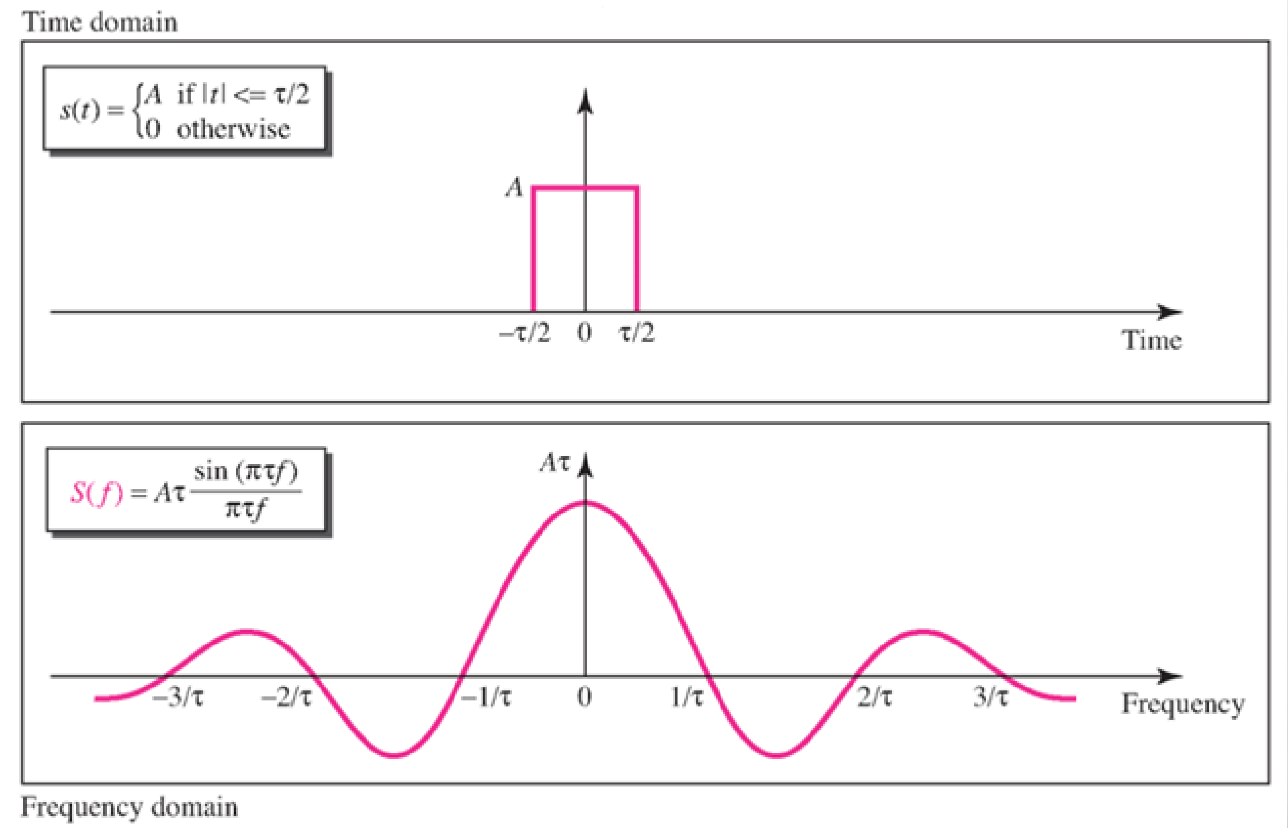
PROBABILITY, SIGNALS & SYSTEMS

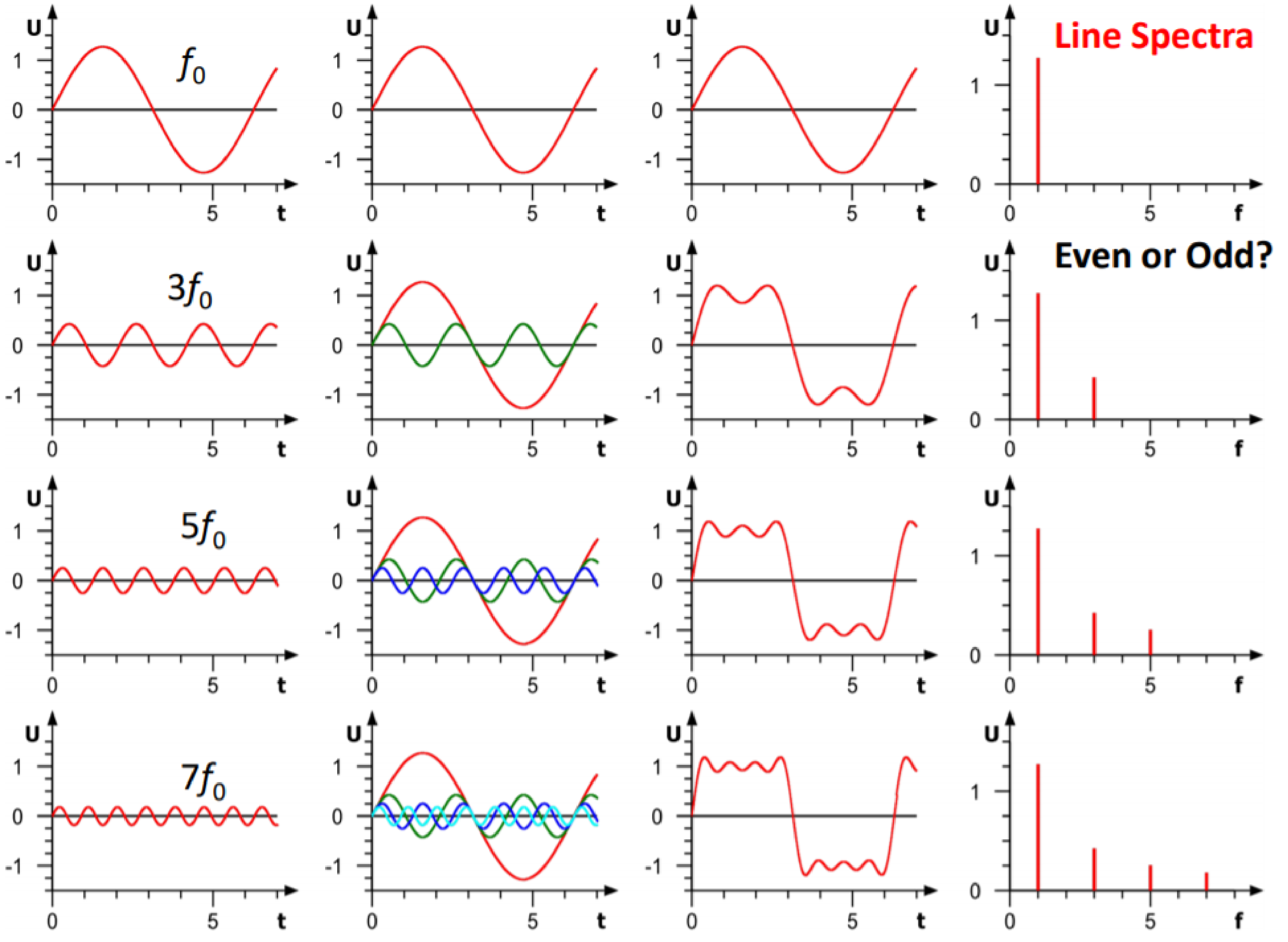
BY: RUAA SHALLAL ANOOZ

NON PERIODIC SIGNALS AND FOURIER TRANSFORM

- *Fourier Transform*
  - Fourier Transform gives the frequency domain of a nonperiodic time domain signal.



