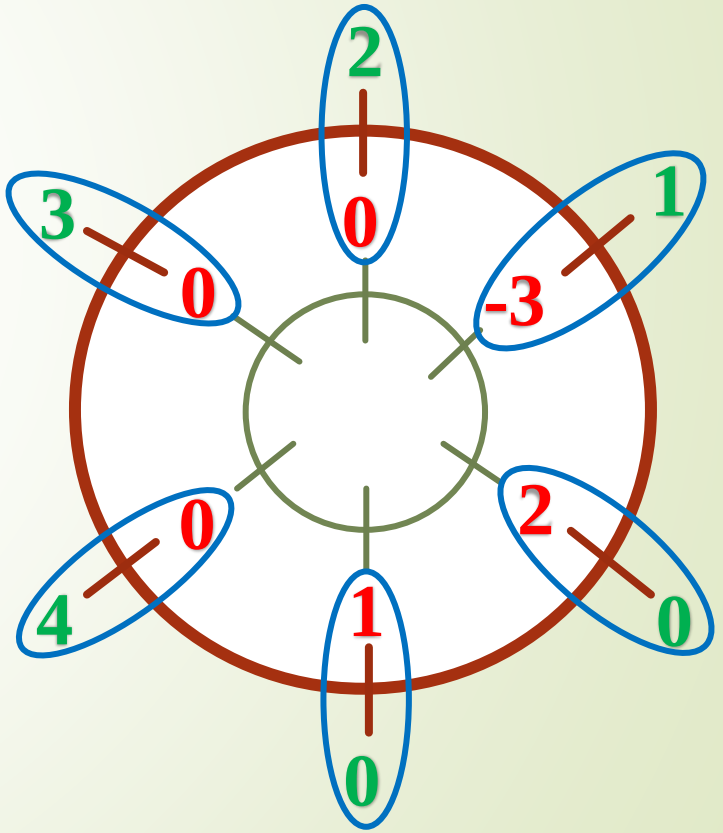
$$y[n]=\{-3,-4,-4,-4,11,4\}$$

$$x[n] = \{1, 2, 3, 4, 0, 0\}$$

$$h[n] = \{-3, 2, 1, 0, 0, 0\}$$

$$y[0] = -3 \times 1 + 0 \times 2 + 0 \times 3 + 0 \times 4 + 1 \times 0 + 2 \times 0$$

