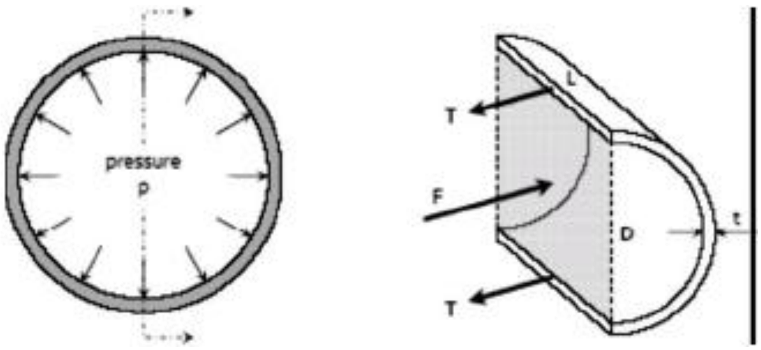
Thin-Walled Pressure Vessels

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

Tangential Stress(Circumferential Stress):

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



$F = pA = pDL$

$T = \sigma_t A_{wall} = \sigma_t tL$

$\sum F_H = 0$

$F = 2T$

$pDL = 2(\sigma_t tL)$

$\sigma_t = pD/2t$

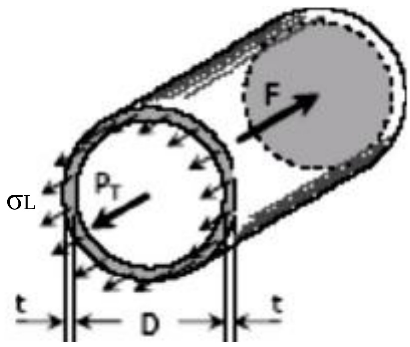
If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_t = \frac{(pi - po)D}{2t}$$

LONGITUDINAL STRESS, σ_L

Consider the free body diagram in the transverse section of the tank:

The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall $P_T = \sigma_L A_{wall}$. Since t is so small compared to D, the area of the wall is close to πDt .



$$F = pA = p\frac{\pi}{4}D^2$$

$$P_T = \sigma_L \pi Dt$$

$$[\,\Sigma F_H = 0\,]$$

$$P_T = F$$

$$\sigma_L \pi Dt = p\frac{\pi}{4}D^2$$

$$\sigma_L = \frac{pD}{4t}$$

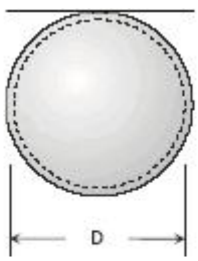
If there exist an external pressure p_o and an internal pressure p_i , the formula may be expressed as:

$$\sigma_L = \frac{(pi - po)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\boldsymbol{\sigma_t = 2\,\sigma_L}$$

Spherical Shell: If a spherical tank of diameter ***D*** and thickness ***t*** contains gas under a pressure of ***p***, the stress at the wall can be expressed as:

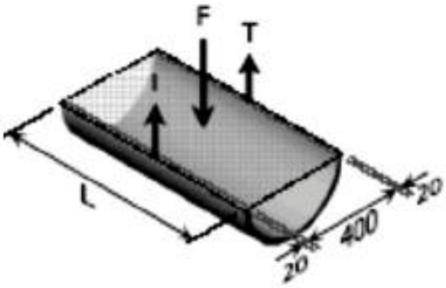


$$\sigma_L = \frac{(pi - po)D}{4t}$$

SOLVED EXAMPLES IN THIN WALLED PREASSURE VESSELS

Example 133: A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2? (c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

Solution



Longitudinal Section

(a) **Tangential stress** (longitudinal section):

$$F = 2T$$

$$pDL = 2(\sigma_t\, tL)$$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$$\sigma_t = 45\ \text{MPa}$$

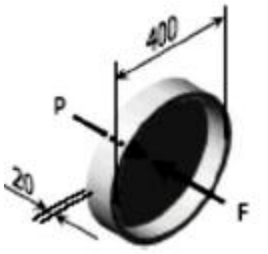
Longitudinal Stress (transverse section)::

$$F = P$$

$$\frac{1}{4}\pi D^2 p = \sigma_l\,(\pi D t)$$

$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

σ_l = **22.5 MPa**



Transverse Section

(b) From (a), $\sigma_t = \dfrac{pD}{2t}$ and $\sigma_l = \dfrac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$,
this shows that tangential stress is the critical.

$$\sigma_t = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

P = **12 MPa**

(c) The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.