AL FURAT AL AWSAT TECHNICAL UNIVERSITY
NAJAF COLLEGE OF TECHNOLOGY
DEPARTMENT OF AVIONICS ENGINEERING

DIGITAL SIGNAL PROCESSING

3rd YEAR

BY

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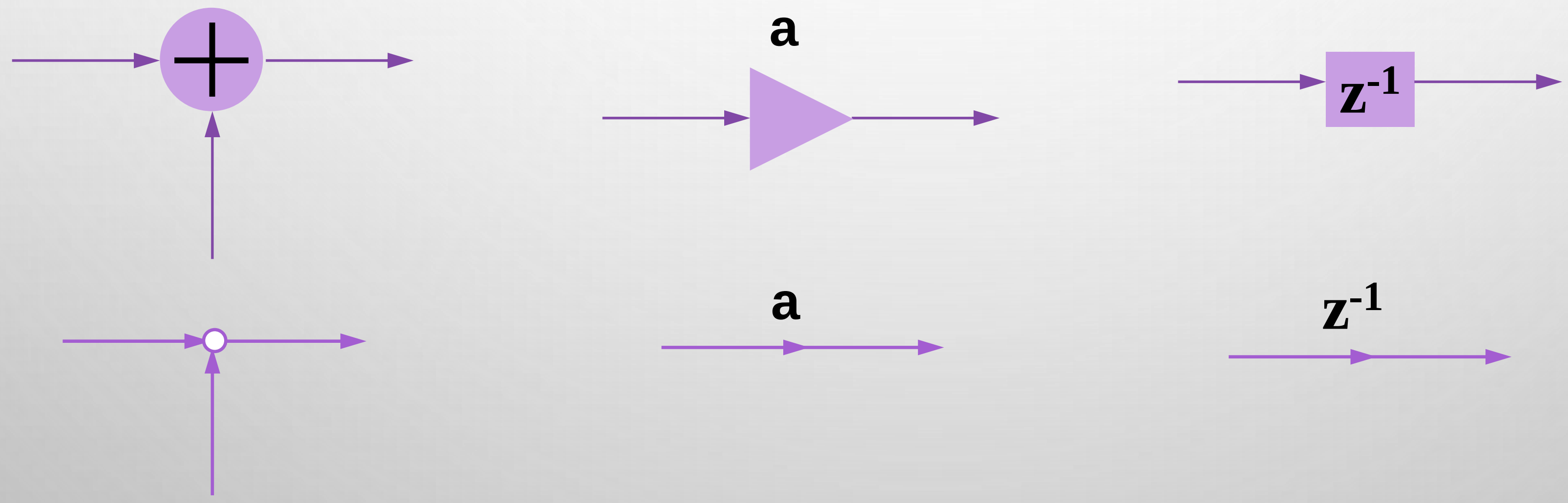
WHAT IS A DIGITAL FILTER

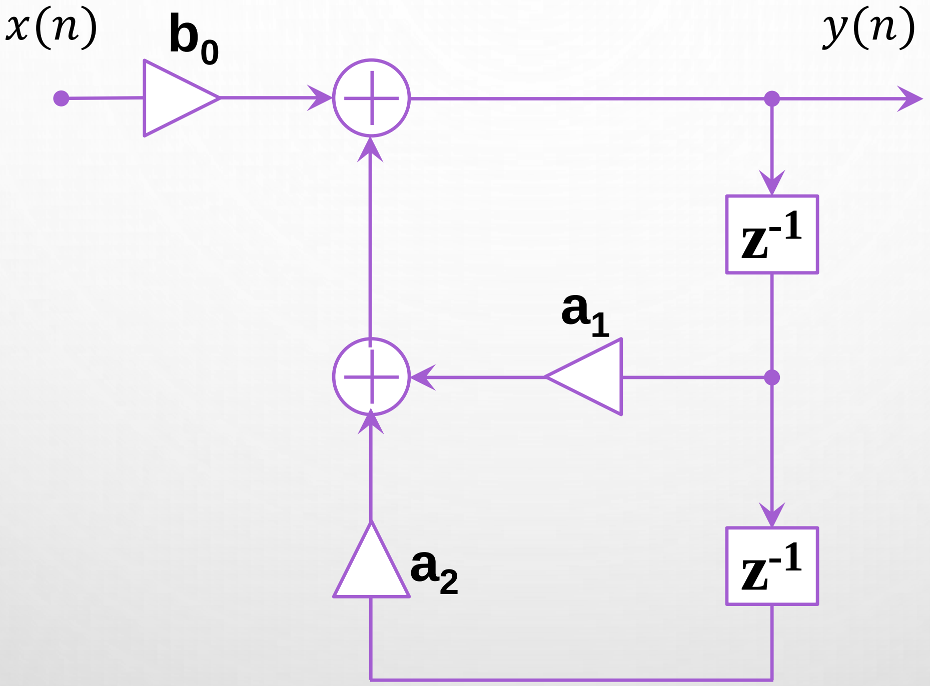
- A ***filter*** is a system that is designed to remove some component or modify some characteristic of a signal
- A digital filter is a discrete-time LTI system which can process the discrete-time signal.
- There are various structures for the implementation of digital filters
- The actual implementation of an LTI digital filter can be either in software or hardware form, depending on applications