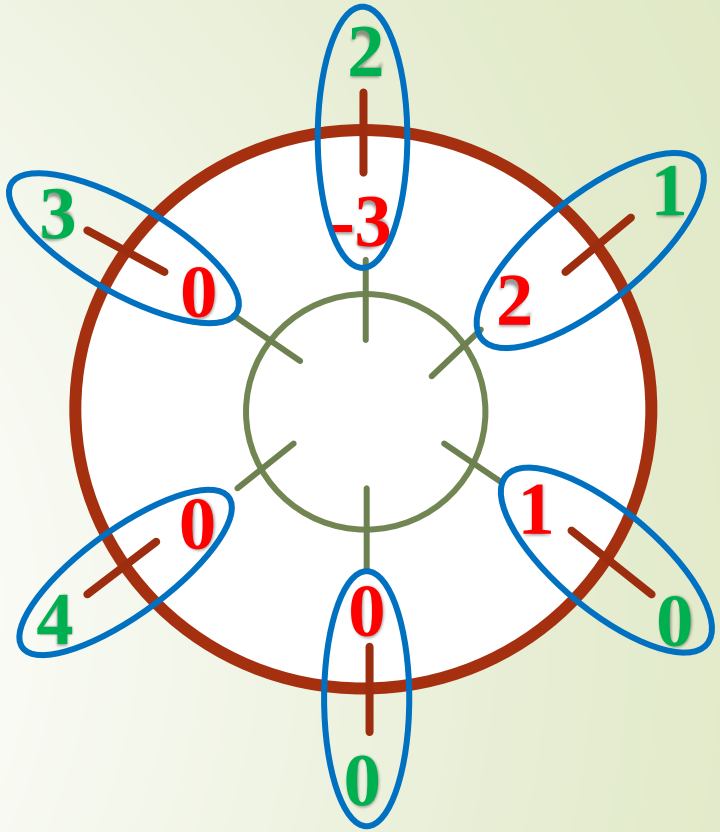
$$= -3$$



$$y[1] = 2\times1 + -3\times2 + 3\times0 + 4\times0 + 0 + 1\times0$$

$$= 2 + -6 + 0$$

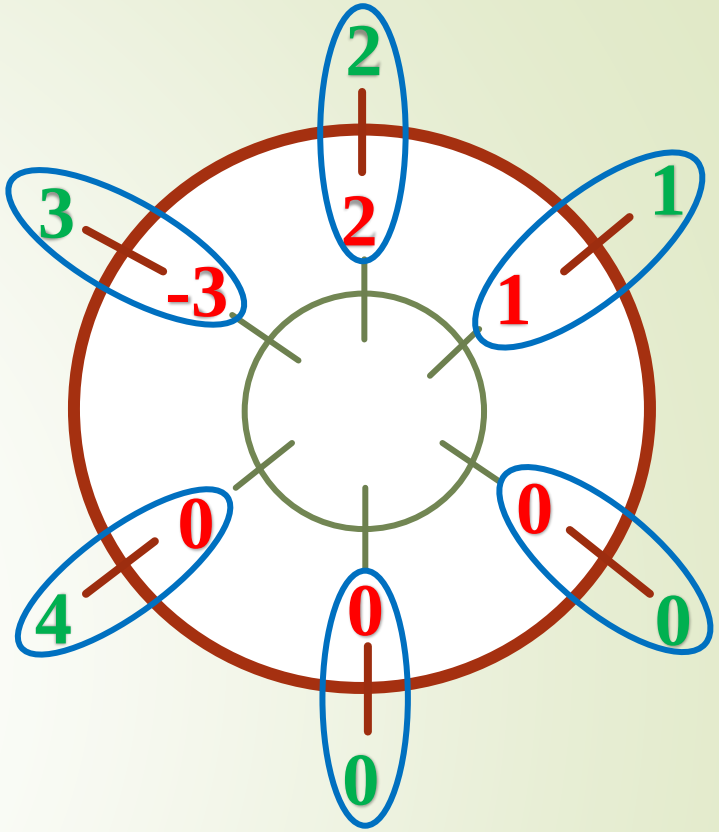
$$= -4$$

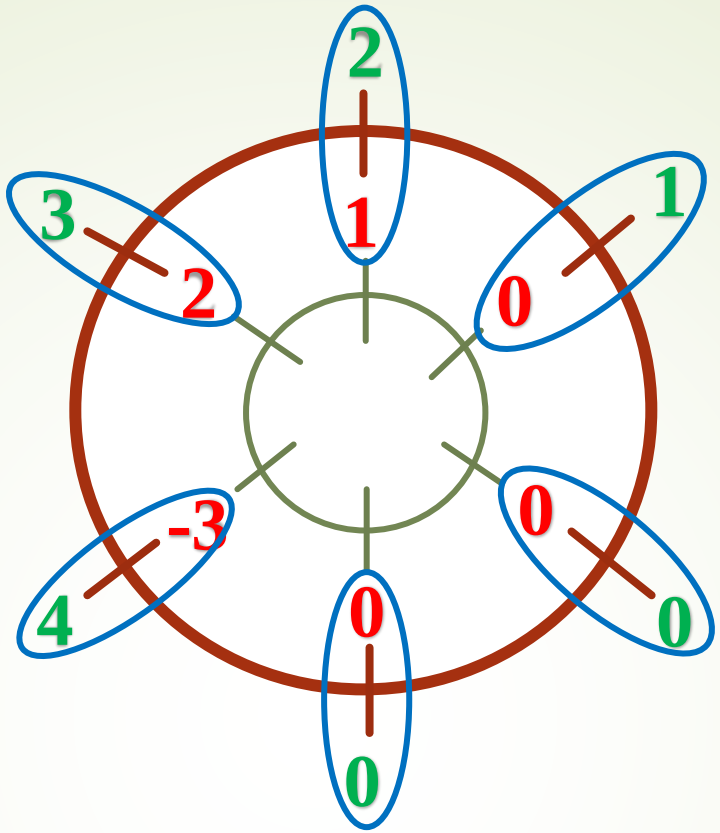


$$y[2] = 1\times1 + 2\times2 + -3\times3 + 0\times4 + 0 + 0$$

$$= 1 + 4 + -9 + 0$$

$$= -4$$

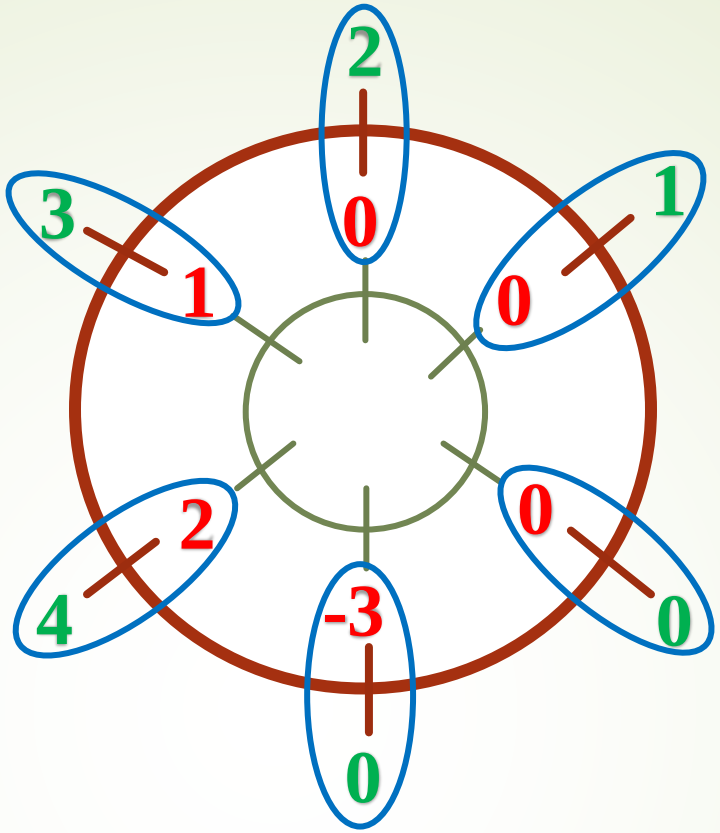




$$y[3] = 1\times0 + 2\times1 + 3\times2 + -3\times4 + 0 + 0$$

$$= 0 + 2 + 6 + -12$$

$$= -4$$

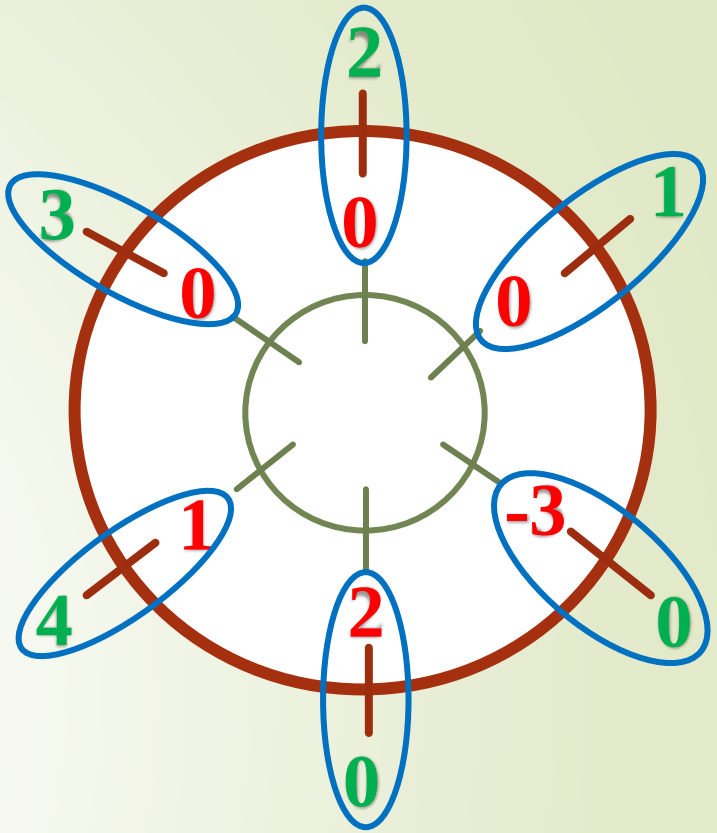


