AL FURAT AL AWSAT TECHNICAL UNIVERSITY
NAJAF COLLEGE OF TECHNOLOGY
DEPARTMENT OF AVIONICS ENGINEERING

DIGITAL SIGNAL PROCESSING

3rd YEAR

BY

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OPERATIONS ON CONTINUOUS TIME SIGNALS

- **Addition of continuous time signals**

1. Point-by-point addition of multiple signals

– Move from left to right (or vice versa), and add the value of each signal together to achieve the final signal y(t) = x_1 (t) + x_2 (t)

2. Graphical solution

– Plot each individual portion of the signal (break into parts)
– Add the signals point by point

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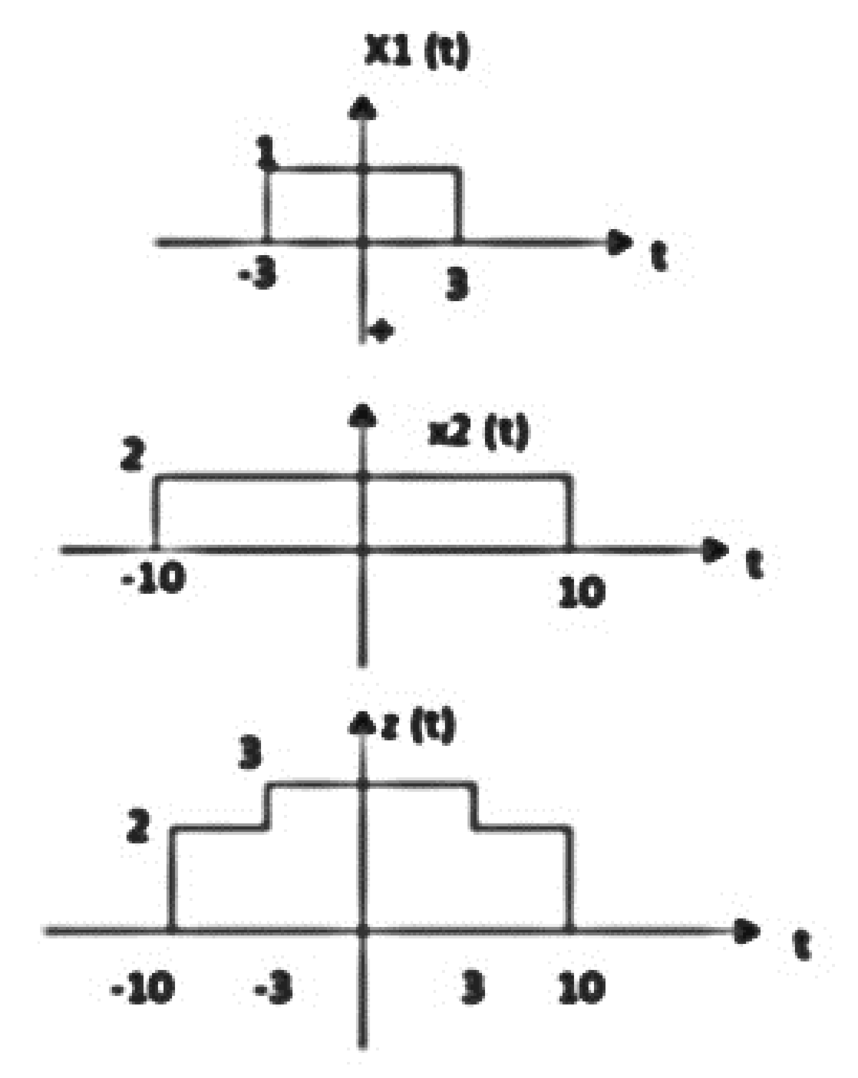
OPERATIONS ON CONTINUOUS TIME SIGNALS

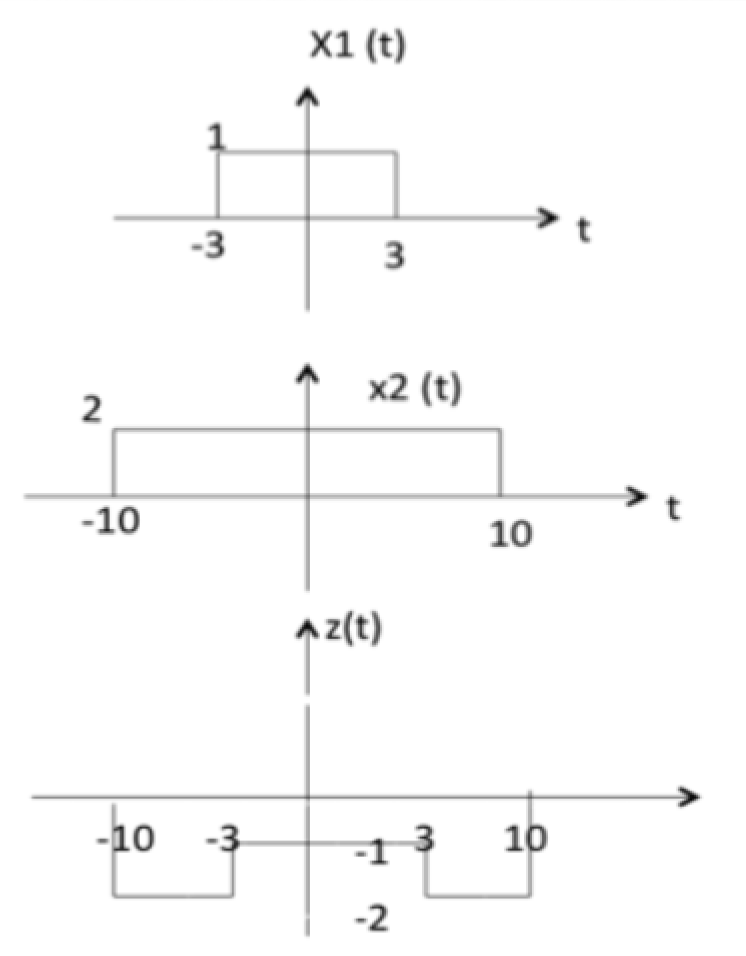
- Example:



