

AL FURAT AL AWSAT TECHNICAL UNIVERSITY
NAJAF COLLEGE OF TECHNOLOGY
DEPARTMENT OF AVIONICS ENGINEERING

DIGITAL SIGNAL PROCESSING
3rd YEAR

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Discrete Correlation

- **Correlation** or “**Co-Relation**” is a measure of **similarity/ relationship** between two signals.
- If $x[n]$ & $y[n]$ are two discrete-time signals, then the correlation of $x[n]$ with respect to $y[n]$ is given as,

$$r[n] = \sum_{k=-\infty}^{\infty} x[k] h[k - n]$$

where L is lag, indicating time-shift.

Relationship b/w Conv. & Corr.

Mathematically, Convolution between $x[n]$ & $h[n]$ is given as:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

▶ Correlation of $x[n]$ with $h[n]$ is given as:

$$r[n] = \sum_{k=-\infty}^{\infty} x[k] h[k-n]$$

Relationship b/w Conv. & Corr.

But if we “time-reversed” the second sequence of the Convolution, we end up with Correlation

$$\begin{aligned}y[n] &= \sum_{k=-\infty}^{\infty} x[k] h[-(n-k)] \\ &= \sum_{k=-\infty}^{\infty} x[k] h[-n+k] \\ r[n] &= \sum_{k=-\infty}^{\infty} x[k] h[k-n]\end{aligned}$$

Where, $r[n]$ is the correlation of $x[n]$ with respect to $h[n]$.

Relationship b/w Conv. & Corr.

- ▶ So, we can say that “Correlation, mathematically, is just Convolution, with the second sequence, time-reversed”

$$r[n] = x[n] * h[-n]$$

- ▶ We can use this property to find Correlation, using the same method we used for Convolution, but the second sequence needs to be time-reversed.
- ▶ This only requires that we don't time-reverse for convolution in the first place!

notes for solving the correlation's problems

length of 1st sequence = L_1

length of 2nd sequence = L_2

length of output sequence = $L_1 + L_2 - 1$

notes for solving the correlation's problems

First sequence start at $n = n_1$

Second sequence start at $n = n_2$

output sequence start at $n_1 - (n_2 + L_2 - 1)$

output sequence ends at $SP + (L_1 + L_2 - 2)$

Types of discrete correlation

► There are two types of correlation:

1. **Cross correlation:**

It is a comparison of two different signals.

2. **Auto correlation:**

It is a comparison of the signal with itself at a different time.

Cross Correlation using Graphical method

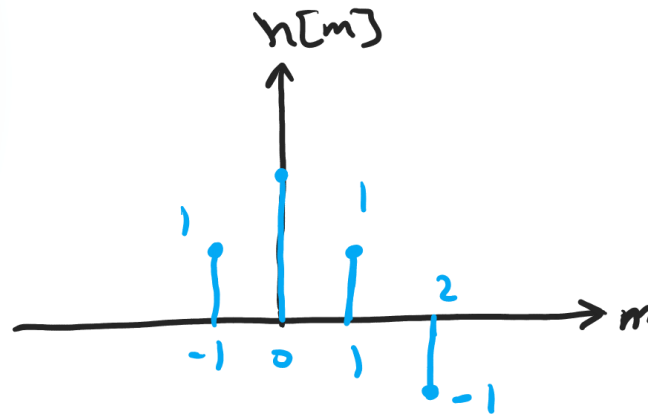
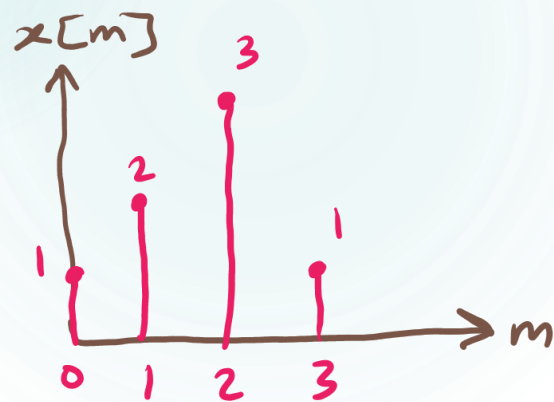
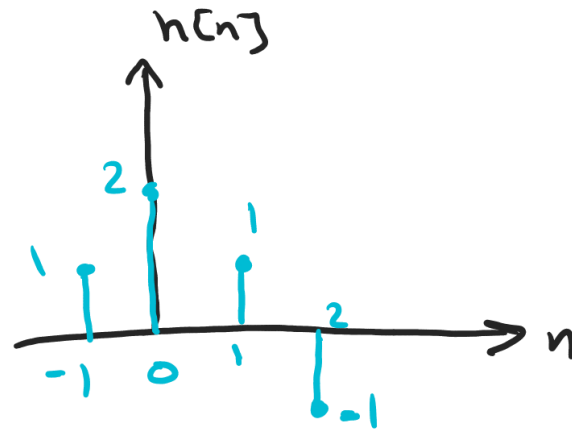
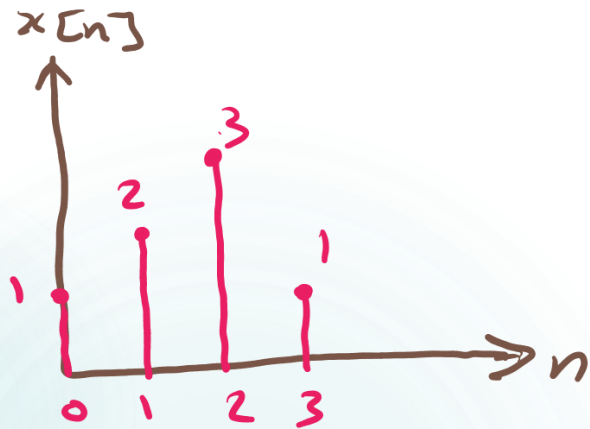
Ex/ Find the cross correlation of the following signals

$$x[n] = \{1, 2, 3, 1\} \quad ; \quad h[n] = \{1, 2, 1, -1\}$$

Cross Correlation

Sol:

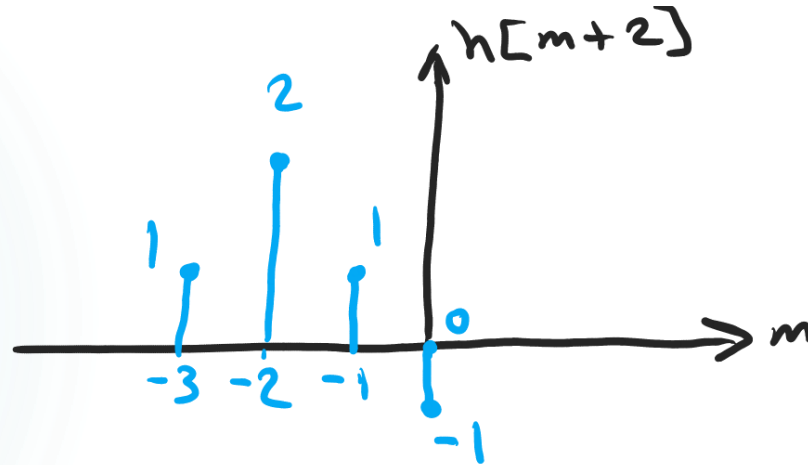
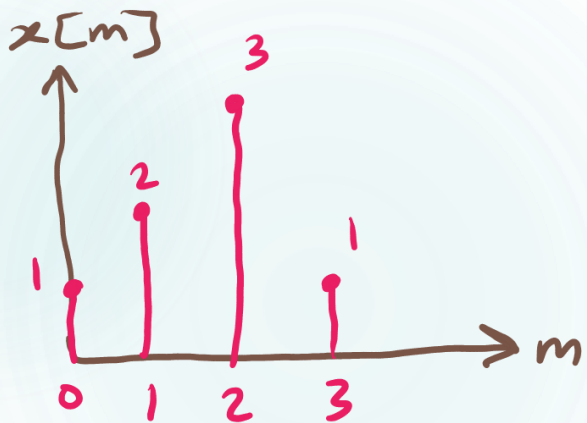
$$L_1=4 \quad L_2=4 \quad L_{\text{output}}=7 \quad n_1=0 \quad n_2=-1 \quad Sp=-2 \quad Ep=4$$