BASIC ELEMENTS OF DIGITAL FILTER STRUCTURES

- Adder has two inputs and one output.
- Multiplier (gain) has single-input, single-output.
- Delay element delays the signal passing through it by one sample. It is implemented by using a shift register.



BASIC ELEMENTS OF DIGITAL FILTER STRUCTURES



$$y(n) = b_0 x(n) + a_1 y(n-1) + a_2 y(n-2)$$

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FIR (FINITE IMPULSE RESPONSE) FILTER STRUCTURES

The characteristics of the FIR filter

- FIR filters have Finite-duration Impulse Responses, thus they can be realized by means of DFT
- The system function *H(z)* has the *ROC* of $|z| > 0$, thus it is a causal system
- An FIR filter is a ***non recursive*** system
- FIR filters can be designed to have a linear-phase response

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

It has *N-1* order poles at $z = 0$

and *N-1* zeros in $|z| > 0$

The *order* of such an FIR filter is *N*-1

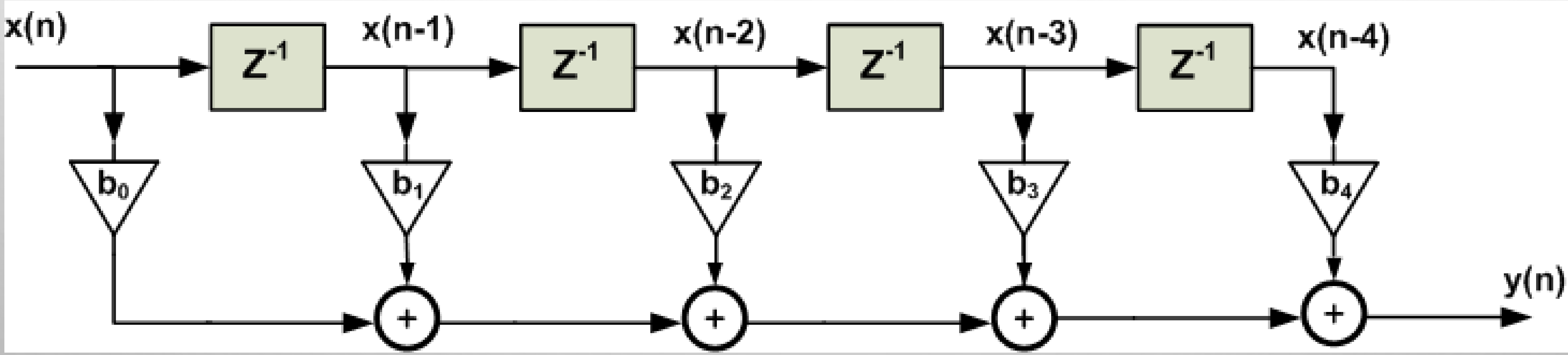
FIR FILTER STRUCTURES

- Direct form

In this form the difference equation is implemented directly as given:

$$y(n) = \sum_{m=0}^{N-1} h(m)x(n-m)$$

It requires *N* multiplications



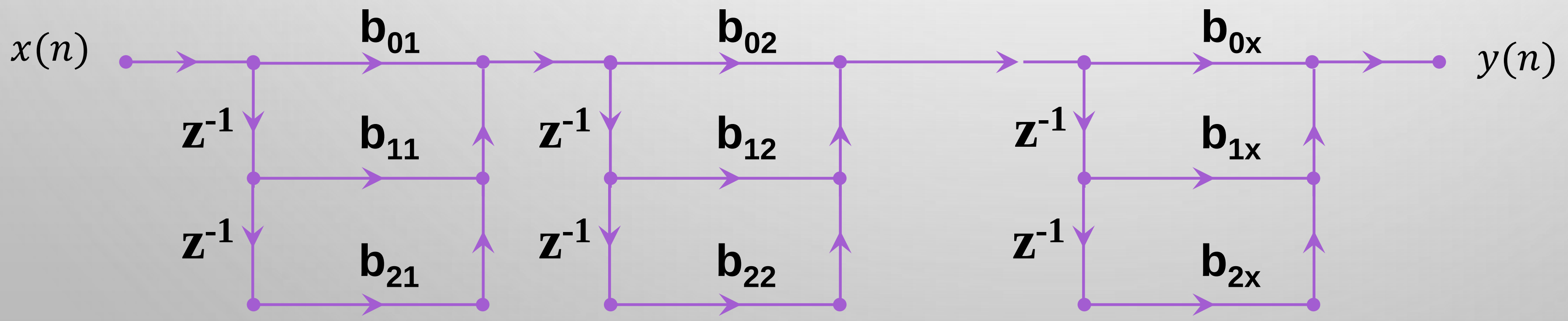
FIR FILTER STRUCTURES

- Cascade form

In this form the system function *H(z)* is converted into products of second-order sections with real coefficients

$$H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}=\prod_{k=1}^{\left[\frac{N}{2}\right]}(b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2})$$

It requires (3*N*/2) multiplications



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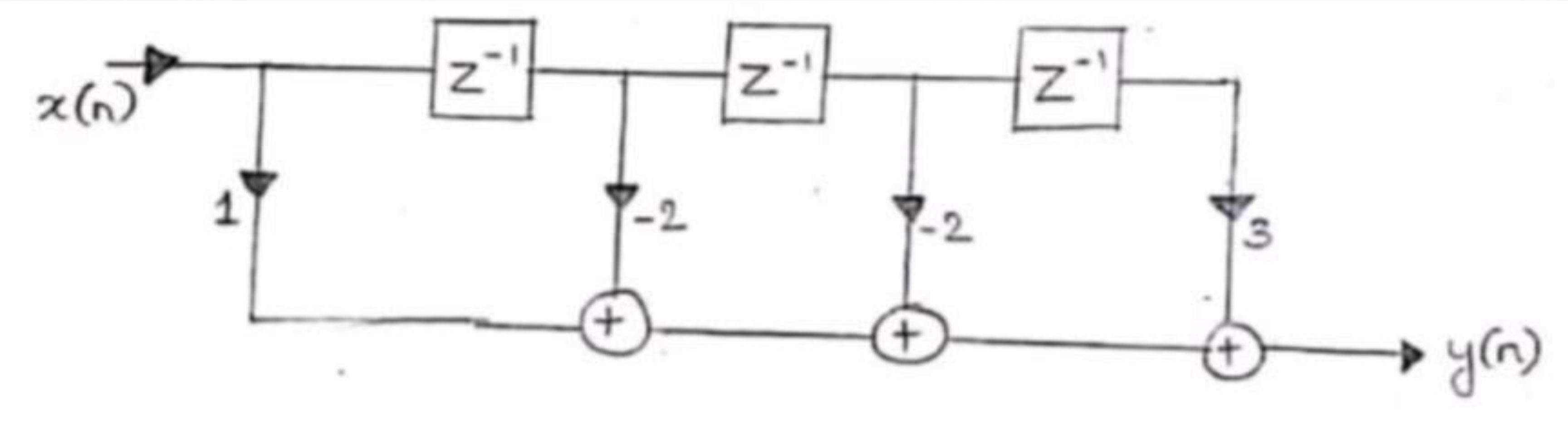
EXAMPLES

Draw the direct form structure for the FIR filter represented by the following difference equation

$$y[n]= x[n]-2x[n-1]-2x[n-2]+3x[n-3]$$

Sol:



EXAMPLES

Draw the direct form structure for the FIR filter represented by the following transfer function

$$H[z]= 4+2z^{-1}-2\ z^{-2}+3\ z^{-3}$$

Sol:

$$H(z)=\frac{Y(z)}{X(z)}=4+2z^{-1}-2z^{-2}+3z^{-3}$$

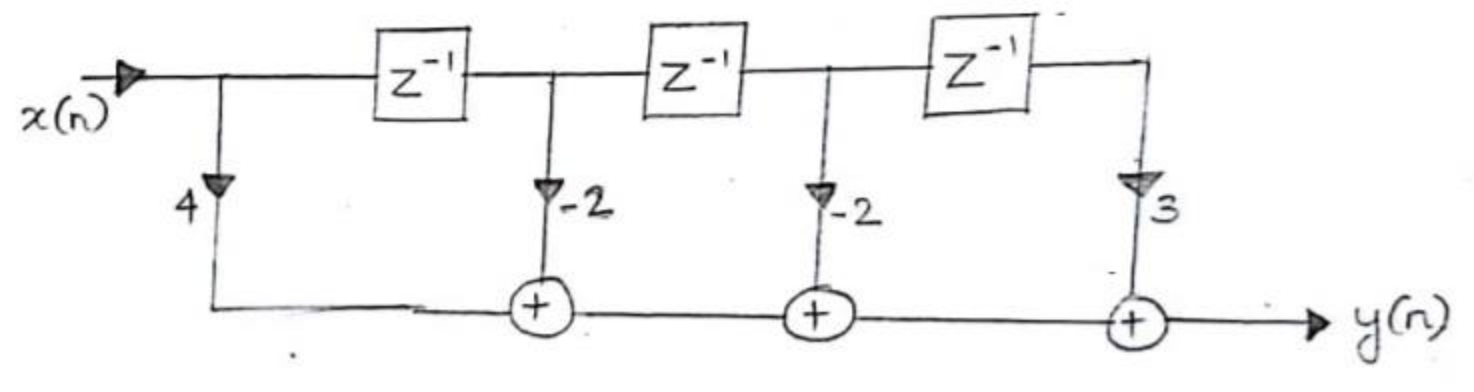
$$Y(z)=4X(z)+2z^{-1}X(z)-2z^{-2}X(z)+3z^{-3}X(z)$$

Taking Inverse z-Transform

$$y(n)=4x(n)+2x(n-1)-2x(n-2)+3x(n-3)$$

Time shifting

$$x(n-k)\ \overset{z}{\leftrightarrow}\ z^{-k}X(z)$$



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EXAMPLES

Using cascade structure realize the FIR filter represented by the following transfer function

$$H[z]= (1+\frac{1}{2}z^{-1}+ z^{-2})\ (1+\frac{1}{4}z^{-1}+ z^{-2})$$

Sol:

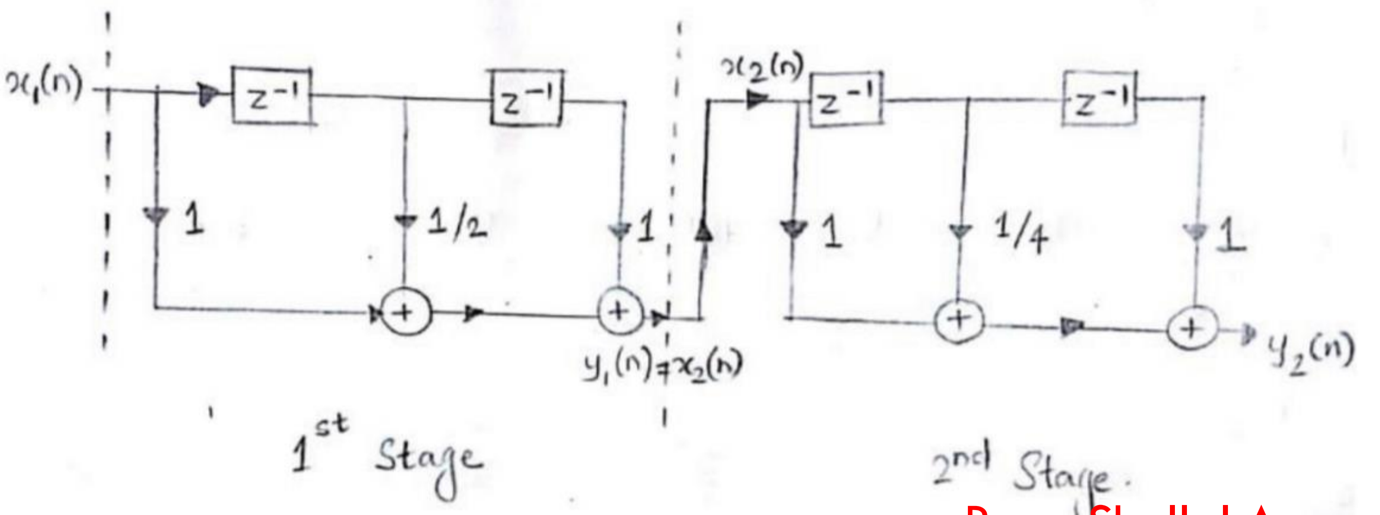
$$H(z) = H_1(z).H_2(z)$$

$$H_1(z) = (1+\frac{1}{2}z^{-1}+z^{-2}) \qquad H_2(z) = (1+\frac{1}{4}z^{-1}+z^{-2})$$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = (1+\frac{1}{2}z^{-1}+z^{-2}) \qquad H_2(z) = \frac{Y_2(z)}{X_2(z)} = (1+\frac{1}{4}z^{-1}+z^{-2})$$