OPERATIONS ON CONTINUOUS TIME SIGNALS

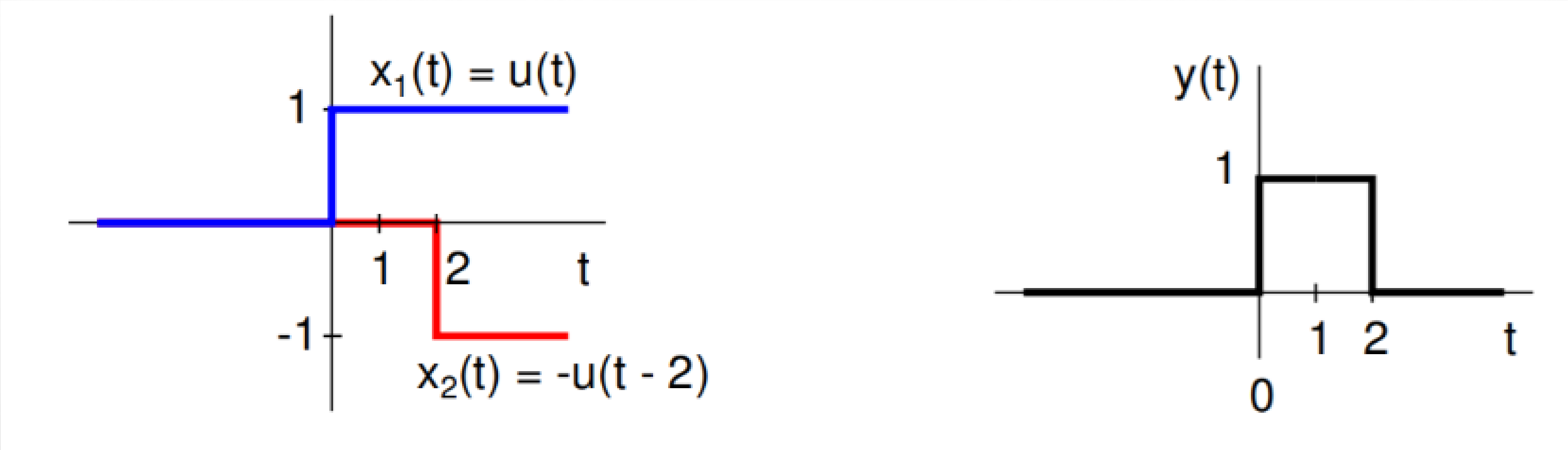
- **subtraction of continuous time signals**

subtraction of two signals is nothing but subtraction of their corresponding amplitudes.



OPERATIONS ON CONTINUOUS TIME SIGNALS

- Example:- Sketch $y(t) = u(t) - u(t-2)$



OPERATIONS ON CONTINUOUS TIME SIGNALS

- **multiplication of continuous time signals**

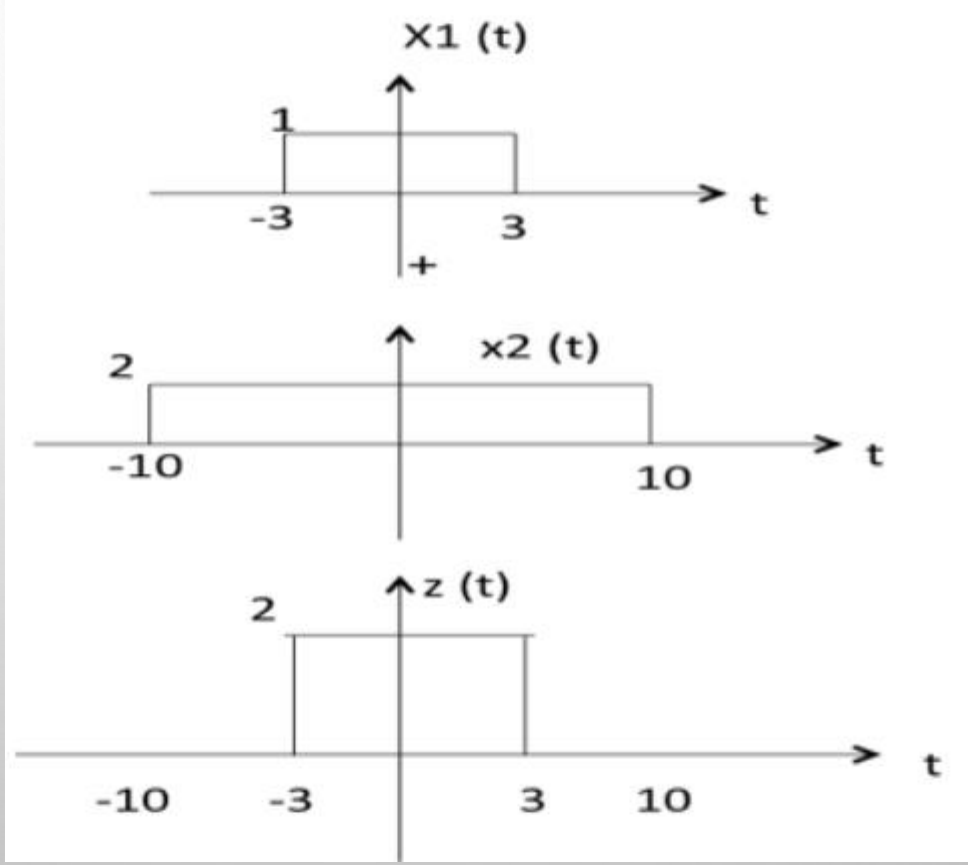
1. Point-by-point multiplication of the values of each signal y(t) = x_1(t) x_2(t)