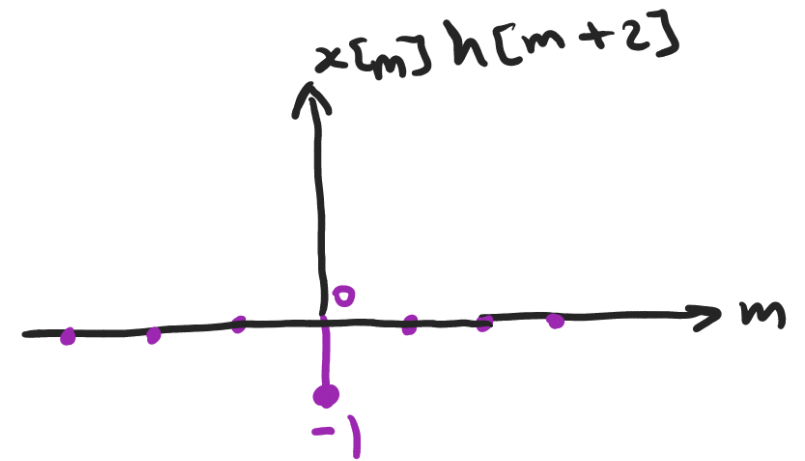
Cross Correlation

$$r_{xh}[\eta] = \sum_{m \rightarrow -\infty}^{\infty} x[m] h[m-\eta]$$

Case 1 when $\eta = -2$

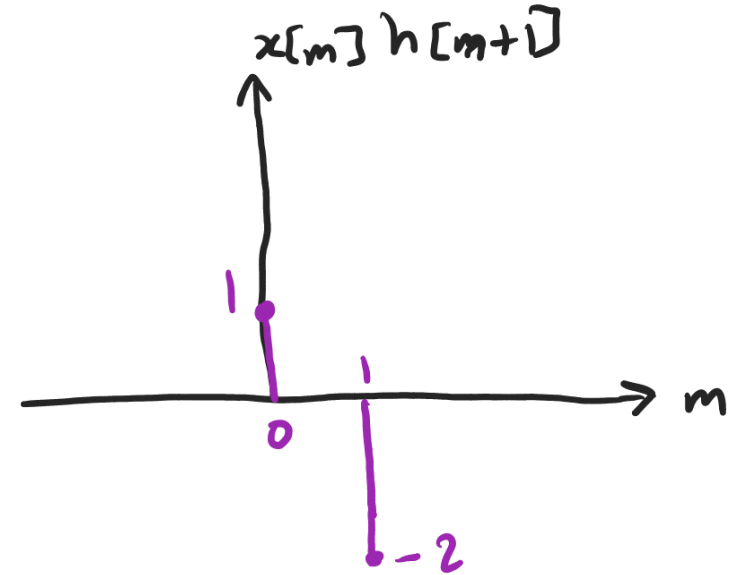
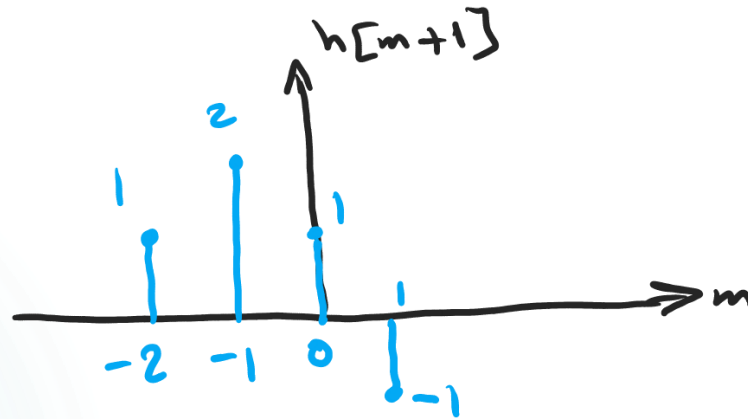
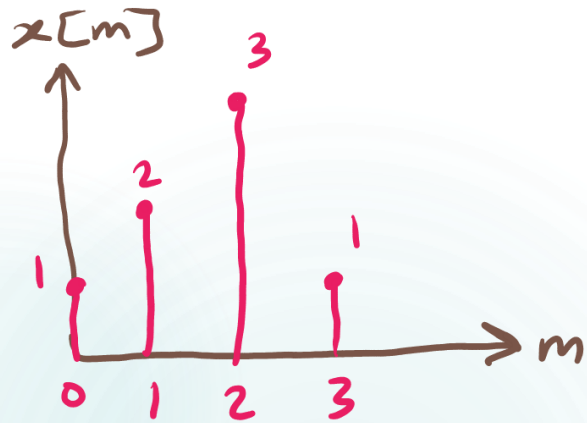


$$r = -1$$



Cross Correlation

Case 2 when $\rho = -1$

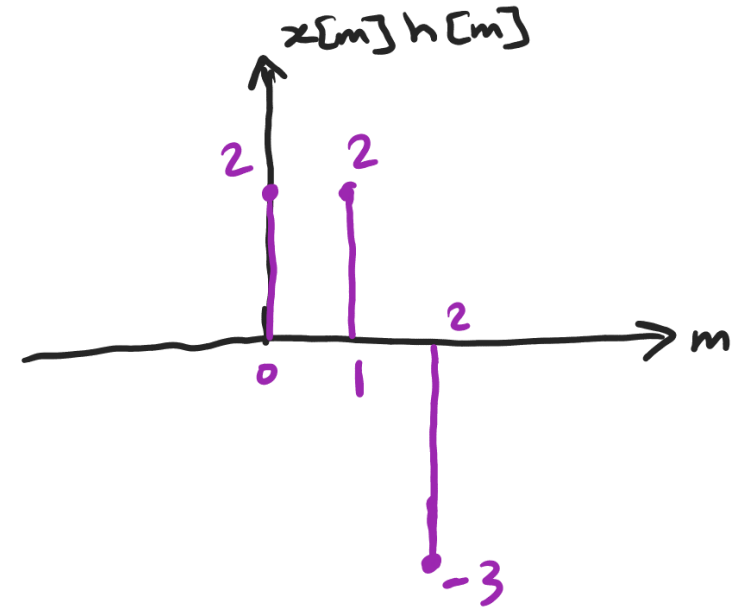
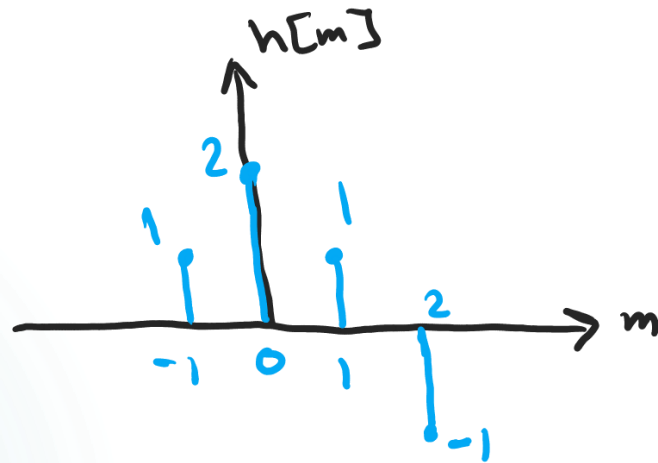
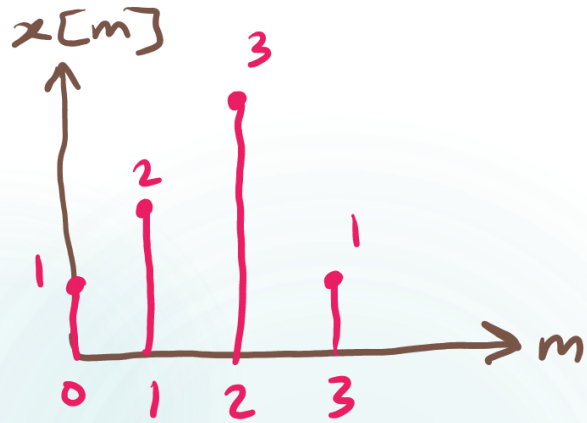


