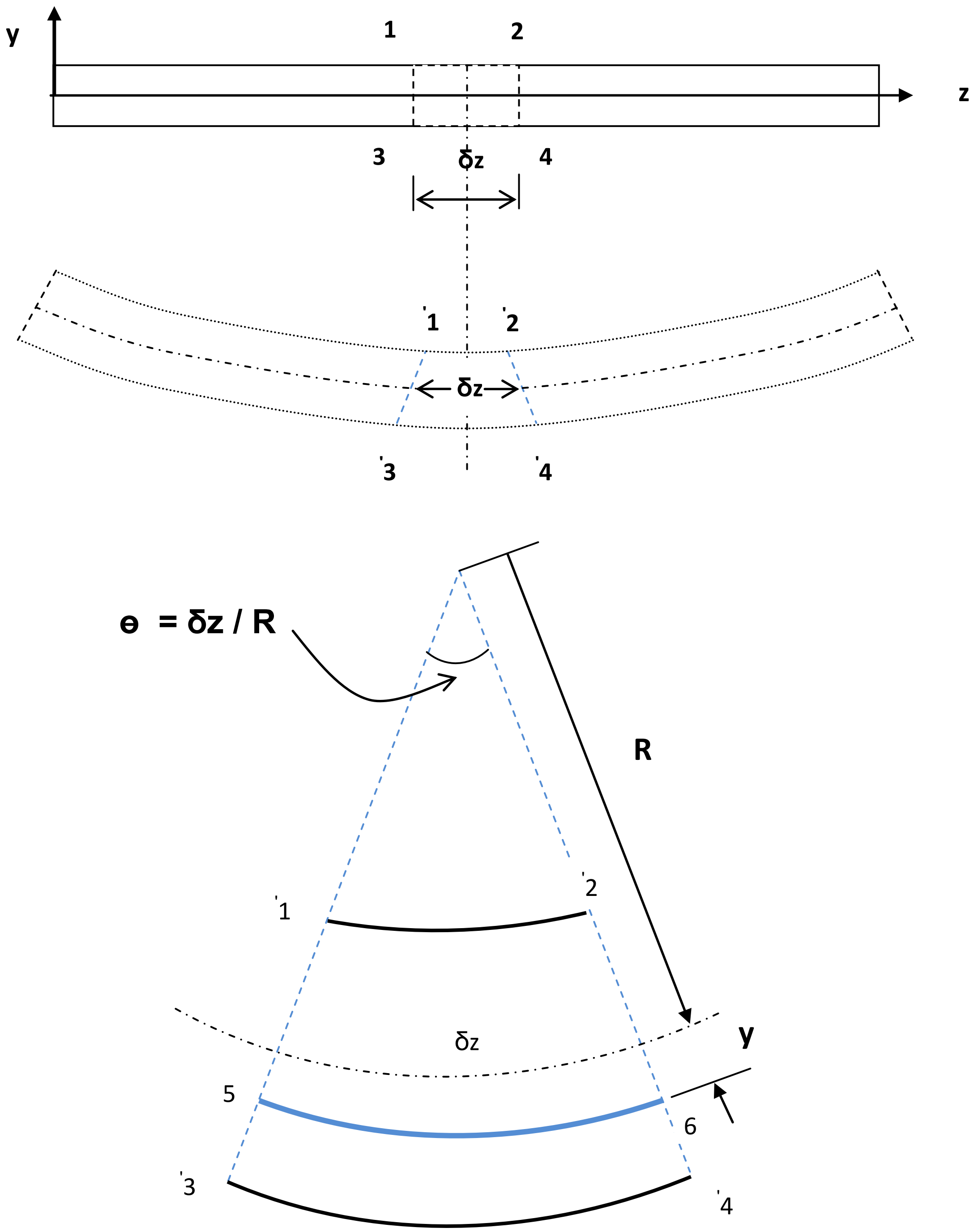
Longitudinal stresses in beams

The figure shown below represents a beam before and after flexural bending :



$$R\theta = \delta z \qquad \rightarrow\rightarrow\rightarrow\rightarrow \theta = \frac{\delta z}{R}$$

$$Length\ of\ (5-6)\ fiber = (R+y)\theta = (R+y)\frac{\delta z}{R}$$

$$Change\ in\ length\ \Delta L = [Length\ of\ (5-6) - \delta z]$$

$$\Delta L = (R+y)\theta - \delta z = (R+y)\frac{\delta z}{R} - \delta z$$

$$\Delta L = \delta z + y\frac{\delta z}{R} - \delta z = y\frac{\delta z}{R}$$

$$\in = \frac{\Delta L}{L} = \frac{y\frac{\delta z}{R}}{\delta z} = \frac{y}{R}$$

$$\sigma = E \in = \frac{Ey}{R}$$

$$But\ M = \int \sigma.b.y.dy = \int \frac{Ey}{R}\ b.y.\in dy = \frac{E}{R}\int b\,y^2 dy = \frac{EI_x}{R}$$

$$\therefore \sigma = \frac{Ey}{R} \quad \rightarrow\rightarrow\rightarrow \frac{\sigma}{\mathrm{y}} = \frac{\mathrm{E}}{\mathrm{R}}$$

$$\boldsymbol{and\ M = \frac{EI_x}{R} \rightarrow\rightarrow\rightarrow\rightarrow \frac{M}{I_x} = \frac{\mathbf{E}}{\mathbf{R}}}$$

$$\therefore \boldsymbol{\frac{\sigma}{y} = \frac{M}{I_x} = \frac{\mathbf{E}}{\mathbf{R}}}$$