2. Graphical solution

– Plot each individual portion of the signal (break into parts)
– Multiply the signals point by point

OPERATIONS ON CONTINUOUS TIME SIGNALS

- Example:-



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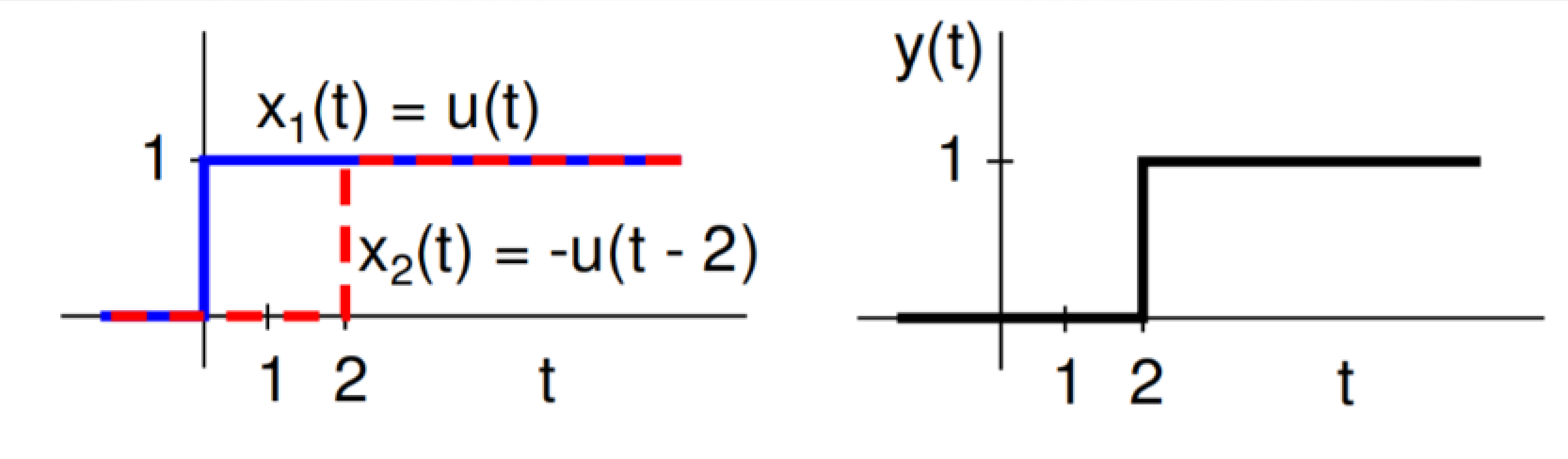
OPERATIONS ON CONTINUOUS TIME SIGNALS

Example:- Sketch $y(t) = u(t) \cdot u(t-2)$



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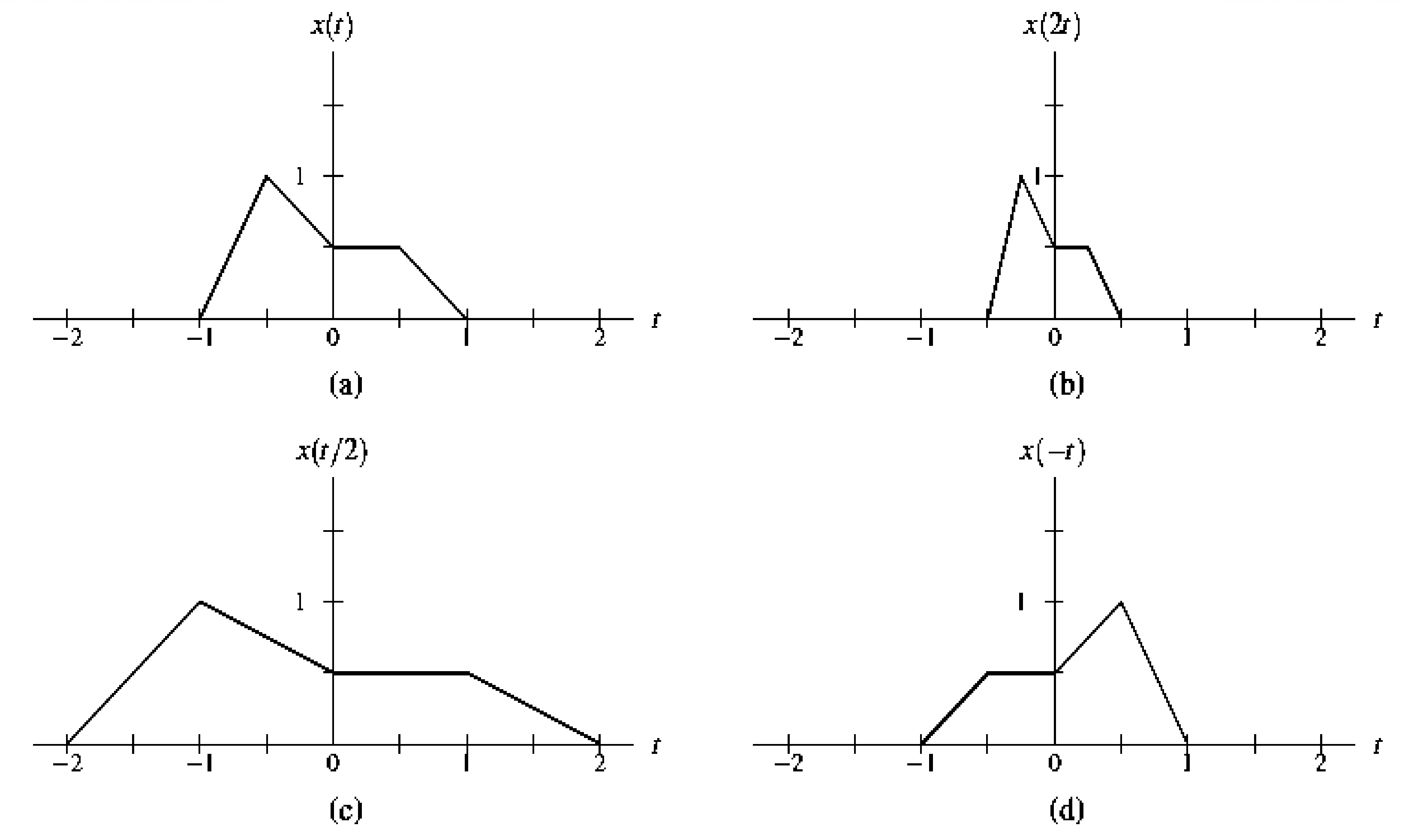
OPERATIONS ON CONTINUOUS TIME SIGNALS

