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DIGITAL SIGNAL PROCESSING

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BY

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INVERSE Z - TRANSFORM

Given a z-transform function X(z), the corresponding time domain sequence x(n) can be obtained using the **inverse z-transform**.

The inverse z-transform may be obtained by the following methods:

1- Using properties.

2- Partial fraction expansion method.

3- Power series expansion
(the solution is obtained by applying long division because the denominator can't be analyzed. It is not accurate method compared with the above methods)

4- Complex inversion integral

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1. USING PROPERTIES

Example:- Find inverse Z transform of:

$X[Z]=2 + 4\left(\frac{Z}{Z-1}\right) - \frac{Z}{Z-0.5}$

Sol:-

$x[n]= 2\,Z^{-1}(1) + 4.Z^{-1}\left(\frac{Z}{Z-1}\right) - Z^{-1}\left(\frac{Z}{Z-0.5}\right)$

$= 2.\delta[n] + 4.U[n] - (0.5)^{n}.U[n]$

Example:- Find inverse Z transform of:

$X[Z]=\frac{1}{1-0.5Z^{-1}}$

Sol:-

$x[n]= (0.5)^{n}.U[n]$

2. USING PARTIAL FRACTION EXPANSION

Example:-

Find the inverse transform of $X(z) = \frac{z}{3z^2-4z+1}$ for the ROC |z| > 1

Sol:-

Dividing both sides by z leads to

$$\frac{X(z)}{z} = \frac{1}{3z^2-4z+1} = \frac{1}{3(z^2-\frac{4}{3}z+\frac{1}{3})} = \frac{1}{3(z-1)(z-\frac{1}{3})} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{3}}$$

$$A= \frac{X(z)}{z}(z-1)\,|\rightarrow \quad z=1 \quad \rightarrow \quad \frac{1}{3(z-\frac{1}{3})}; \qquad A=1/2$$

$$B= \frac{X(z)}{z}(z-\frac{1}{3})\,|\rightarrow \; z=\frac{1}{3} \quad \rightarrow \quad \frac{1}{3(z-1)}, \qquad B= -1/2$$

Therefore,

$$\frac{X(z)}{z} = \frac{1/2}{z-1} + \frac{-1/2}{z-\frac{1}{3}}, \qquad X(z) = \frac{\frac{1}{2}z}{z-1} + \frac{\left(-\frac{1}{2}\right)z}{z-\frac{1}{3}}$$

∴ $x(n)= \frac{1}{2}u(n) - \frac{1}{2}\left(\frac{1}{3}\right)^n u(n)$

3. USING POWER SERIES EXPANSION (LONG DIVISION METHOD)

Example:-

Find the inverse z-transform sequence of the following signal:

$$X(z) = \frac{z^2 + z}{z^2 - 3z + 4}$$

Sol:-

Represent the z-transform function X(z) in terms of z^{-1} by dividing z^2 for both numerator and denominator.

$$X(z) = \frac{z^2+z}{z^2-3z+4} = \frac{1+z^{-1}}{1-3z^{-1}+4z^{-2}}$$

$$1+4z^{-1}+8z^{-2}+8z^{-3}$$

$$1-3z^{-1}+4z^{-2}\,)\; 1+z^{-1}$$

$$1-3z^{-1}+4z^{-2}$$

$$4z^{-1}-4z^{-2}$$

$$4z^{-1}-12z^{-2}+16z^{-3}$$

$$8z^{-2}-16z^{-3}$$

$$8z^{-2}-24z^{-3}+32z^{-4}$$

$$8z^{-3}-32z^{-4}$$

By examination, the sequence x(n) is;

X(0) = 1, x(1) = 4, x(2) = 8, x(3) = 8, ……

The long division procedure used in the example above can be carried out to any desired number of steps.

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4. USING COMPLEX INVERSION INTEGRAL

The integral is a contour integral over a closed path C that must
- enclose the origin,
- lie in the ROC of X(z).

$$x[n] = \frac{1}{2\pi j}\oint X(z)\, z^{n-1}\, dz$$

Practical problems requiring inverse z-transform:

- Given a system function H(z), e.g., described by a pole-zero plot, find h[n].
  This is particularly important since we will design filters in the z-domain.

- When performing convolution via z-transforms: Y (z) = H(z) X(z), leading to y[n].

Poles and Zeros

- When X(z) is represented as a numerator and denominator of polynomial (N(z)/D(z)).

$$X(z)=\frac{N(z)}{D(z)}=\frac{b_0 z^0+b_1 z^{-1}+\ldots+b_M z^{-M}}{a_0 z^0+a_1 z^{-1}+\ldots+a_N z^{-N}}=\frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

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Zero (z) is the value of z where X(z) = 0 ; Pole (p) is the value of z at X(z) = ∞.

$$X(z)=\frac{b_0}{a_0}z^{-M+N}\frac{(z-z_1)(z-z_2)...(z-z_M)}{(z-p_1)(z-p_2)...(z-p_N)}$$

Zero of X(z) at $z = z_1, z_2, \ldots, z_M$; Pole at $z = p_1, p_2, \ldots, p_N$.