$$r = -2 + 1$$

$$r = -1$$

Cross Correlation

Case 3 when $\tau = 0$

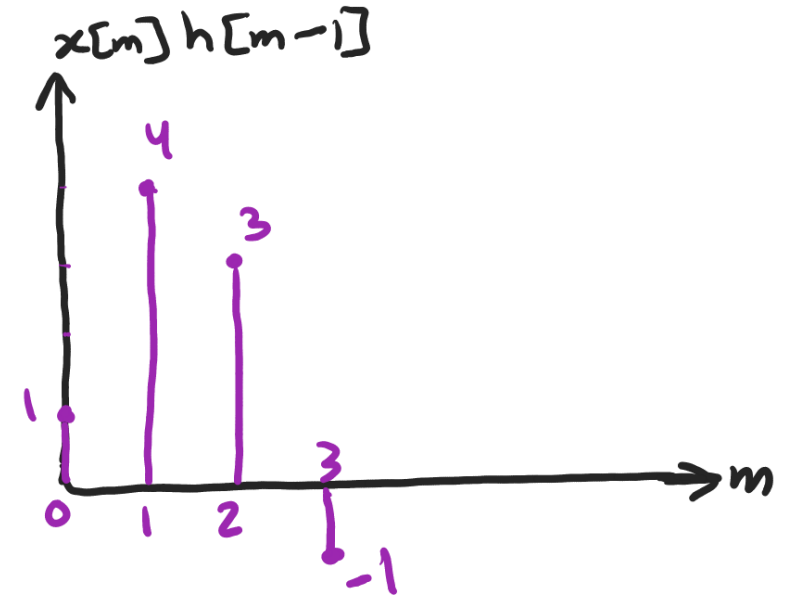
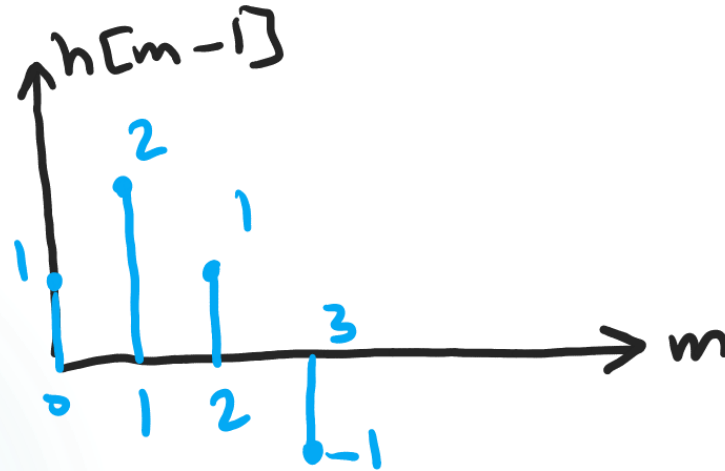
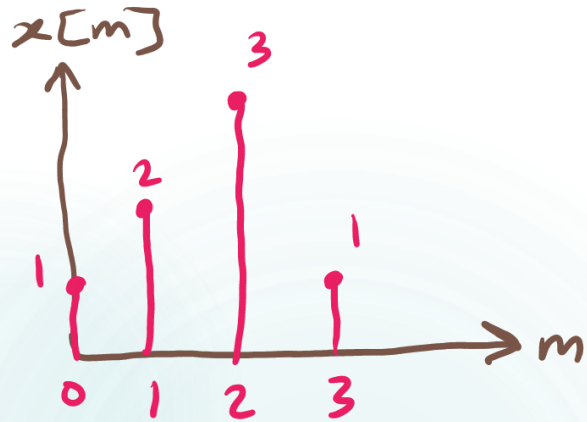


$$r = -3 + 2 + 2$$

$$r = 1$$

Cross Correlation

Case 4 when $\gamma = 1$

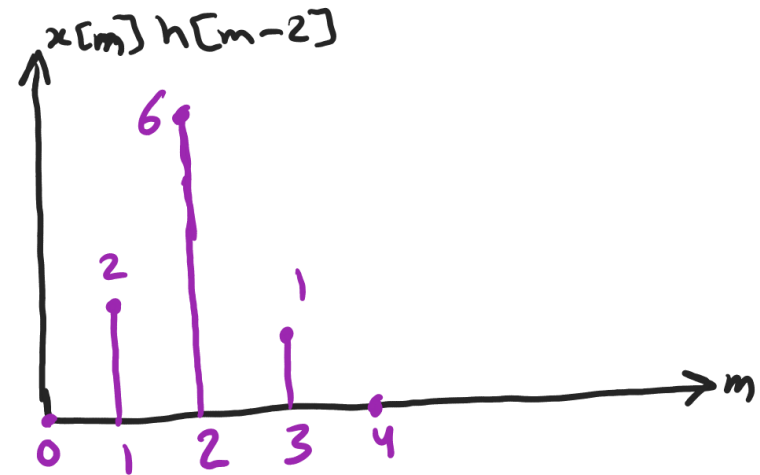
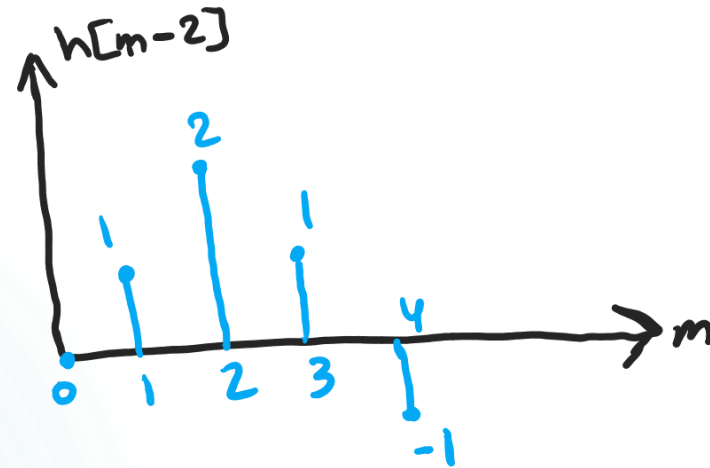
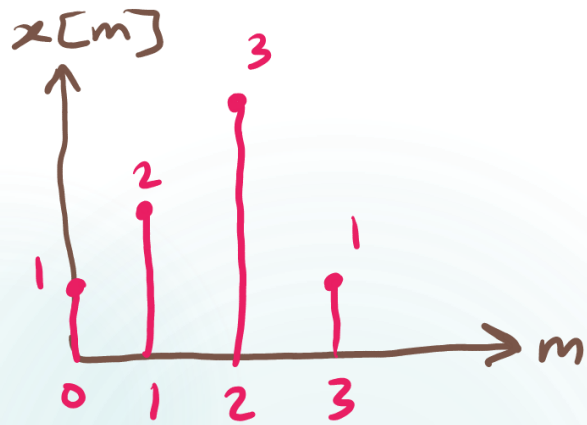