- **Scaling** → **Time scaling**
  → **Amplitude scaling**

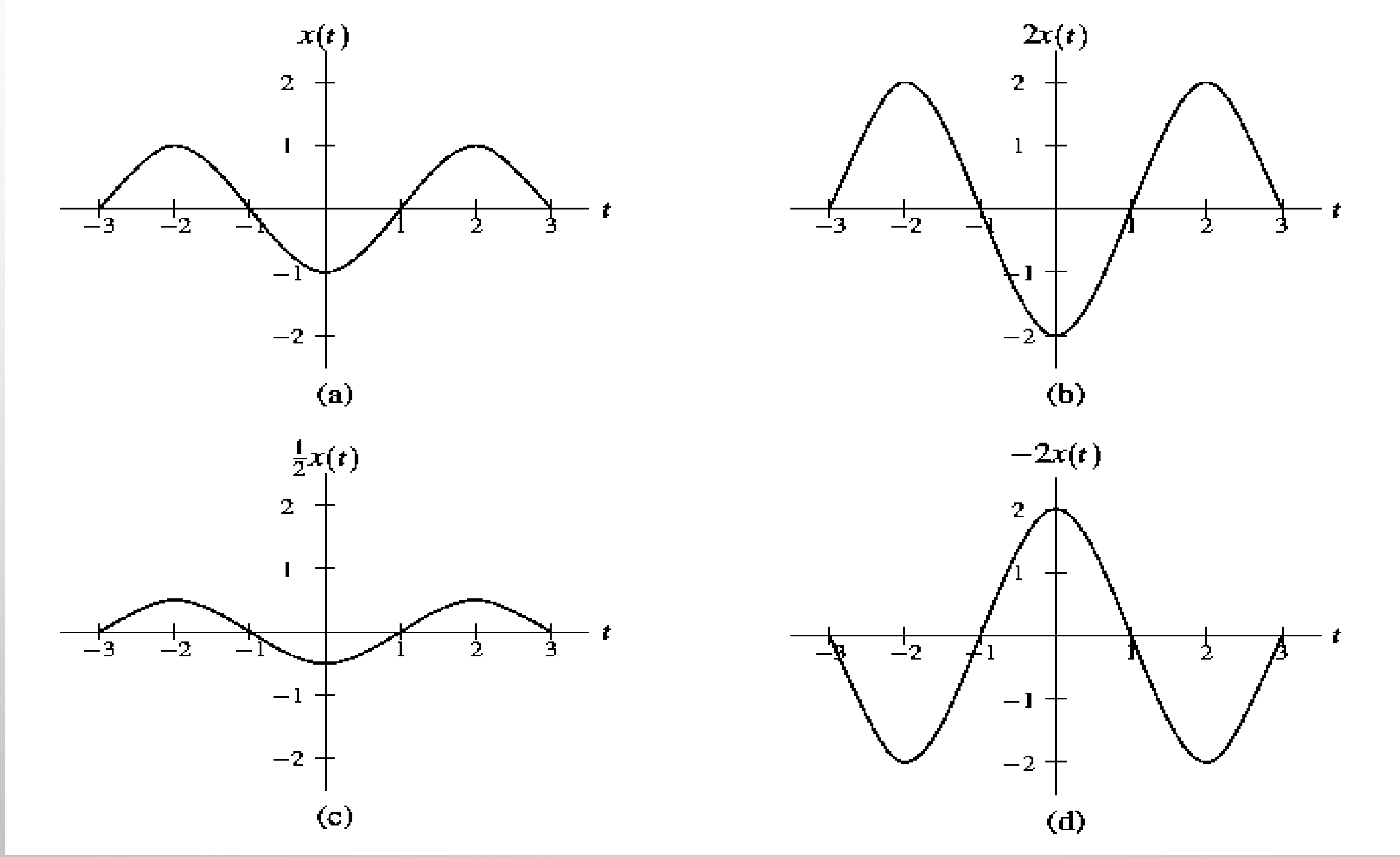
- **Time scaling:** is simply the multiplication of a time of a signal by a positive or a negative constant. In the case of analog signals, this operation is usually called ***Compression*** if the magnitude of the multiplying constant is greater than one. If the magnitude of the multiplying constant is less than one, the operation is called ***Expansion*****.** Thus, if $x(t)$ is an analog signal, the scaling operation generates a signal $y(t)=x(\alpha t)$, where α is the multiplying constant and not equal zero.