$$Y_1(z) = X_1(z)+\frac{1}{2}z^{-1}X_1(z)+z^{-2}X_1(z) \qquad Y_2(z) = X_2(z)+\frac{1}{4}z^{-1}X_2(z)+z^{-2}X_2(z)$$

$$y_1(n) = x_1(n)+\frac{1}{2}x_1(n-1)+x_1(n-2) \qquad y_2(n) = x_2(n)+\frac{1}{4}x_2(n-1)+x_2(n-2)$$



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IIR(INFINITE IMPULSE RESPONSE) FILTER STRUCTURES

- The characteristics of the IIR filter

- IIR filters have Infinite-duration Impulse Responses
- The system function *H(z)* has poles in $0 < |z| < \infty$
- An IIR filter is a ***recursive*** system

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 - (a_1 z^{-1} + \cdots + a_N z^{-N})}$$

$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

The *order* of such an IIR filter is called *N* if $a_N \neq 0$

IIR FILTER STRUCTURES

- Direct form

In this form the difference equation is implemented directly as given. There are two parts to this filter, namely the moving average part and the recursive part (or the numerator and denominator parts). Therefore this implementation leads to two versions: direct form I and direct form II structures

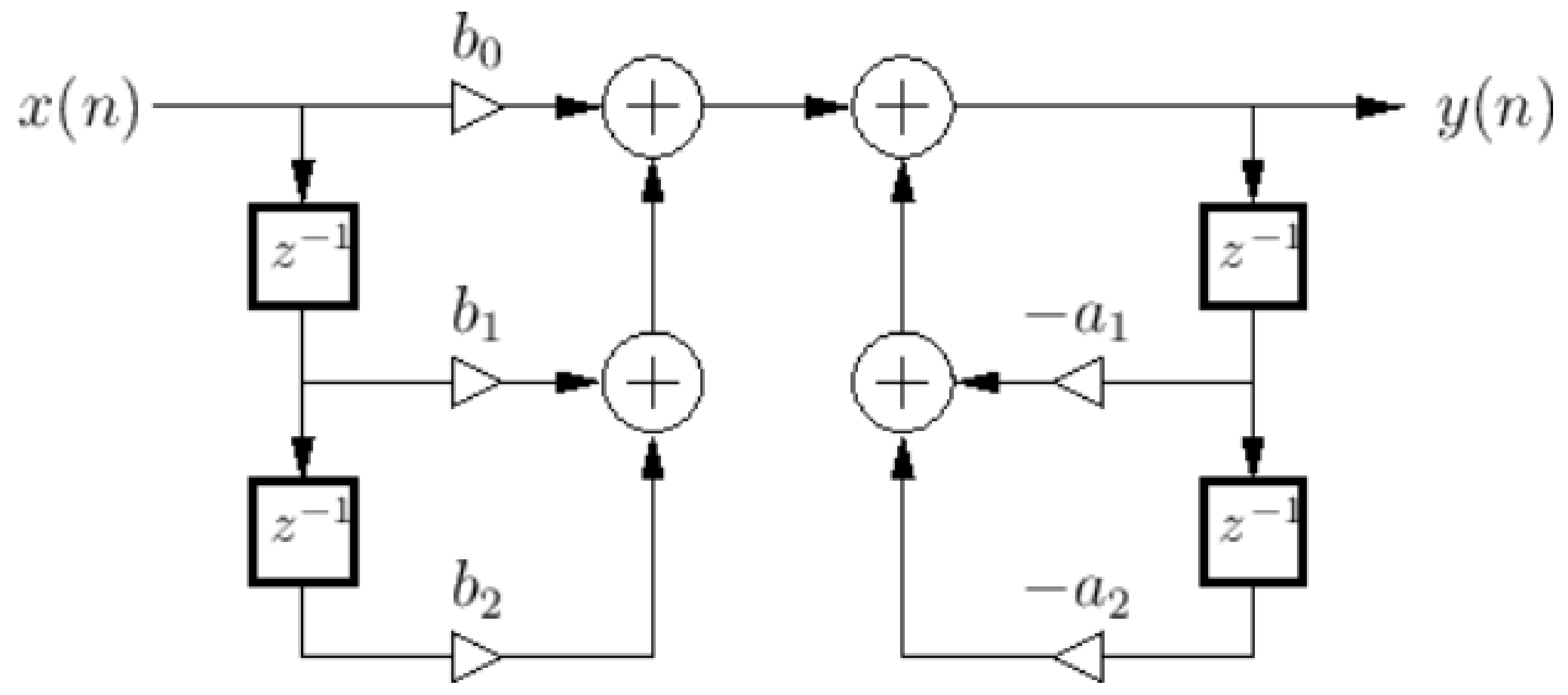
$$y(n) = \sum_{k=0}^{M} b_k x(n-k) + \sum_{k=1}^{N} a_k y(n-k)$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