$$r = 1 + 4 + 3 - 1$$

$$r = 7$$

Cross Correlation

Case 5 when $\gamma = 2$

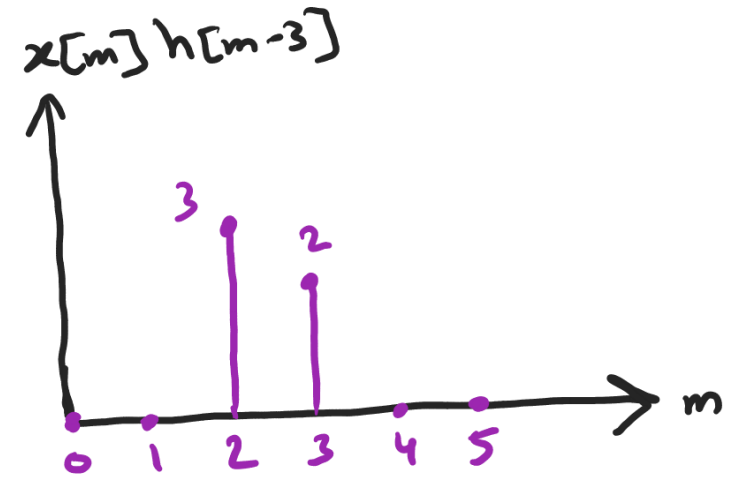
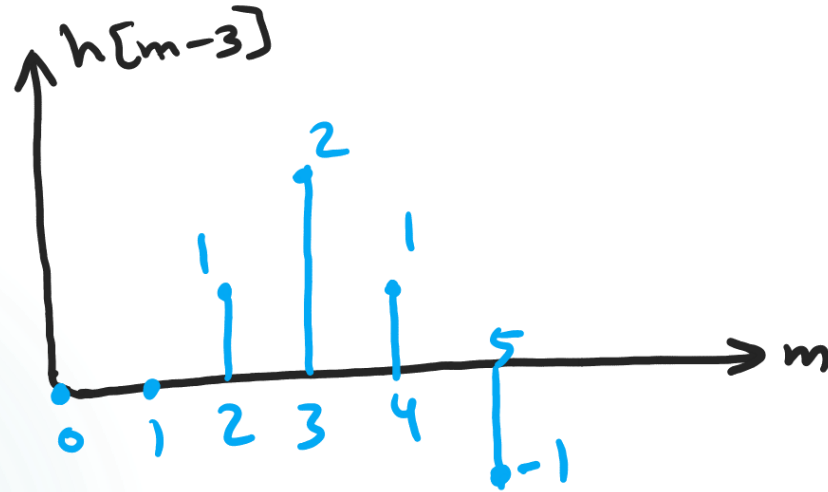
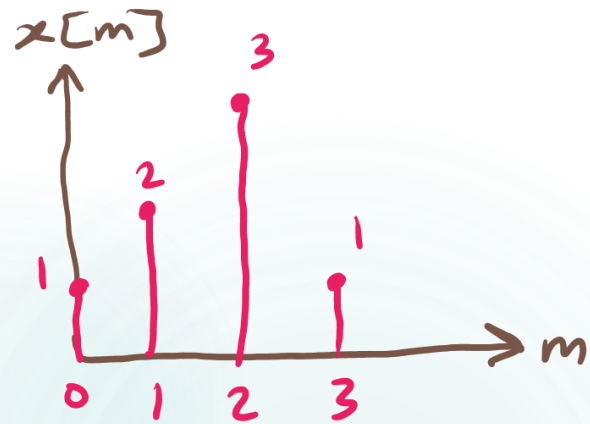


$$r = 2 + 6 + 1$$

$$r = 9$$

Cross Correlation

Case 6 when $\tau = 3$

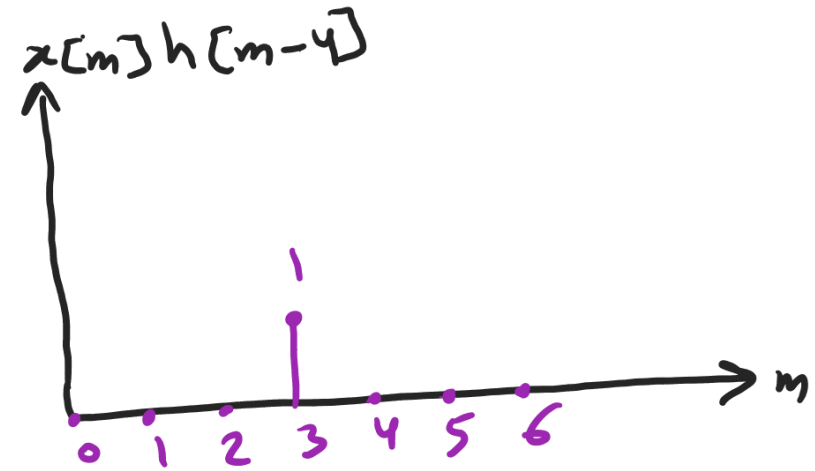
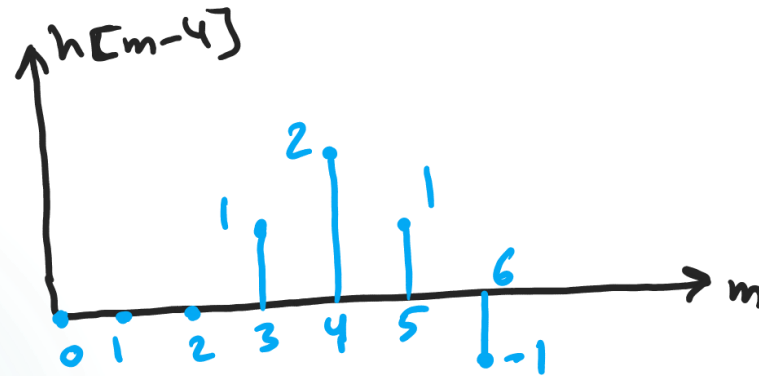
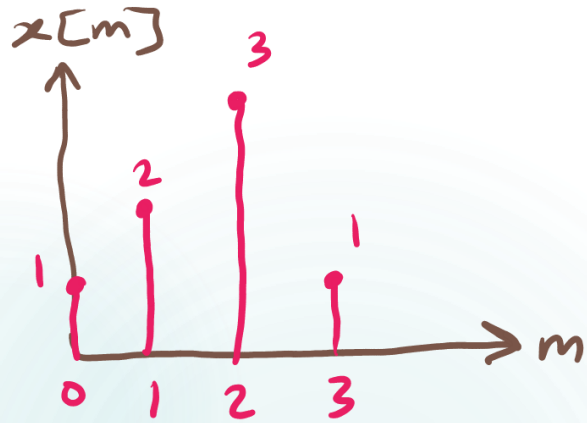