- **Amplitude scaling:** is simply the multiplication of the amplitude of a signal by a positive or a negative constant. In the case of analog signals, this operation is usually called ***amplification*** if the magnitude of the multiplying constant, called ***gain***, is greater than one. If the magnitude of the multiplying constant is less than one, the operation is called ***attenuation*****.** Thus, if $x(t)$ is an analog signal, the scaling operation generates a signal $y(t)= \beta\, x(t)$, where β is the multiplying constant.

EXAMPLES



EXAMPLES



OPERATIONS ON CONTINUOUS TIME SIGNALS

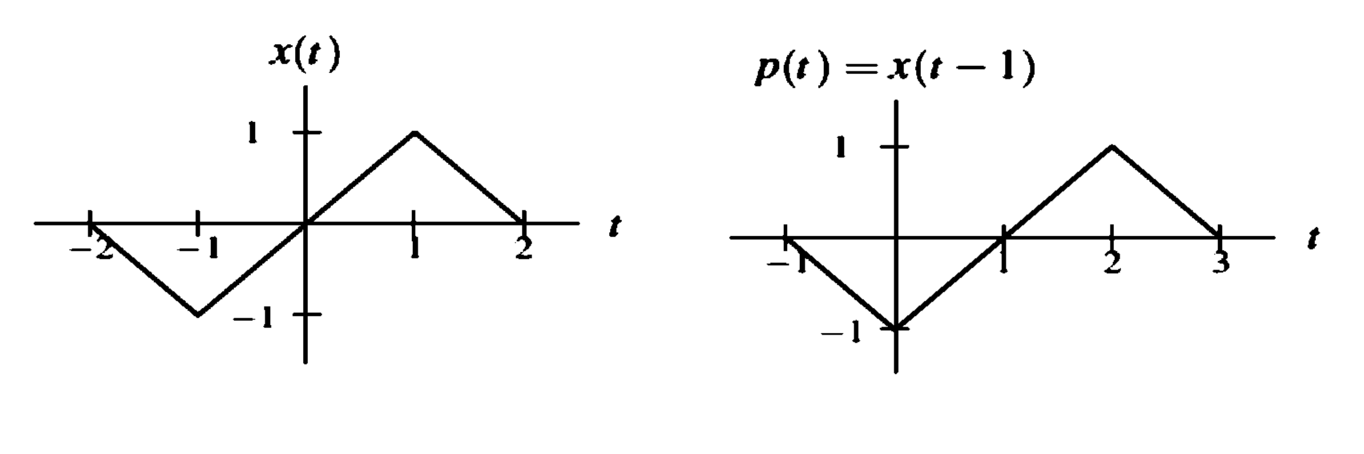
- **Shifting** → **Time Shifting**
  → **Amplitude Shifting**

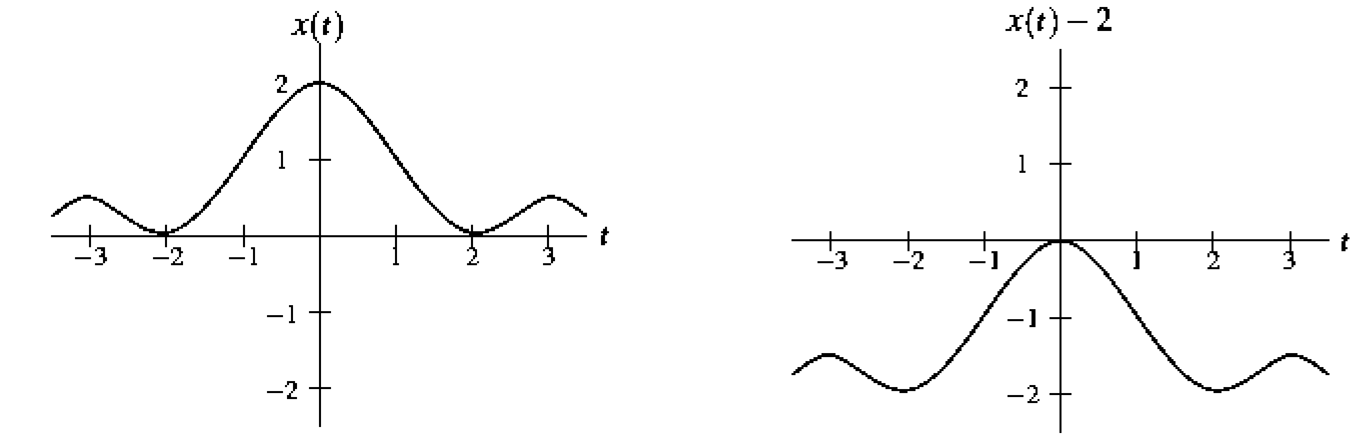
- **Time shifting:** it is mean add a constant to the time of a signal. In the case of analog signals, this operation is usually called ***Left Shifting (Time Advance)*** if the magnitude of constant is less than zero. If the magnitude of the constant is greater than zero, the operation is called ***Right Shifting (Time Delay).***

  $y(t)=x(t-\alpha)$, where α is the constant.

- **Amplitude Shifting:** it is mean add a constant to a signal. $y(t)=x(t)+\alpha$, where α is the constant.