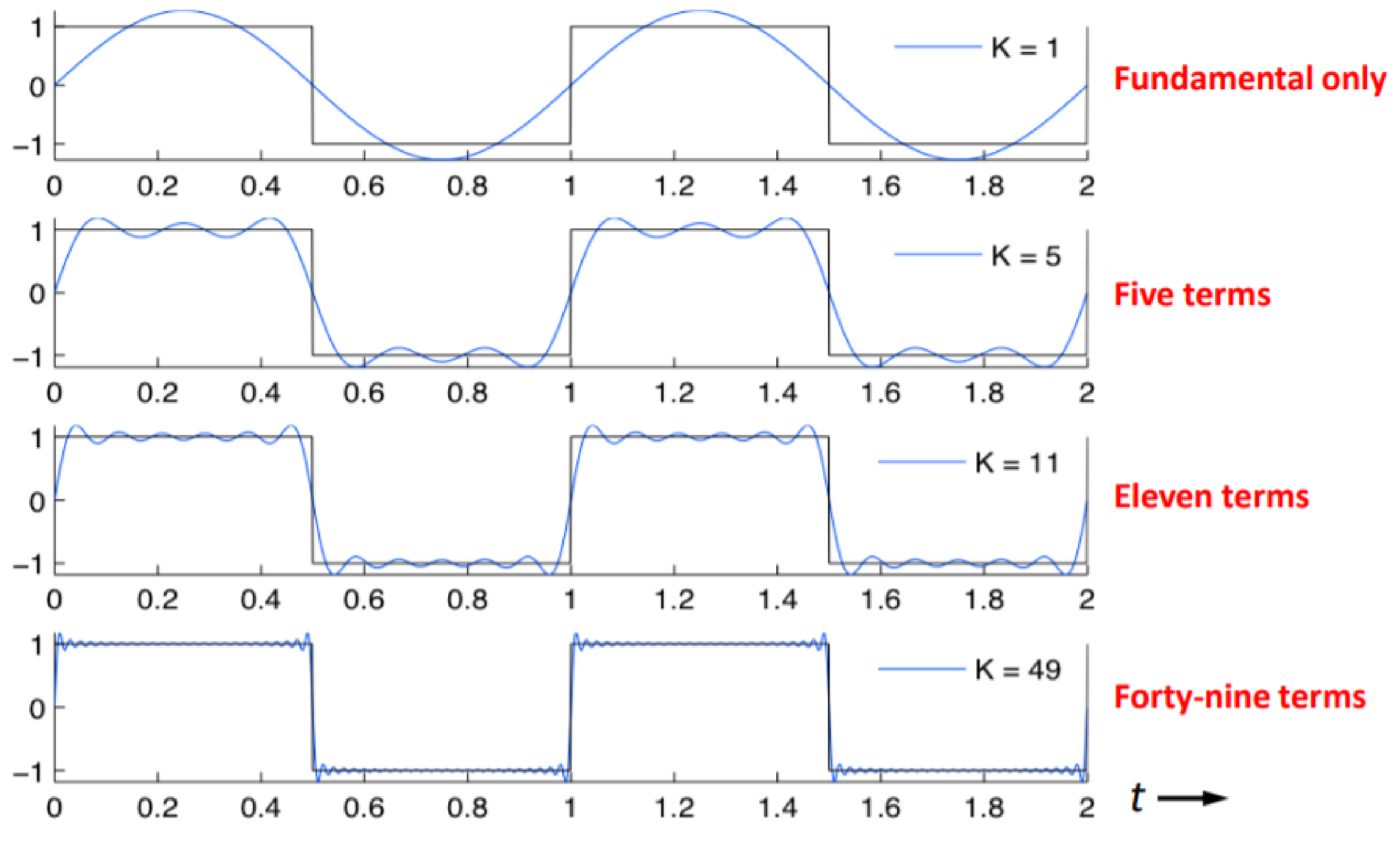
PERIODIC SQUARE WAVE IS SUM OF SINUSOIDS