$$r = 3 + 2$$

$$r = 5$$

Cross Correlation

Case 7 when $q=4$



$$r=1$$

$$r_{xh}[q] = \{-1, -1, 1, 7, 9, 5, 13\}$$

↑

Cross Correlation using tabular method

$x[m]$	$h[m]$	1	2	1	-1
1	1	1	2	1	-1
2	2	2	4	2	-2
3	3	3	6	3	-3
1	1	1	2	1	-1

$$r_{xh}[q] = \{-1, -1, 1, 7, 9, 5, 13\}$$

↑

Auto-correlation using graphical method

Ex/ Find the auto correlation of the following sequence

$$x[n] = \{ \underset{\uparrow}{1}, 2, 3, 1 \}$$

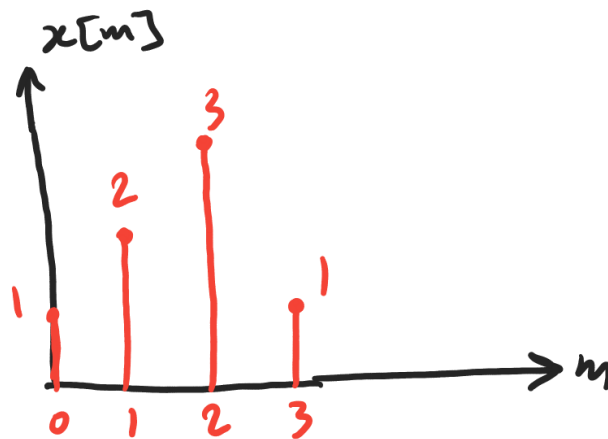
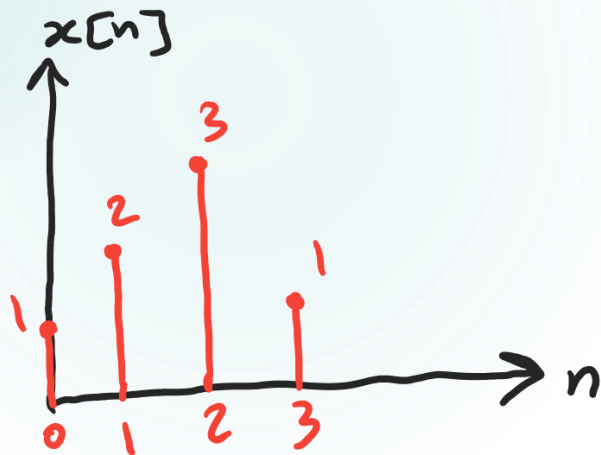
Sol.

$$L_1 = L_2 = 4$$

$$\text{length of o/p} = 7 \quad n_1 = n_2 = 0$$

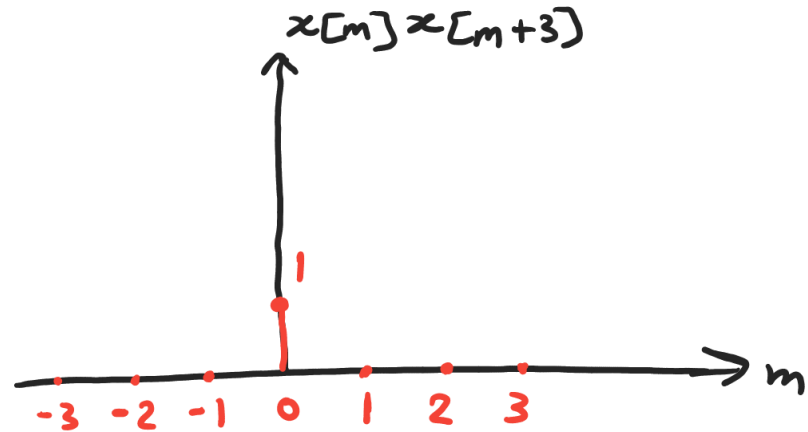
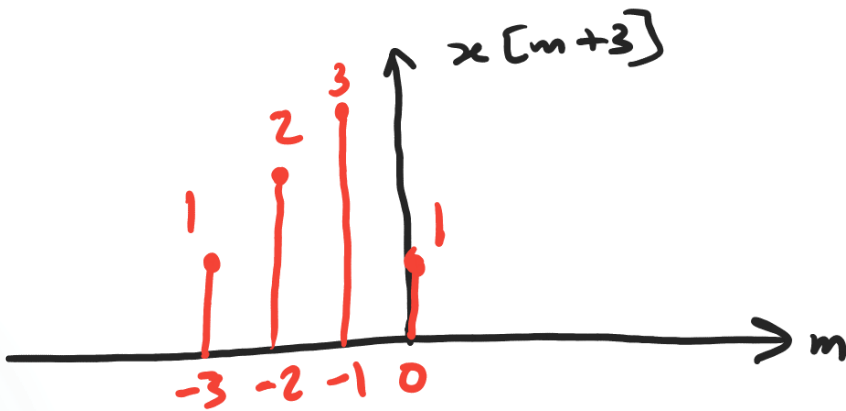
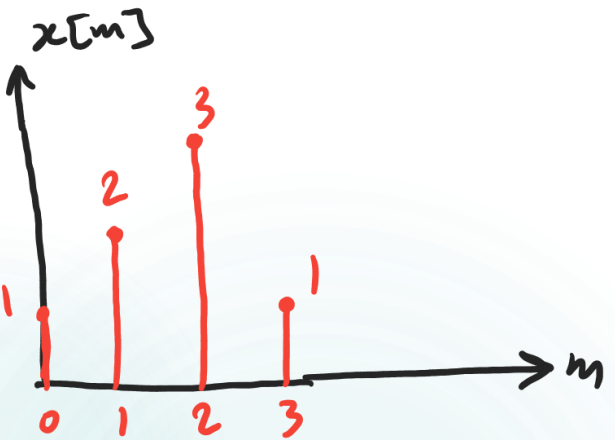
$$SP = -3$$