EXAMPLES





(a)

(b)

OPERATION ON CONTINUOUS TIME SIGNALS

- **Reversal (fold)** → **Time Reversal**
  → **Amplitude Reversal**

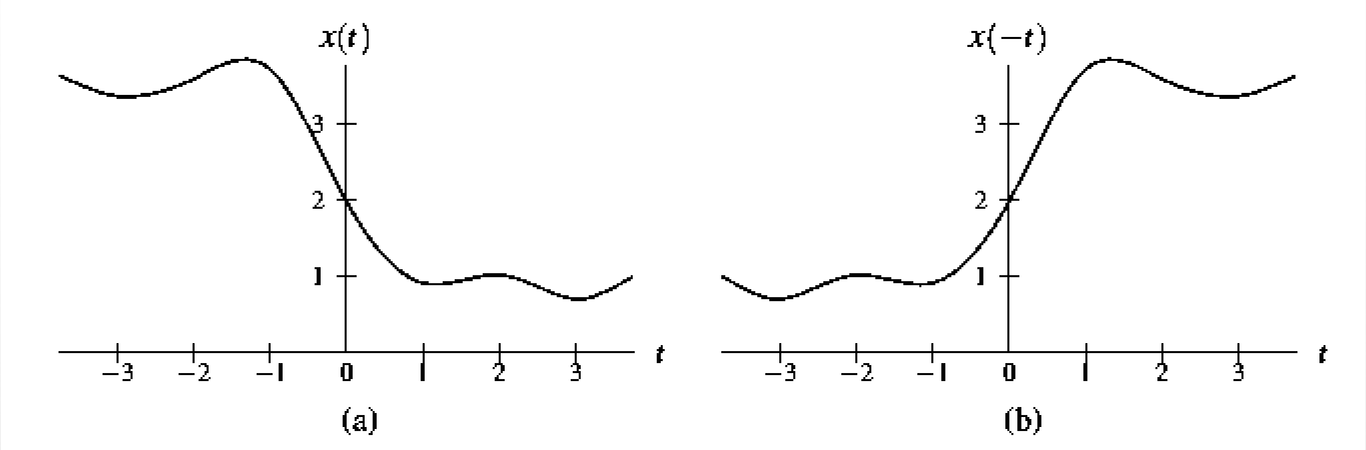
- **Time Reversal: It is a special case of time scaling when** $\alpha = -1$, $y(t) = x(\alpha t)$

  $y(t) = x(-t)$.

- **Amplitude Reversal: It is a special case of amplitude scaling when** $\beta = -1$, $y(t) = \beta\, x(t)$

$y(t) = -\, x(t)$.

EXAMPLE



EXAMPLE

1. Consider the signal x(t) shown in Figure (a). Let us now determine the transformed signal y(t) = x(at −b) where a = 2 and b = 1.

Sol:

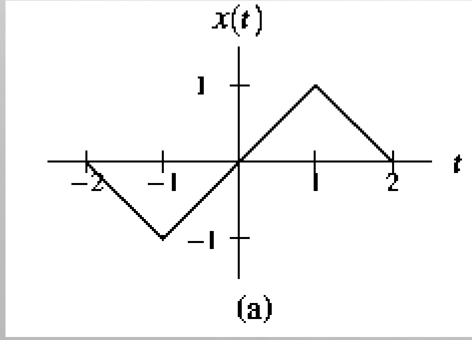
First, we consider the shift-then-scale method. In this case, we first shift the signal *x*(*t*) by *b*=*1*. This yields the signal in Figure (b).

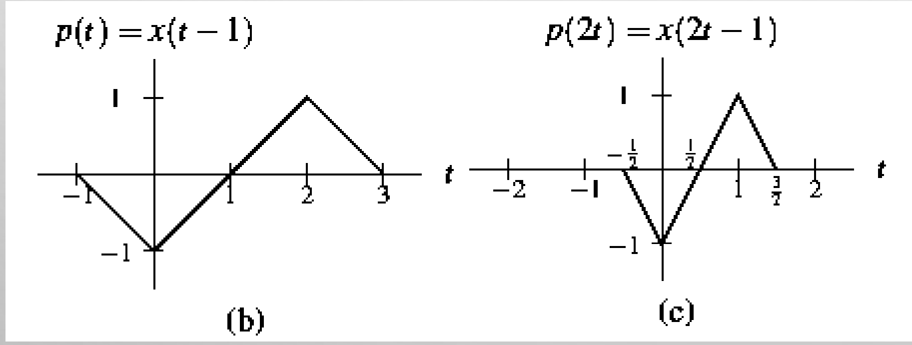
Then, we scale this new signal by *a*=*2*. *in order to obtain y(t) as shown in Figure (c).*

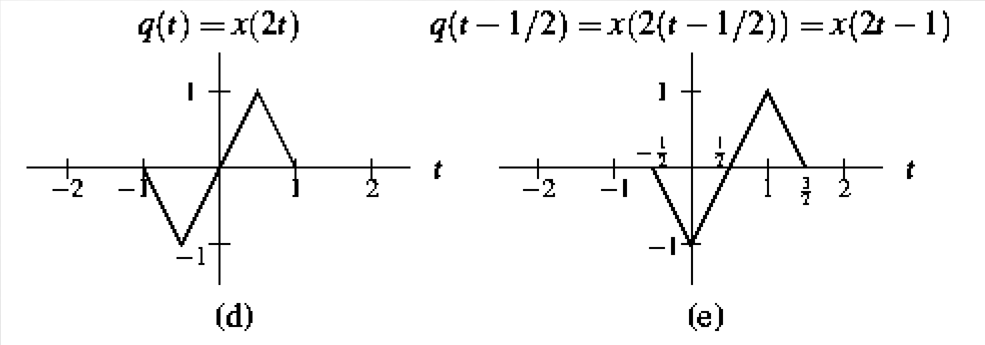
Another sol:

First, we consider the scale-then-shift method. In this case, we first scale the signal *x*(*t*) by *a*=*2*. This yields the signal in Figure (d).