IIR FILTER STRUCTURES

- Direct form I

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) + \sum_{k=1}^{N} a_k y(n-k)$$

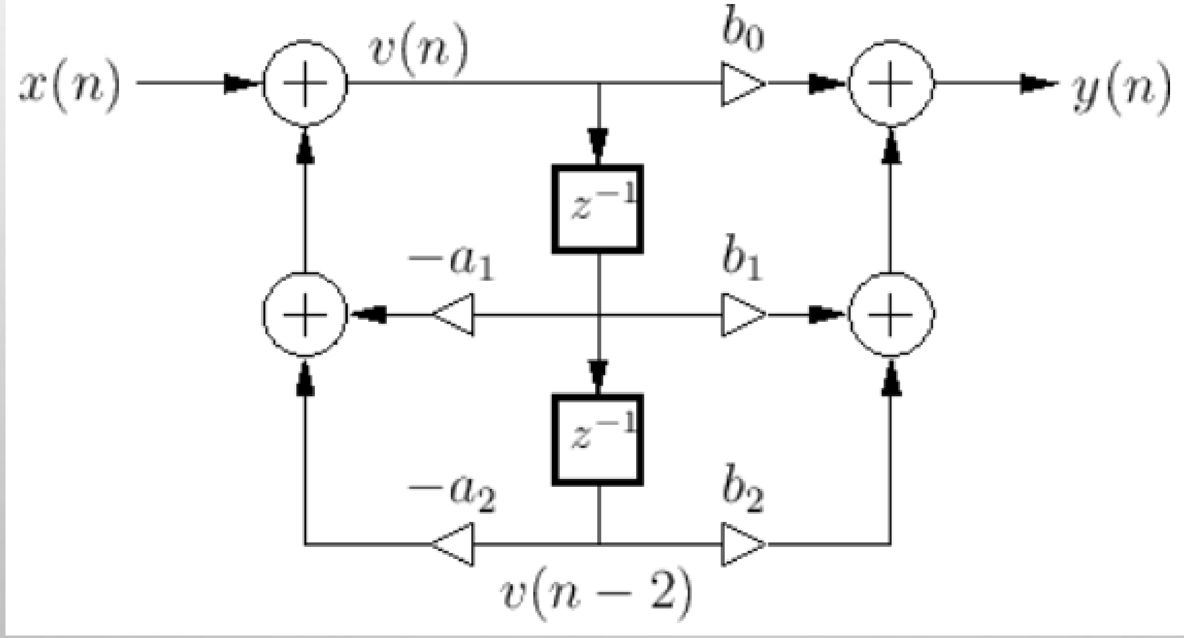


IIR FILTER STRUCTURES

- Direct form II

For an LTI cascade system, we can change the order of the systems without changing the overall system response

$$
\begin{aligned}
v(n) &= x(n) - a_1 v(n-1) - a_2 v(n-2) \\
y(n) &= b_0 v(n) + b_1 v(n-1) + b_2 v(n-2)
\end{aligned}
$$



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IIR FILTER STRUCTURES

- Cascade form

In this form the system function *H(z)* is written as a product of second-order sections with real coefficients