$$EP = 3$$



Auto-correlation

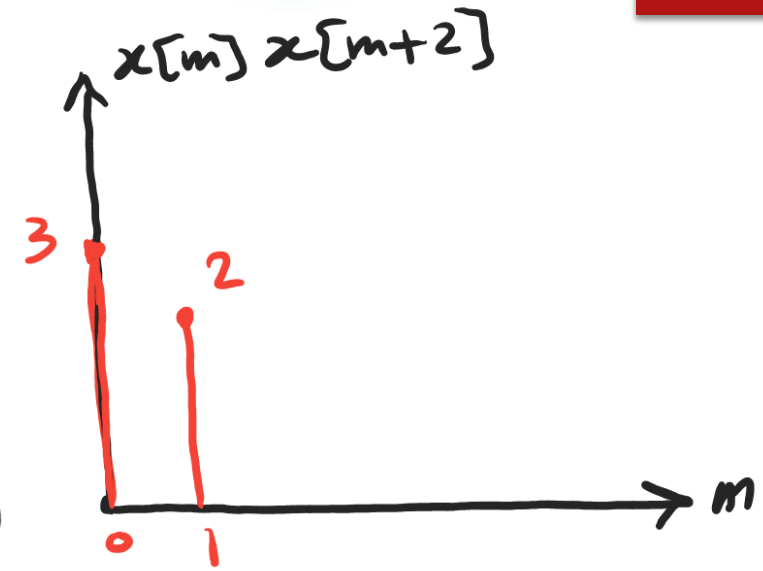
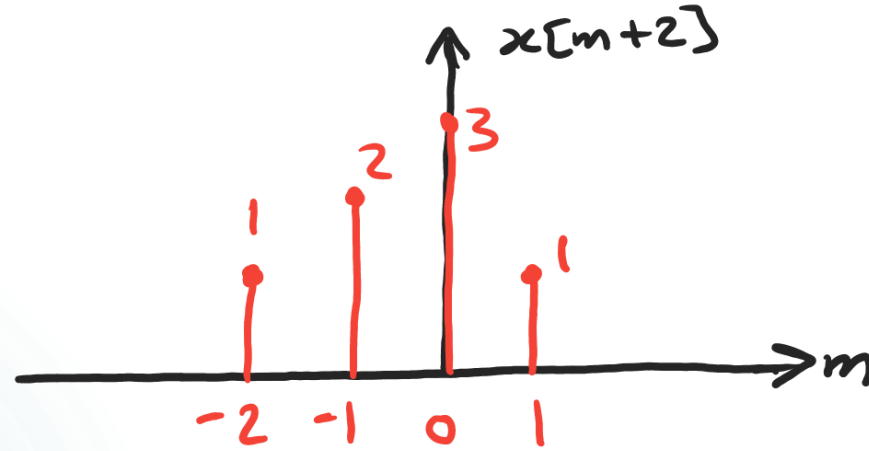
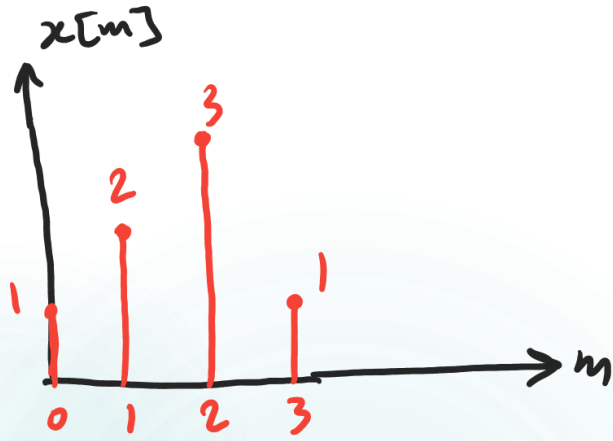
Case 1 when $\tau = -3$



$$r = 1$$

Auto-correlation

Case 2 when $\rho = -2$

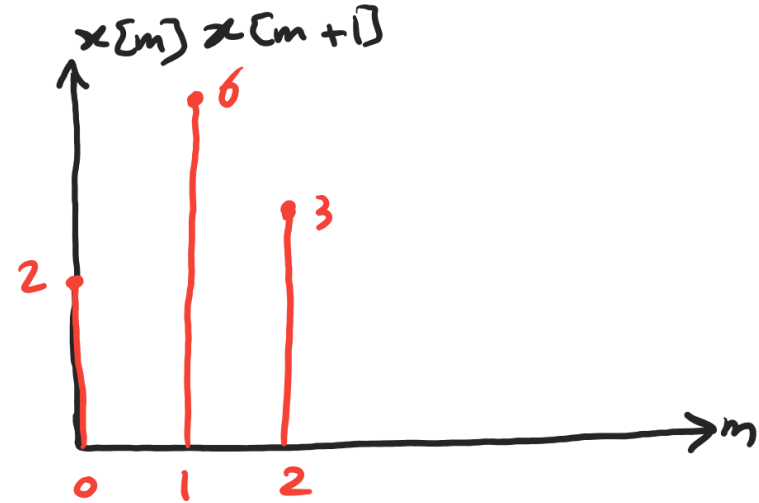
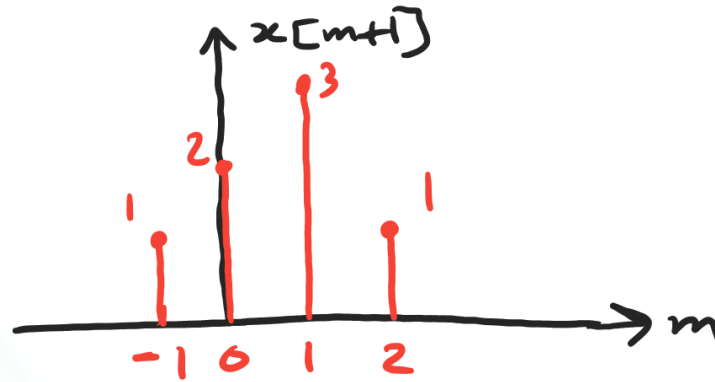
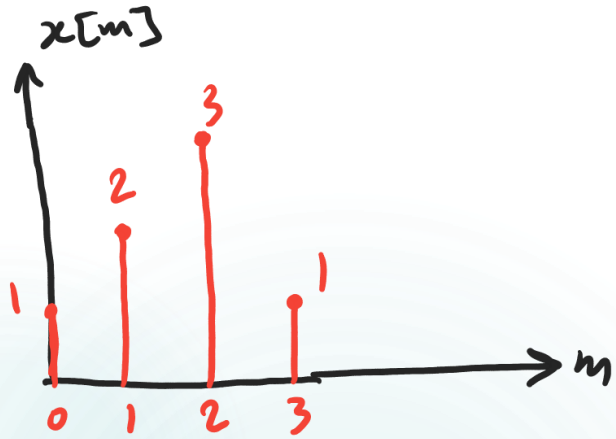