Ruaa Shallal Abbas

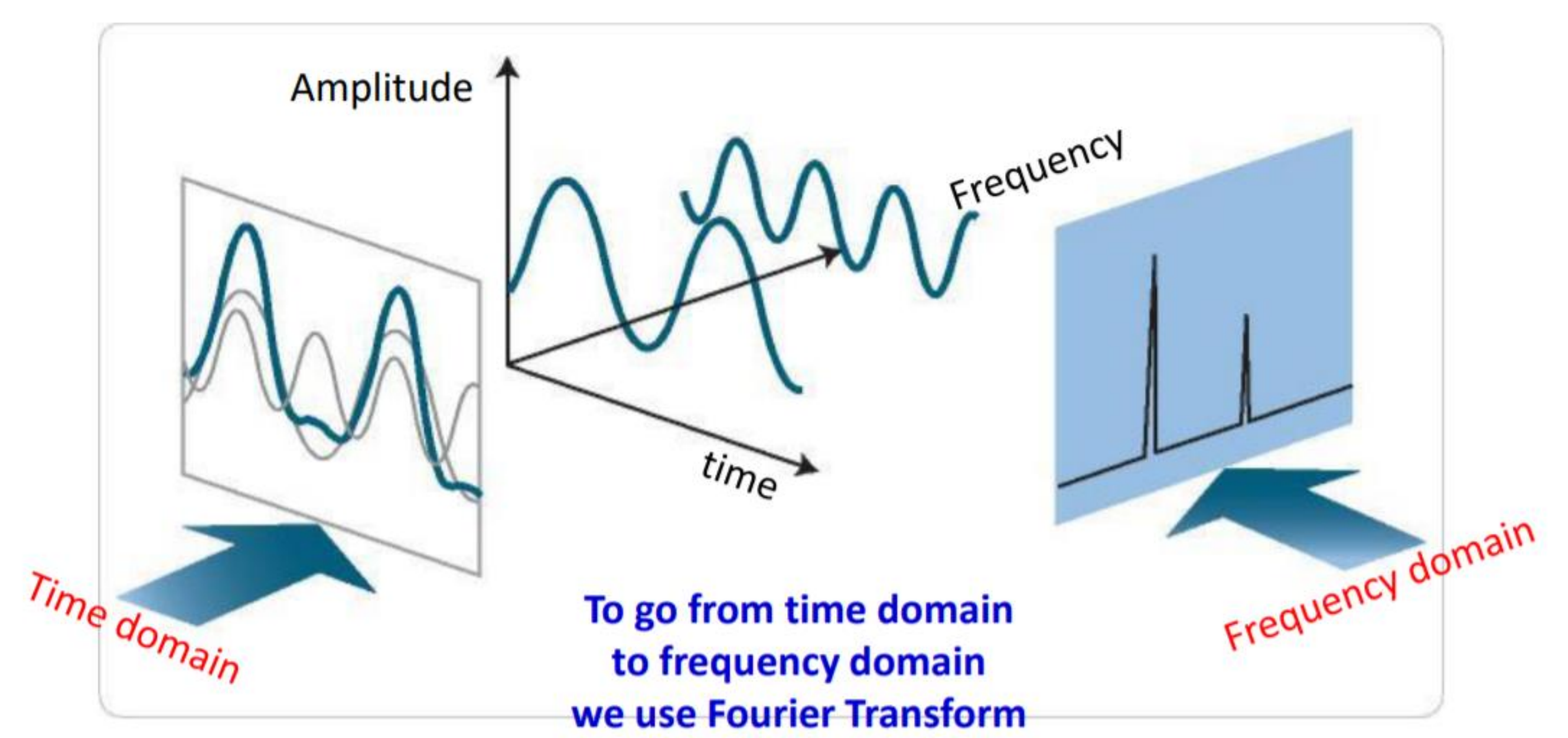
PERIODIC SQUARE WAVE

$$f(t)=\frac{4}{\pi}\left[\sin(\pi t)+\tfrac{1}{3}\sin(3\pi t)+\tfrac{1}{5}\sin(5\pi t)+\tfrac{1}{7}\sin(7\pi t)+\cdots\right]$$



This is an odd function

VISUALIZING A SIGNAL – TIME DOMAIN & FREQUENCY DOMAIN



Ruaa Shallal Abbas

FOURIER SERIES VERSUS FOURIER TRANSFORM

	Continuous time	Discrete time
Periodic	Fourier Series	Discrete Fourier Transform
Aperiodic	Fourier Transform	Discrete Fourier Transform

- Fourier series for continuous-time periodic signals → discrete spectra
- Fourier transform for continuous aperiodic signals → continuous spectra