Then, we shift this new signal by *b*=*1*. *in order to obtain y(t) as shown in Figure (e).*







OPERATIONS ON DISCRETE TIME SIGNALS

- **Addition and Subtraction of Discrete-Time Signals**
- In the discrete time case, the sum of two sequences $x_1(n)$ and $x_2(n)$ can be obtained by adding the corresponding sample values. Similarly, the difference of the two sequences $x_1(n)$ and $x_2(n)$ can be obtained by subtracting each sample of one signal from the corresponding sample of the other signal.
- Example:- let $x_1(n) = \{2, 1, 3, 5, 2\}$ and $x_2(n) = \{1, 4, 2, 1, -3\}$ then the addition of discrete-time signals is,

$x_1(n) + x_2(n) = \{2 + 1, 1 + 4, 3 + 2, 5 + 1, 2 - 3\} = \{3, 5, 5, 6, -1\}$

- Similarly, the subtraction of discrete-time signals is,

$x_1(n) - x_2(n) = \{2 - 1, 1 - 4, 3 - 2, 5 - 1, 2 + 3\} = \{1, -3, 1, 4, 5\}$

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OPERATIONS ON DISCRETE TIME SIGNALS

- **Multiplication of Discrete-Time Signals**
- In the discrete time case, the multiplication of two sequences $x_1(n)$ and $x_2(n)$ can be obtained by multiply the corresponding sample values.
- Example:- let $x_1(n) = \{2, 1, 3, 5, 2\}$ and $x_2(n) = \{1, 4, 2, 1, -3\}$ then the multiplication of discrete-time signals is,

$x_1(n)\, x_2(n)$ = {2 * 1, 1 * 4, 3 * 2, 5 * 1, 2 * -3} = {2, 4, 6, 5, −6}

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OPERATIONS ON DISCRETE TIME SIGNALS

- **Scaling** → **Time scaling**
  → **Amplitude scaling**

- **Time scaling:** is simply the multiplication of a time of a signal by a positive or a negative constant. In the case of discrete signals, this operation is usually called ***Compression*** if the magnitude of the multiplying constant is greater than one. If the magnitude of the multiplying constant is less than one, the operation is called ***Expansion*****.** Thus, if $x[n]$ is an discrete signal, the scaling operation generates a signal $y[n]=x[\alpha n]$, where α is the multiplying constant.

- **Amplitude scaling:** is simply the multiplication of the amplitude of a signal by a positive or a negative constant. In the case of discrete signals, this operation is usually called ***amplification*** if the magnitude of the multiplying constant is greater than one. If the magnitude of the multiplying constant is less than one, the operation is called ***attenuation*****.** Thus, if $x[n]$ is an discrete signal, the scaling operation generates a signal $y[n]= \beta\, x[n]$, where β is the multiplying constant.

OPERATIONS ON DISCRETE TIME SIGNALS

1. Time compression (decimation)

EX: X[n]={5,3,7,**<u>8</u>**,-2,4,9} a=2

Sol: x[2n] when n=0 ⟶ x[2*0]=x[0]=8

When n=1 ⟶ x[2*1]=x[2]=4, when n=2 ⟶ x[2*2]=x[4]=0

When n=-1 ⟶ x[2*-1]=x[-2]=3, when n=-2 ⟶ x[2*-2]=x[-4]=0 ⟶ X[2n]={3,**<u>8</u>**,4}

2. Time Expansion (Interpolation)

Ex: x[n]={4,**<u>3</u>**,5} a=0.5

Sol: x[0.5n] when n=0 ⟶ x[0.5*0]=x[0]=3

when n=1 ⟶ x[0.5*1]=x[0.5]=0, when n=2 ⟶ x[0.5*2]=x[1]=5

When n=-1 ⟶ x[0.5*-1]=x[-0.5]=0, when n=-2 ⟶ x[0.5*-2]=x[-1]=4

When n=-3 ⟶ x[0.5*-3]=x[-1.5]=0

We stopped here because the result be always zeros

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OPERATIONS ON DISCRETE TIME SIGNALS

1. Amplitude amplification

EX: X[n]={5,3,7,**8**,-2,4,9} a=2

Sol: ax[n] 2x[n]={10,6,14,**16**,-4,8,18}

2. Amplitude attenuation

EX: X[n]={5,3,7,**8**,-2,4,9} a=0.5

Sol: ax[n] 0.5x[n]={2.5,1.5,3.5,**4**,-1,2,4.5}

OPERATIONS ON DISCRETE TIME SIGNALS

- **Shifting** → **Time Shifting**
  → **Amplitude Shifting**

- **Time shifting:** it is mean add a constant to the time of a signal. In the case of discrete signals, this operation is usually called ***Left Shifting (Time Advance)*** if the magnitude of constant is less than zero. If the magnitude of the constant is greater than zero, the operation is called ***Right Shifting (Time Delay).***

  $y[n]=x[n-\alpha]$, where α is the constant.

- **Amplitude Shifting:** it is mean add a constant to a signal. $Y[n]=x[n]+\alpha$, where α is the constant.