$$r = 3 + 2$$

$$r = 5$$

Auto-correlation

Case 3 when $\rho = -1$

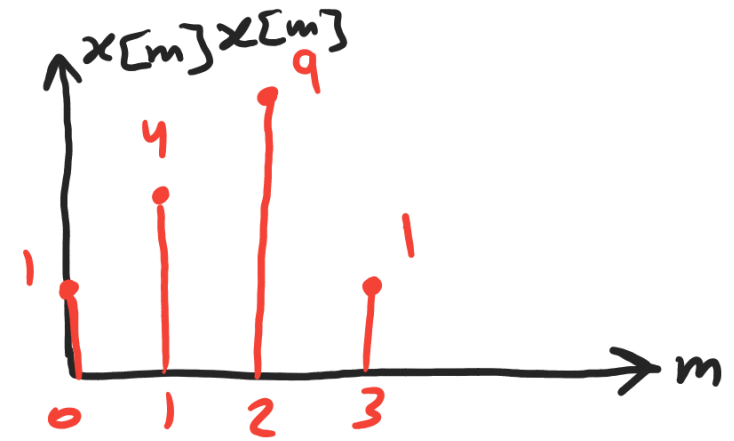
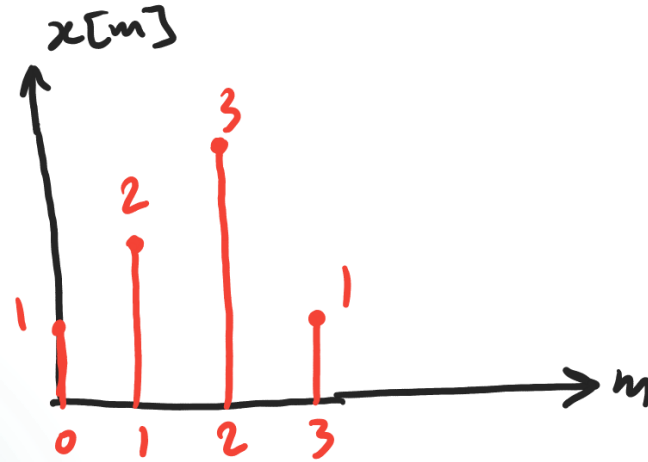
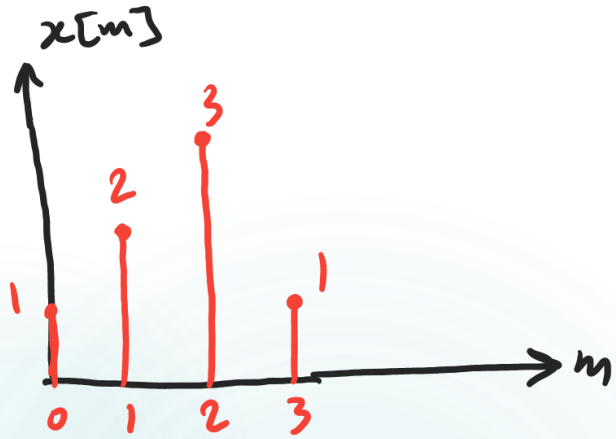


$$r = 6 + 3 + 2$$

$$r = 11$$

Auto-correlation

Case 4 when $\gamma = 0$

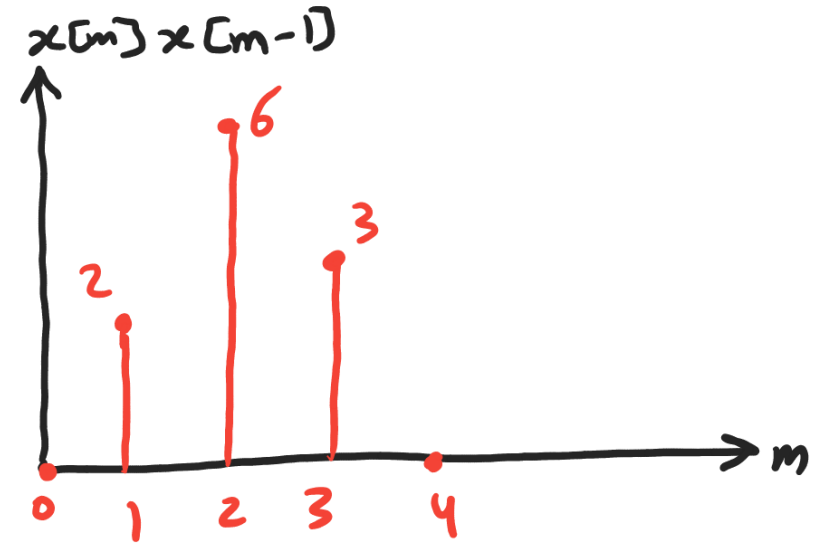
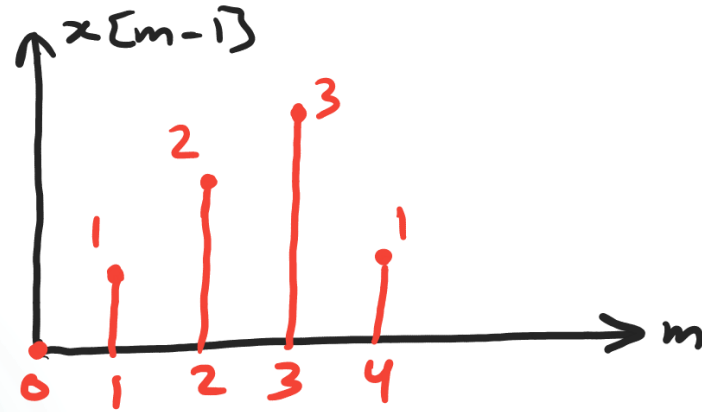
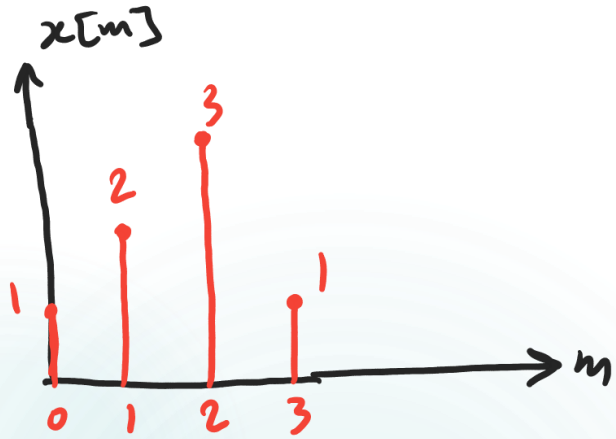


$$r = 1 + 4 + 9 + 1$$

$$r = 15$$

Auto-correlation

Case 5 when $\rho = 1$

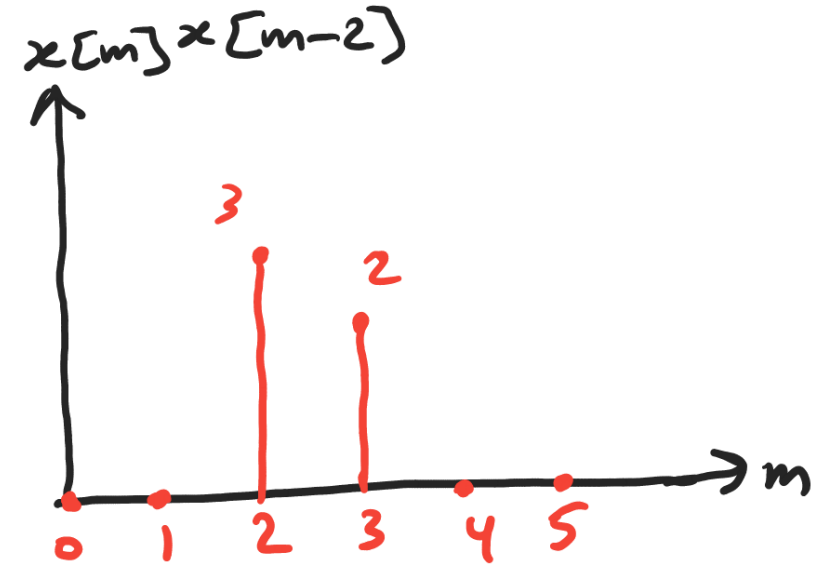
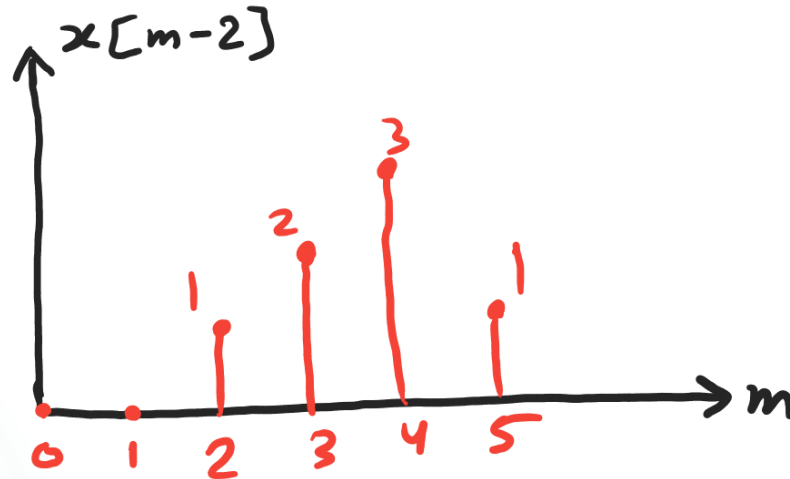
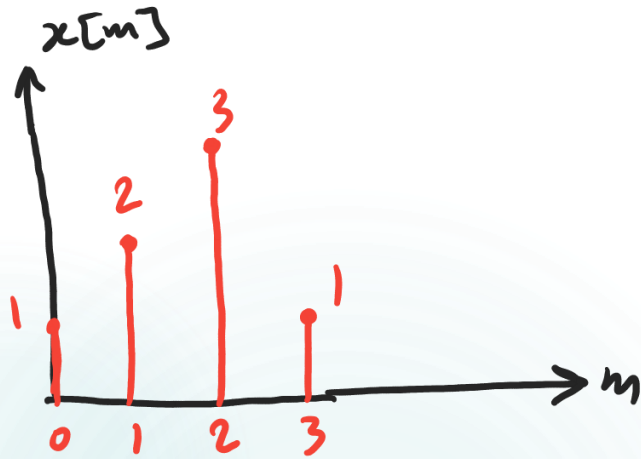