DEFINITION OF FOURIER TRANSFORM

- The Fourier transform (spectrum) of f(t) is F(w):

$$F(\omega) = \mathfrak{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

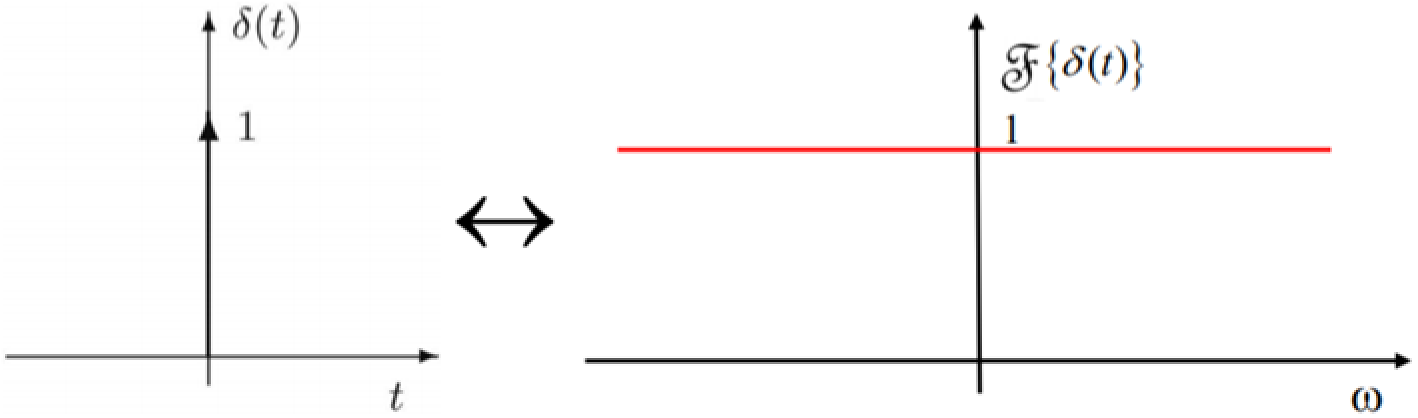
$$f(t) = \mathfrak{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

- Note: Remember $\omega = 2\pi f$

EXAMPLE: IMPULSE FUNCTION δ (T)

$$F(\omega) = \mathfrak{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = e^{-j\omega t}\Big|_{t=0} = e^{j0} = 1$$

$$\delta(t) \Leftrightarrow 1$$
$$1 \Leftrightarrow 2\pi\delta(\omega)$$



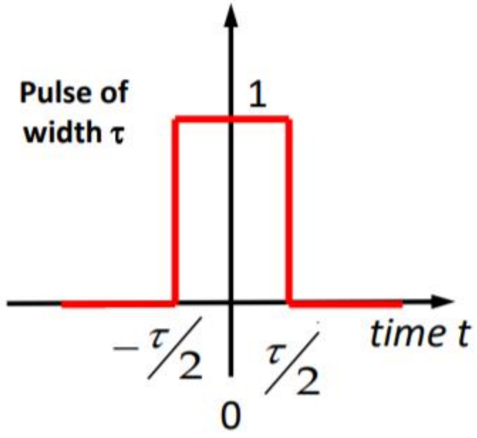
$$\delta(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

Delta function has unity area.

EXAMPLE: FOURIER TRANSFORM OF SINGLE RECTANGULAR PULSE

$$f(t) = \text{rect}(t) = \text{II}(t/\tau) = \begin{cases} 1 & \text{for } -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0 & \text{for all } |t| > \frac{\tau}{2} \end{cases}$$

$$f(t) = \text{rect}(t) = \text{II}(t/\tau)$$



Remember $\omega = 2\pi f$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_{-\tau/2}^{\tau/2} e^{-j\omega t} dt$$

