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### DIGITAL SIGNAL PROCESSING 3<sup>rd</sup> YEAR

BY

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# CONTINUOUS AND DISCRETE-TIME SYSTEMS

- A **continuous-time system** is a system in which continuous-time input signals are applied and result in continuous-time output signals.
- A **discrete-time system** is a system in which discrete-time input signals are applied and result in discrete-time output signals.
- Systems might be with one input and one output i.e. single-input, single-output (SISO) systems.



# CONTINUOUS AND DISCRETE-TIME SYSTEMS

- y = Sx or y = S(x), meaning the system S acts on an input signal x to produce output signal y.
- y = Sx does not (in general) mean multiplication!
- Systems often denoted by block diagram:
- Examples (with input signal x and output signal y)
- **1- Scaling system:** y(t) = ax(t)
- Called an amplifier if |a| > 1.
- Called an attenuator if |a| < 1.
- Called inverting if a < 0.

a is called the gain or scale factor. SometimeS DENOTED BY TRIANGLE OR CIRCLE IN BLOCK DIAGRAM:





## CONTINUOUS AND DISCRETE-TIME SYSTEMS

**2- Differentiator System:** 

 $\mathbf{y}(\mathbf{t}) = \mathbf{x}'(\mathbf{t})$ 

**3- Integrator System:** 





 $y(t) = \int_{a}^{t} x(\tau) d\tau$ , (a is often 0 or  $-\infty$ )

4- Time Shift System:

 $\mathbf{y}(\mathbf{t}) = \mathbf{x}(\mathbf{t} - \mathbf{T})$ 



• called a **predictor system** if T > 0



#### **5-** Convolution system:

$$y(t) = \int x(t-\tau)h(\tau)d\tau$$

Where h is a given function (you'll be hearing much more about this!).

**Examples (with multiple inputs,** inputs  $x_1(t)$ ,  $x_2(t)$ , and Output y(t))

**1- Summing system:** 

 $y(t) = x_1(t) + x_2(t)$ 

**2- Difference system:** 

 $y(t) = x_1(t) - x_2(t)$ 

**3- Multiplier system:** 

 $\mathbf{y}(t) = \mathbf{x}_1(t)\mathbf{x}_2(t)$ 







# INTERCONNECTION OF SYSTEMS

**1-** Series or cascade interconnection.

The output of System 1 is the input to System 2.



#### 2- Parallel interconnection.

The same input signal is applied to Systems 1 and 2.



## INTERCONNECTION OF SYSTEMS

**3-** Combination of both cascade and parallel interconnection.



4- Feedback interconnection.

The output of System 2 is fed back and added to the external input to produce the actual input to System 1.



#### 1. Systems with and without memory

a. Static systems or memory less system. if the output depends only on the present input. Similar to combinational circuits.

**Example:** y(t) = 2 x(t)

For present value t=0, the system output is y(0) = 2x(0). Here, the output is only dependent upon present input.

Hence the system is memory less or static.

**b**. **Dynamic systems or memory.** If the output signal that depends on inputs in the past or future in addition to the instant value. Similar to sequential circuits.

**Example:** y(t) = 2 x(t) + 3 x(t-3)

For present value t=0, the system output is y(0) = 2x(0) + 3x(-3). Here x(-3) is past value for the present input for which the system requires memory to get this output.

Hence, the system is a dynamic system.

#### 2. Invertible and non-invertible systems

a. <u>Invertible systems</u>. A system is called invertible, if another system exists that is called inverse that, when cascaded with the original system, yields an output equal to the input to the first system. if the input signal can be recovered from the output signal.

#### Example

y(t) = x(t) + 2

Sol:- if we subtitute any number in x we get a number in y without repeated this called invertible system

#### **Invertible systems property:**

If we multiply the system with the inverse of it the result be the same input and then the gain equal to 1. as shown bellow:

 $Y(S) = X(S) \cdot H1(S) \cdot H2(S) = X(S) H1(S) \cdot 1/(H1(S))$ Since H2(S) = 1/(H1(S))

 $\therefore \mathbf{Y}(\mathbf{S}) = \mathbf{X}(\mathbf{S})$ 

Hence, the system is invertible.



**b.** <u>Non-invertible systems.</u> A system is called non-invertible, if no system exists that, when cascaded with the original system, yields an output equal to the input to the first system.

#### **Example**

Consider the system  $y[n] = x^2[n]$ . This is non-invertible, since from knowledge of the output we cannot determine the sign of the input.

#### **3-** Causal and non-causal systems

a. <u>Causal or non-anticipative systems</u>. The output at instant  $\mathbf{t}$  for continuous-time systems or at instant  $\mathbf{n}$  for discretetime systems depends on the value of input at the <u>same</u> instant  $\mathbf{t}$  or  $\mathbf{n}$  and on <u>past</u> values of the input but <u>not on future</u> values of the input.

**b.** <u>Non-causal or anticipative systems.</u> The output at instant **t** for continuous-time systems or at instant **n** for discrete-time systems depends <u>on future</u> values of the input.