$$r = 2 + 6 + 3$$

$$r = 11$$

Auto-correlation

Case 6 when $q = 2$

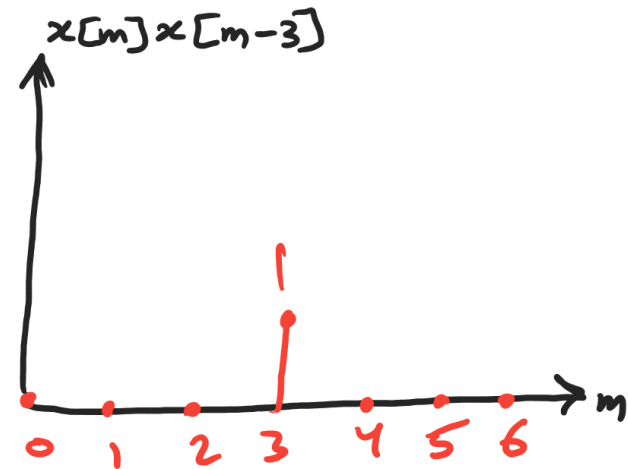
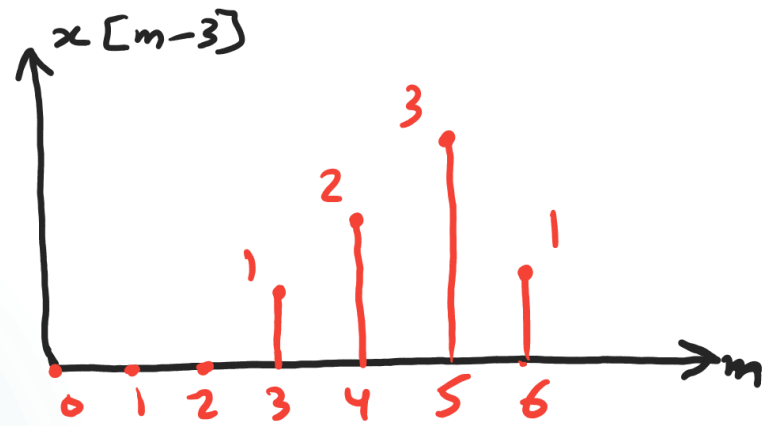
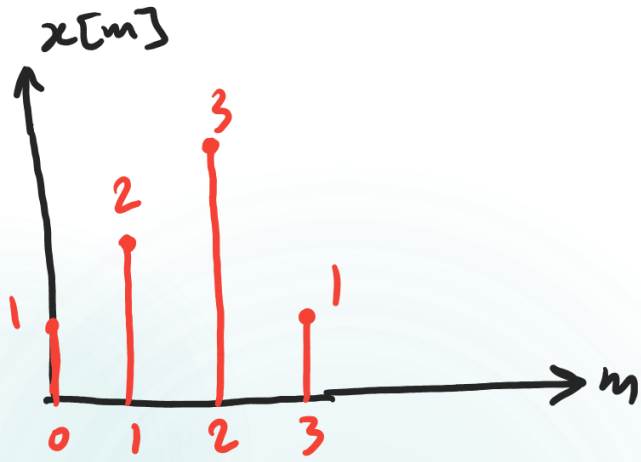


$$r = 3 + 2$$

$$r = 5$$

Auto-correlation

Case 7 when $\gamma = 3$

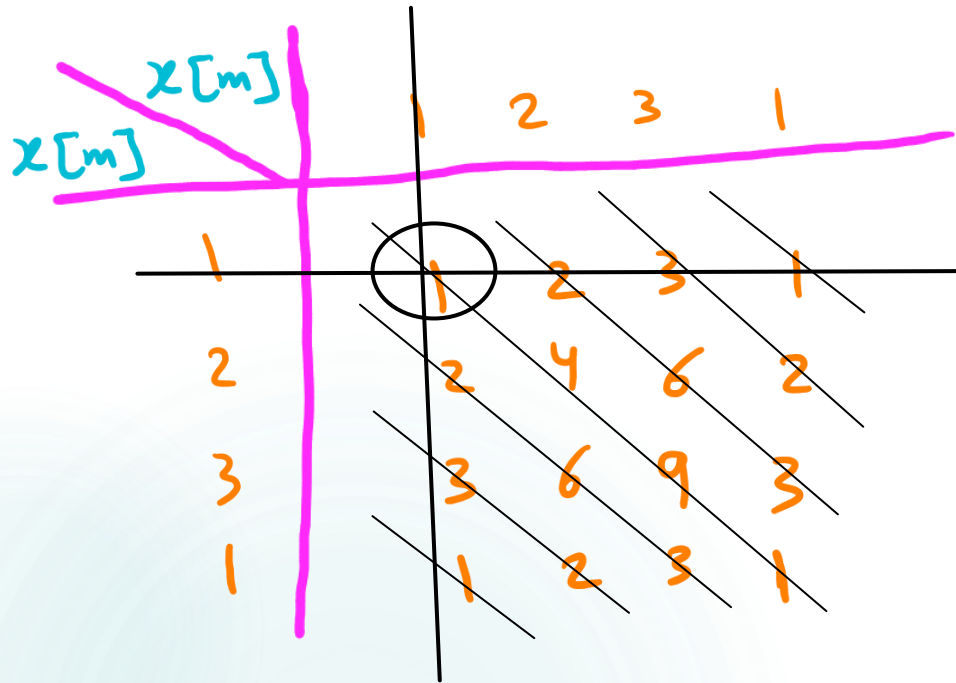


$$r = 1$$

$$r_{xx}[\gamma] = \{1, 5, 11, 15, 11, 5, 1\}$$

↑

Auto-correlation using tabular method



$$r_{xx}[q] = \{1, 5, 11, 15, 11, 5, 1\}$$

↑