$$y[4] = 1\times0 + 2\times0 + 3\times1 + 4\times2 + 0 + 0$$

$$= 0 + 0 + 3 + 8 + 0 + 0$$

$$= 11$$

$$y[n] = \{-3, -4, -4, -4, 11, 4\}$$



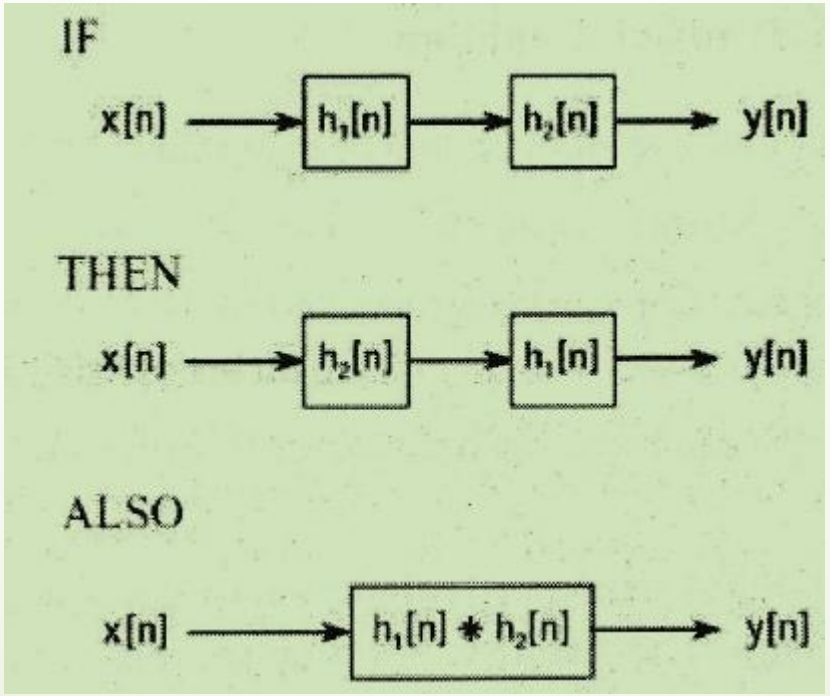
$$y[5] = 1\times0 + 2\times0 + 3\times0 + 4\times1 + 0\times2 + 0\times-3$$

$$= 4$$

Properties of Convolution

1. Commutative property:

$$x(n) * h(n) = h(n) * x(n)$$



Properties of Convolution

2. Associative property:

$$(x(n) * h(n)) * g(n) = x(n) * (h(n) * g(n))$$

3. Distributive property:

$$x(n) * [h(n) + g(n)] = x(n) * h(n) + x(n) * g(n)$$