OPERATIONS ON DISCRETE TIME SIGNALS

1. Right Shifting (Time Delay)

Ex: x[n]={2,3,**<u>2</u>**,1,3}

Sol: x[n-2] when n=0 x[0-2]=x[-2]=2, when n=1 x[1-2]=x[-1]=3

When n=2 x[2-2]=x[0]=2, when n= 3 x[3-2]=x[1]=1

When n=4 x[4-2]=x[2]=3, when n=-1 x[-1-2]=x[-3]=0

x[n-2]={**<u>2</u>**,3,2,1,3}

2. Left Shifting (Time Advance)

Ex: x[n]={2,3,**<u>2</u>**,1,3}

Sol: x[n+2] when n=0 x[0+2]=x[2]=3, when n=1 x[1+2]=x[3]=0

When n= -1 x[-1+2]=x[1]=1, when n= -2 x[-2+2]=x[0]=2

When n= -3 x[-3+2]=x[-1]=3, when n= -4 x[-4+2]=x[-2]=2

x[n+2]={2,3,2,1,**<u>3</u>**}

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OPERATIONS ON DISCRETE TIME SIGNALS

1. Upward shifting

EX: X[n]={5,3,7,**8**,-2,4,9} a=2

Sol: x[n]+a x[n]+2 ={7,5,9,**10**,0,6,11}

2. Downward shifting

EX: X[n]={5,3,7,**8**,-2,4,9} a= -2

Sol: x[n]-a x[n]-2 ={3,1,5,**6**,-4,2,7}

OPERATIONS ON DISCRETE TIME SIGNALS

- **Reversal (fold)** → **Time Reversal**
  → **Amplitude Reversal**

- **Time Reversal:** It is a special case of time scaling when $\alpha = -1$, $y[n] = x[\alpha\, n]$

  $y[n] = x[-n]$.

- **Amplitude Reversal:** It is a special case of amplitude scaling when $\beta = -1$, $y[n] = \beta\, x[n]$

  $y[n] = -\, x[n]$.

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OPERATIONS ON DISCRETE TIME SIGNALS

- TIME REVERSAL when a = -1

Ex: x[n]={1,**2**,4,5}

Sol:

x[a*n] → x[-n] → when n=0 → x[-1*0]=x[0]=2

When n=1 → x[-1*1]=x[-1]=1, when n=2 → x[-1*2]=x[-2]=0

When n= -1 → x[-1*-1]=x[1]=4, when n=-2 → x[-1*-2]=x[2]=5

x[-n]={5,4,**2**,1}

- AMPLITUDE REVERSAL when a = -1

EX: X[n]={5,3,7,**8**,-2,4,9} a= -1

Sol: ax[n] -x[n]={-5,-3,-7,**-8**,2,-4,-9}

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EXAMPLE

IF X(N) = {2, 3 ,4 , **5**, 6, 7} . FIND 1. Y(N)=X(N-3) 2. X(N+2) 3. X(-N) 4. X(-N+1) 5. X(-N-2)

SOL:

1. Y(N)= X(N-3) = {**2**,3,4,5,6,7} SHIFT X(N) RIGHT 3 UNITS.

2. X(N+2) = { 2,3,4,5,6,**7**} SHIFT X(N) LEFT 2 UNITS.

3. X(-N) = { 7,6,**5**,4,3,2} FOLD X(N) ABOUT N=0.

4. X(-N+1) = { 7,6,5,**4**,3,2} FOLD X(N), DELAY BY 1.

5. X(-N-2) = { **7**,6,5,4,3,2} FOLD X(N), ADVANCED BY 2.

PARSEVAL'S THEOREM

$$\int_{-\infty}^{\infty} w_1(t) w_2 {}^*(t)\,dt = \int_{-\infty}^{\infty} W_1(f) W_2 {}^*(f)\,df$$

If $w_1(t) = w_2(t) = w(t), \ \int_{-\infty}^{\infty} |w(t)|^2\,dt = \int_{-\infty}^{\infty} |W(f)|^2\,df \equiv E.$

E is the energy of $w(t)$.

Energy spectral density (ESD) : $\mathcal{E}(f) = |W(f)|^2$ (unit = joules per hertz)

Total normalized energy : $E = \int_{-\infty}^{\infty} |\mathcal{E}(f)|^2\,df$

ENERGY AND POWER SIGNALS

We will use abroad definition of power and energy that applies to any signal x(t) or x[n]

***Signal Energy**

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$ and in discrete time $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

- Has $\mathbf{0} < E < \infty$; $P = 0$ A signal will have finite amount of energy if it is absolutely integrable

***Signal Power**

$P = \lim_{t \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$ and in discrete time $P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} |x[n]|^2$

Has $\mathbf{0} < p < \infty$; $E = \infty$

- A signal can be an energy signal, a power signal or neither type.

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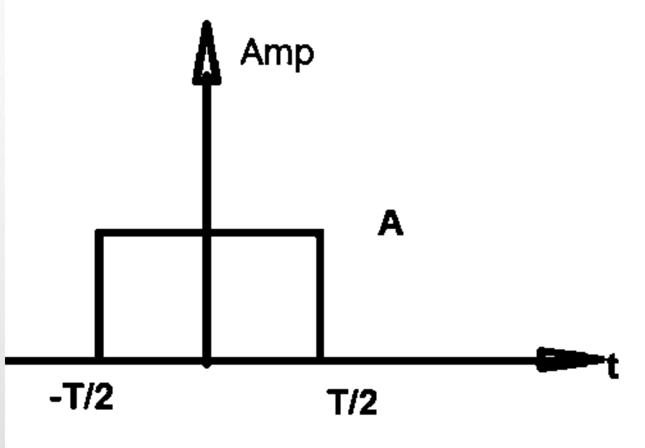
SOLVED PROBLEMS

Example: find the energy of a rectangular plus shown in figure below :-

$$x(t) = \begin{cases} A & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & other\ wise \end{cases}$$



Sol:-

$$E=\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2\, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} |A|^2\, dt$$

$$= A^2 . T \Big|_{-T/2}^{T/2} = A^2 [t/2-(-t/2)]$$

$$E = A^2 . T$$

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