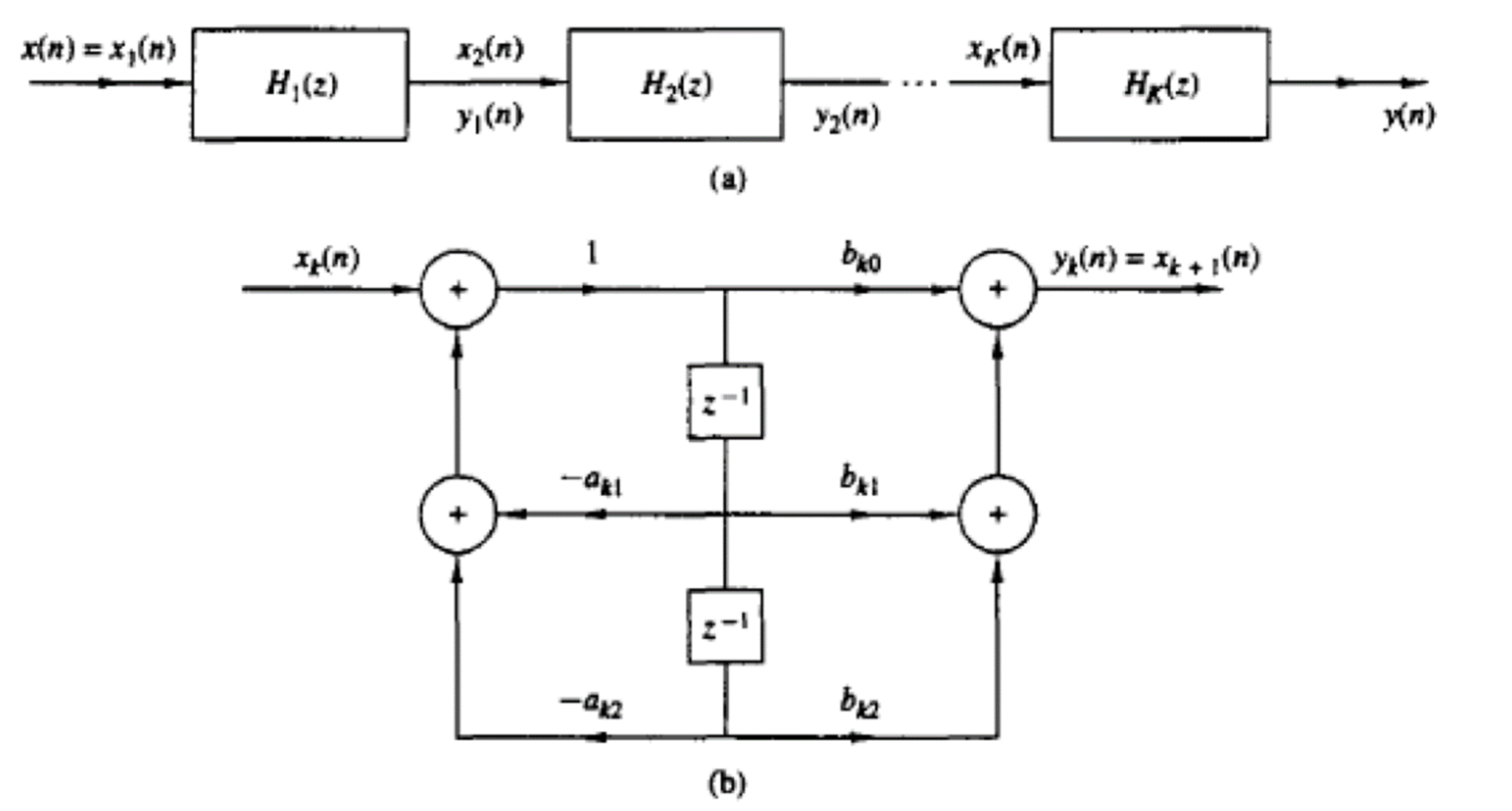
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = A \frac{\prod_{k=1}^{M_1}(1 - p_k z^{-1}) \prod_{k=1}^{M_2}(1 - q_k z^{-1})(1 - q_k^* z^{-1})}{\prod_{k=1}^{N_1}(1 - c_k z^{-1}) \prod_{k=1}^{N_2}(1 - d_k z^{-1})(1 - d_k^* z^{-1})}$$

$$H(z) = A \frac{\prod_{k=1}^{M_1}(1 - p_k z^{-1}) \prod_{k=1}^{M_2}(1 + b_{1k} z^{-1} + b_{2k} z^{-2})}{\prod_{k=1}^{N_1}(1 - c_k z^{-1}) \prod_{k=1}^{N_2}(1 - a_{1k} z^{-1} - a_{2k} z^{-2})}$$