### EXAMPLES

1. y(T) = 2 x(t) + 3 x(t-3) For present value t=1. Is it causal or not?

#### Sol:

the system output is y(1) = 2x(1) + 3x(-2). Here, the system output only depends upon present and past inputs. Hence, the system is causal.

2. y(T) = 2 x(t) + 3 x(t+3) For present value t=1. Is it causal or not ?

#### Sol:

the system output is y(1) = 2x(1) + 3x(4). Here, the system output only depends upon present and future inputs.

Hence, the system is non causal.

#### 4- Stable and Unstable Systems

**a.** <u>Stable systems.</u> A stable system is one in which all bounded inputs lead to bounded outputs.
<u>Example</u>

consider the system y(t) = x(t)+2

Let the input is u(t) (unit step bounded input) then the output  $y(t) = u^2(t) = u(t) = bounded$  output.

Hence, the system is stable.

**b.** <u>Unstable systems.</u> An unstable system is one in which not all bounded inputs lead to bounded outputs.
<u>Example</u>

consider the system y(t) = t.u(t), with u(t) the unit step function. In that case the input u(t)=1 but the output y(t) increases without bound as n increases.

#### 5- Time-invariant and time- variant systems

a. <u>Time-invariant (TI) systems.</u> A system is time invariant if the system's output is the same, given the same input signal, regardless of time.

Conceptually, a system is TI if the behavior and the characteristics of the system are fixed over time. For example the system  $y(t) = \alpha x(t)$ .

**b.** <u>Time-variant systems.</u> A time-varying system is one in which if y(t) is the output when the input x(t) is applied, then  $y(t-t_0)$  is not necessarily the output when  $x(t-t_0)$  is applied.

**Example** consider the system y(t) = t x(t). In that case if  $y_1(t)$  is the output to the input  $x_1(t)$ , then the output to the input  $x_2(t) = x_1(t-t_0)$  is  $y_2(t) = t x_2(t) = t x_1(t-t_0) \neq (t-t_0) x_1(t-t_0) = y(t-t_0)$ . Thus, the system is not time-invariant.

#### **6-** Linear and non-linear systems

a. <u>Linear systems.</u> A system is linear if the following two properties hold:

1. Additivity property. If the response to  $x_1(t)$  is  $y_1(t)$  and the response to  $x_2(t)$  is  $y_2(t)$  then the response to the signal  $x_1(t) + x_2(t)$  is  $y_1(t) + y_2(t)$ .

**2. Homogeneity property.** If the response to  $x_1(t)$  is  $y_1(t)$ , then the response to the signal  $\alpha x_1(t)$  is  $\alpha y_1(t)$ . **In another words, linearity means:** 

- Scaling before or after the system is the same.
- Summing before or after the system is the same.

b. Non-linear systems. At least one of the above properties does not hold.

### LINEAR AND NON-LINEAR SYSTEMS



Linear a. Multiplication by a constant



Nonlinear b. Multiplication of two signals

### EXAMPLES OF LINEAR SYSTEMS

- Wave propagation such as sound and electromagnetic waves
- Electrical circuits composed of resistors, capacitors, and inductors
- Electronic circuits, such as amplifiers and filters
- Mechanical motion from the interaction of masses, springs, and dashpots (dampeners)
- Systems described by differential equations such as resistor-capacitor-inductor networks
- Multiplication by a constant, that is, amplification or attenuation of the signal
- Signal changes, such as echoes, resonances, and image blurring
- The unity system where the output is always equal to the input
- The null system where the output is always equal to the zero, regardless of the input
- Differentiation and integration, and the analogous operations of *first difference* and *running sum* for discrete signals
- Small perturbations in an otherwise nonlinear system, for instance, a small signal being amplified by a properly biased transistor
- Convolution, a mathematical operation where each value in the output is expressed as the sum of values in the input multiplied by a set of weighing coefficients.
- Recursion, a technique similar to convolution, except previously calculated values in the output are used in addition to values from the input

### EXAMPLES OF NONLINEAR SYSTEMS

Systems that do not have static linearity, for instance, the voltage and power in a resistor:  $P = V^2R$ , the radiant energy emission of a hot object depending on its temperature:  $R = kT^4$ , the intensity of light transmitted through a thickness of translucent material:  $I = e^{-\alpha T}$ , etc.

Systems that do not have sinusoidal fidelity, such as electronics circuits for: peak detection, squaring, sine wave to square wave conversion, frequency doubling, etc.

Common electronic distortion, such as clipping, crossover distortion and slewing

Multiplication of one signal by another signal, such as in amplitude modulation and automatic gain controls

Hysteresis phenomena, such as magnetic flux density versus magnetic intensity in iron, or mechanical stress versus strain in vulcanized rubber

Saturation, such as electronic amplifiers and transformers driven too hard

Systems with a threshold, for example, digital logic gates, or seismic vibrations that are strong enough to pulverize the intervening rock

### H.W

A discrete-time system is characterized by the following equation  $y(nT + 2T) = e^{nT} + 5x(nT + 2T)$ 

Check the system for (a) linearity, (b) time invariance, and (c) causality.