Pole-zero is plotted on z-plane with notation (X) as pole and (0) as zero.

Example:-

Determine the pole-zero of signal:

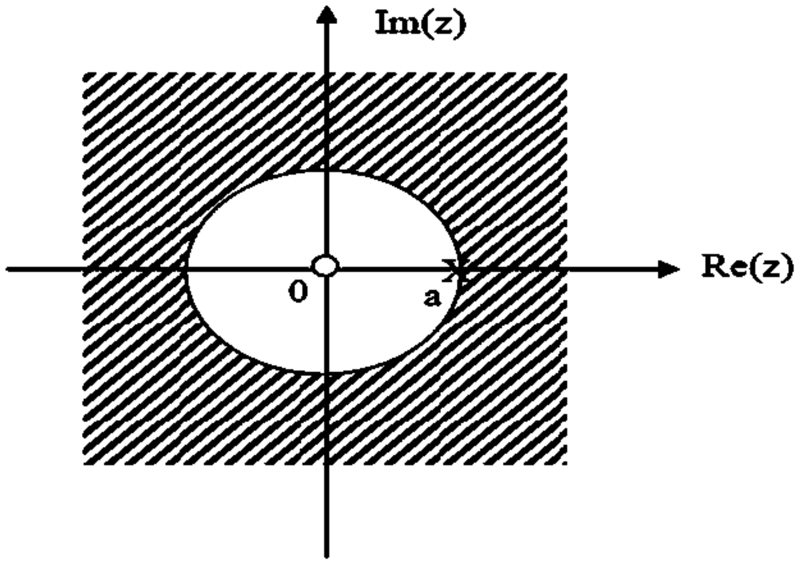
$x(n) = a^n u(n)$; a>0

Sol:-

Transform x(n) into z-equation:

$$x(z) = \frac{1}{1 - az^{-1}} = \frac{z}{z - a} \quad ; \qquad \text{ROC: } |z| > a$$

x(z) has one zero $z_1=0$ and one pole at $p_1=a$.



Pole-zero untuk $x(n) = a^n u(n)$

Pole-Zero Cancellation

If a pole is at the same location as a zero, the characteristics of poles can be cancelled by zero.

Example: Determine the convolution of the two sequences with Z-transform x(n)={2,1,0,0.5} and h(n)={2,2,1,1}

Sol:

Tacking Z-transform of the two sequences

X(z) = 2+Z^{-1}+0.5 Z^{-3}

H(z) = 2+2 Z^{-1}+ Z^{-2}+ Z^{-3}

Y(z)= X(z) H(z)

= (2+Z^{-1}+0.5 Z^{-3})(2+2 Z^{-1}+ Z^{-2}+ Z^{-3})

= 4+6 Z^{-1}+4 Z^{-2}+4 Z^{-3}+2 Z^{-4}+0.5 Z^{-5}+0.5 Z^{-6}

By tacking inverse Z-transform, we get

y(n) = 4$\delta(n)$+6 $\delta(n$-1)+4 $\delta(n$-2)+4 $\delta(n$-3)+2 $\delta(n$-4)+0.5 $\delta(n$-5)+0.5 $\delta(n$-6)

Then y(n) ={4,6,4,4,2,0.5,0.5}

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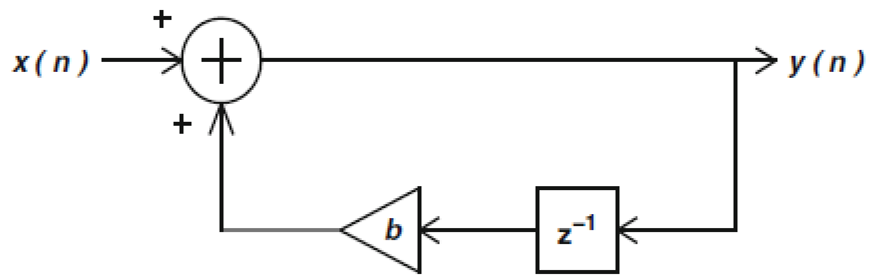
Example: Find the transfer function with Z-transform for the system in fig.



Sol:

$$y(n) = x(n) + by(n-1)$$

where, b is a multiplicative constant (gain). Taking the ZT of both sides

$$Y(z) = X(z) + bz^{-1}Y(z)$$

$$[1 - bz^{-1}]Y(z) = X(z).$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - bz^{-1}} = \frac{z}{z - b}$$

From Tables (z-Transform Pairs), the IZT is

$$h(n) = b^n u(n)$$