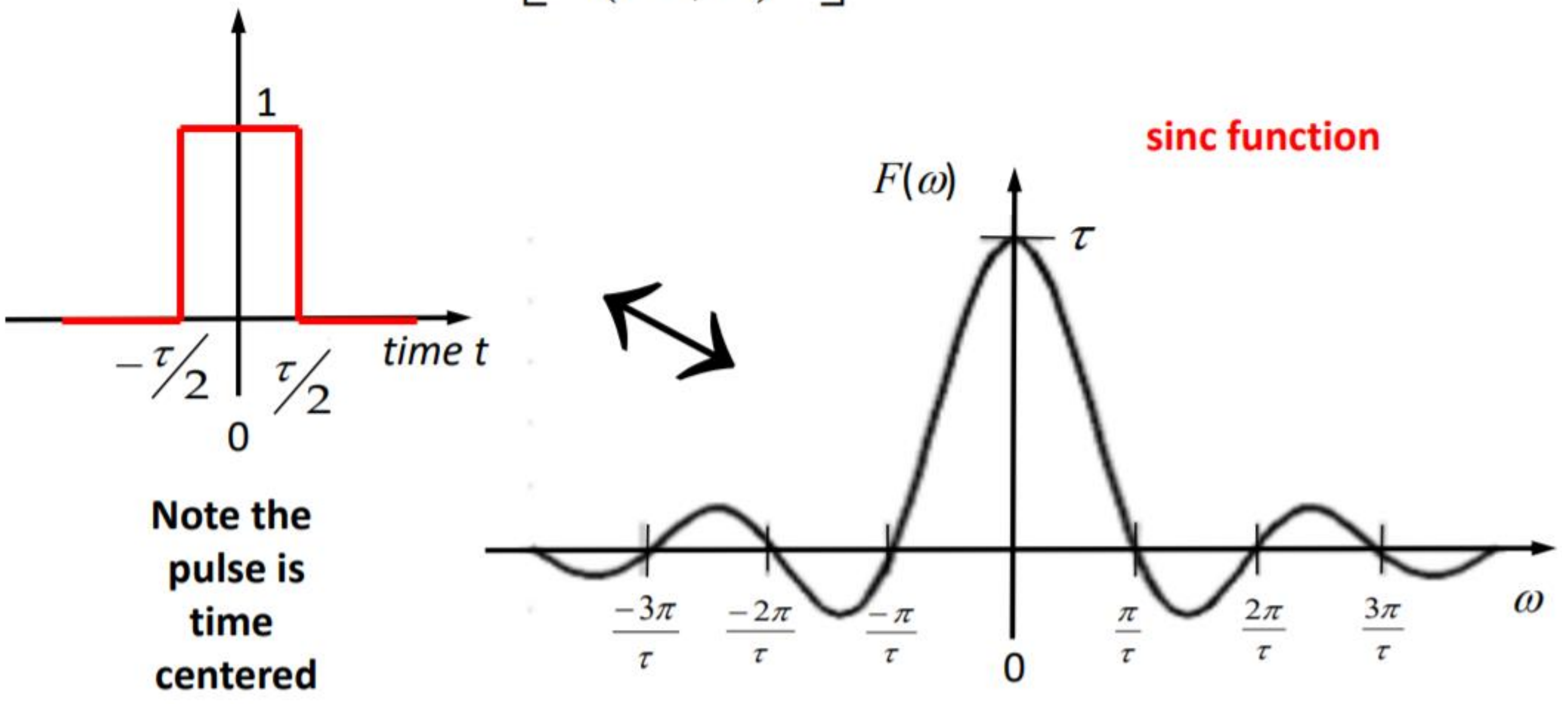
$$= \left(\frac{e^{-j\omega t}}{-j\omega} \right) \Bigg|_{-\tau/2}^{\tau/2} = \frac{e^{-j\omega\tau/2} - e^{j\omega\tau/2}}{-j\omega}$$

$$= \frac{-j2\sin(\omega\tau/2)}{-j\omega} = \tau \cdot \left[\frac{\sin(\omega\tau/2)}{(\omega\tau/2)} \right]$$

Ruaa Shallal Abbas

EXAMPLE: FOURIER TRANSFORM OF SINGLE RECTANGULAR PULSE

$$F(\omega) = \tau \cdot \left[\frac{\sin(\omega\tau/2)}{(\omega\tau/2)} \right] = \tau \cdot \mathrm{sinc}(\pi f \tau)$$



PROPERTIES OF THE SINC FUNCTION

- Definitions of the sinc function:

$$\text{sinc}(x) = \frac{\sin(x)}{x} \quad \text{and} \quad \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

- Sinc Properties:

1. sinc(x) is an even function of x.
2. sinc(x) = 0 at points where sin(x) = 0, that is, sinc(x) = 0 when $x = \pm\pi, \pm2\pi, \pm3\pi, \ldots$.
3. Using L'Hôpital's rule, it can be shown that sinc(0) = 1.
4. sinc(x) oscillates as sin(x) oscillates and monotonically decreases as 1/ x decreases as | x | increases.
5. sinc(x) is the Fourier transform of a single rectangular pulse.