$$M = M_1 + 2M_2$$
$$N = N_1 + 2N_2$$

IIR FILTER STRUCTURES

- Cascade form



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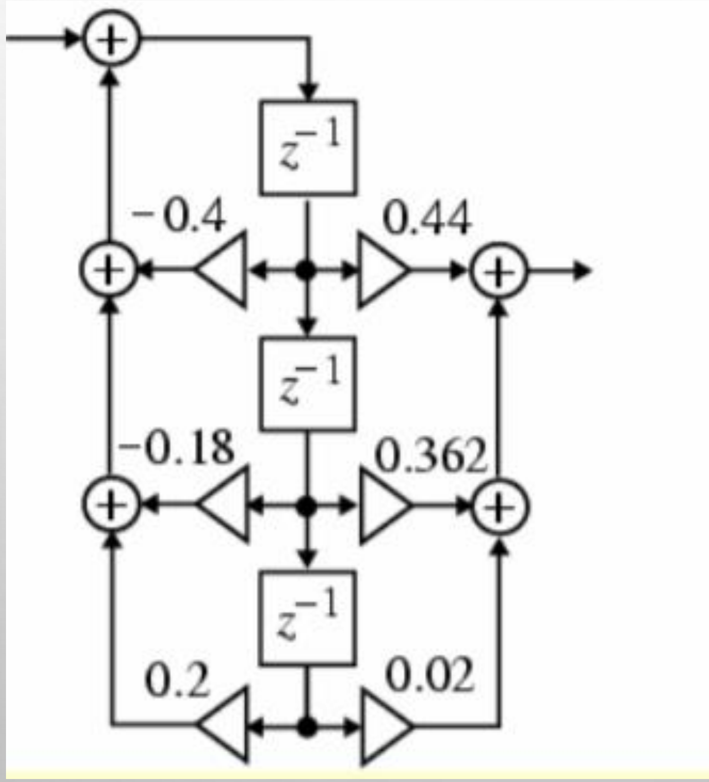
EXAMPLE

Draw the direct form II realization of the following transfer function

$$H(z)=\frac{0.44z^{-1}+0.362z^{-2}+0.02z^{-3}}{1+0.4z^{-1}+0.18z^{-2}-0.2z^{-3}}$$

Sol:



IIR FILTER STRUCTURES

Parallel form

- In this structure, the input signal is processed separately by a different subsystems.
- An IIR Transfer function can be realized in a parallel form by making use of the partial fraction of expansion of the Transfer function.

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = \frac{\prod_{k=1}^{M}(1-\beta_k z^{-1})}{\prod_{k=1}^{N}(1-\alpha_k z^{-1})}$$

EXAMPLE

Obtain Cascade and parallel structures for the following system:

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$$

<u>Sol</u>:

Taking Z-Transform on both sides results in

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z) + \frac{1}{3}z^{-1}X(z)$$

The corresponding Transfer Function is

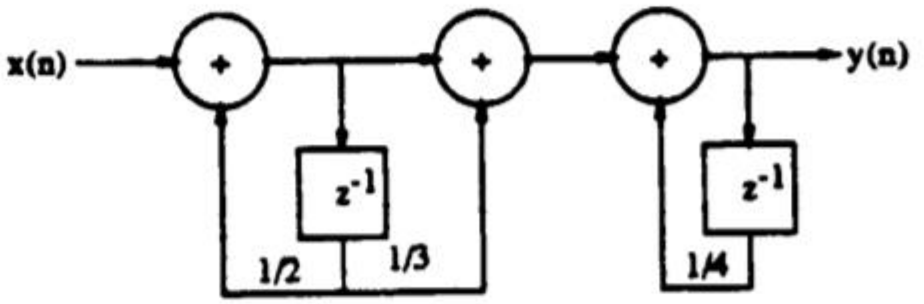
$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

EXAMPLE

Cascade Form:

$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})} \cdot \frac{1}{(1-\frac{1}{4}z^{-1})}$$

$$= H_1(z).H_2(z)$$



EXAMPLE

Parallel Form:

By using partial fraction expansion

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{A}{(1 - \frac{1}{2}z^{-1})} + \frac{B}{(1 - \frac{1}{4}z^{-1})}$$

Solving for A and B,

$$H(z) = \frac{10}{3}\frac{1}{(1-\frac{1}{2}z^{-1})} - \frac{7}{3}\frac{1}{(1-\frac{1}{4}z^{-1})}$$