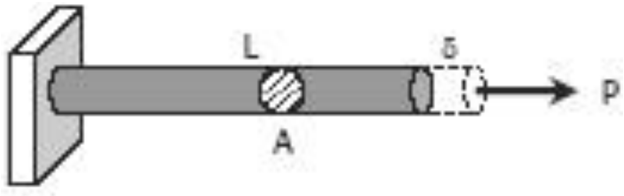
CHAPTER 2

STRAIN

Simple Strain

Strain (ϵ) is the ratio of the change in length caused by the applied force, to the original length.(Also known as unit deformation).



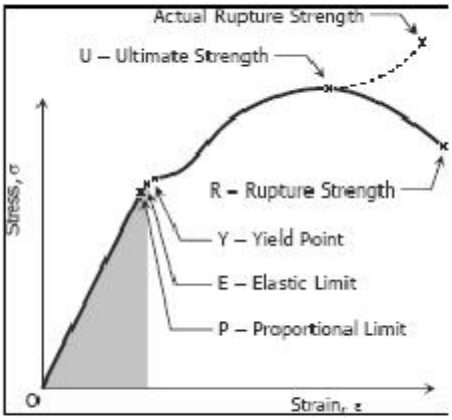
$$\varepsilon = \frac{\delta}{L}$$

where δ is the deformation and L is the original length, thus **ε** is dimensionless.

Stress-Strain Diagram

Suppose that a metal specimen be placed in tension-compression testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel.

Metallic engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rupture like cast iron and concrete. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes.



Proportional Limit (Hooke's Law)

From the origin o to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called **Hooke's Law that within the proportional limit, the stress is directly proportional to strain or:**

$$\sigma \propto \epsilon \ \ or$$

$$\sigma = k\epsilon$$

The constant of proportionality **k** is called the Modulus of Elasticity **E** or Young's Modulus and is equal to the slope of the stress-strain diagram from O to P. Then :

$$\sigma = E\epsilon$$

Elastic Limit

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may be developed such that there is **nonpermanent** (or **residual**) deformation when the load is entirely removed.

Elastic and Plastic Ranges

The region in stress-strain diagram from O to P is called the elastic range. The region from P to R is called the plastic range.

Yield Point

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

Ultimate Strength

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

Rupture Strength

Rupture strength is the strength of the material at rupture. This is also known as the breaking strength.

Modulus Of Resilience

Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from O to P, in Nm/m3. This may be calculated as the area under the stress-strain curve from the origin O to up to the elastic limit E (the shaded area in the figure). The resilience of the material is its ability to absorb energy without creating a permanent distortion.

Modulus Of Toughness

Modulus of toughness is the work done on a unit volume of material as the force is gradually increased from O to R, in Nm/m3. This may be calculated as the area under the entire stress-strain curve (from O to R). The toughness of a material is its ability to absorb energy without causing it to break.

STIFFNESS, k

Stiffness is the ratio of the steady force acting on an elastic body to the resulting displacement. It has the unit of N/mm.

$$k = P / \delta$$

Working Stress, Allowable Stress, And Factor Of Safety

Working stress is defined as the actual stress of a material under a given loading. The maximum safe stress that a material can carry is termed as the allowable stress. The allowable stress should be limited to values not exceeding the proportional limit. However, since proportional limit is difficult to determine accurately, the allowable tress is taken as either the yield point or ultimate strength divided by a factor of safety. The ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety.

AXIAL DEFORMATION

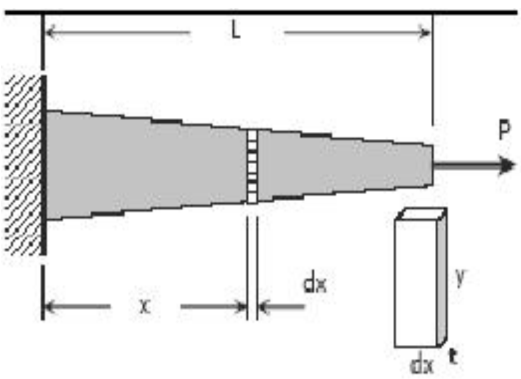
In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by:**$\sigma = E\varepsilon$**

since $\sigma = P / A$ and $\varepsilon = \delta / L$, then $P / A = E\,\delta / L$. Solving for δ,

$$\boldsymbol{\delta = \frac{PL}{EI} = \frac{\sigma L}{E}}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit. If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.

where A = t*y and y and t, if variable, must be expressed in terms of x.



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is :

$$\delta = \frac{\rho g L^2}{2E} = \frac{MgL}{2AE}$$

where ρ is in kg/m^3, L is the length of the rod in mm, M is the total mass of the rod in kg, A is the cross-sectional area of the rod in mm^2, and $g = 9.81\ m/s^2$.

SOLVED EXAMPLES ON STRAIN & AXIAL DEFORMATION

Concept Application 2.1

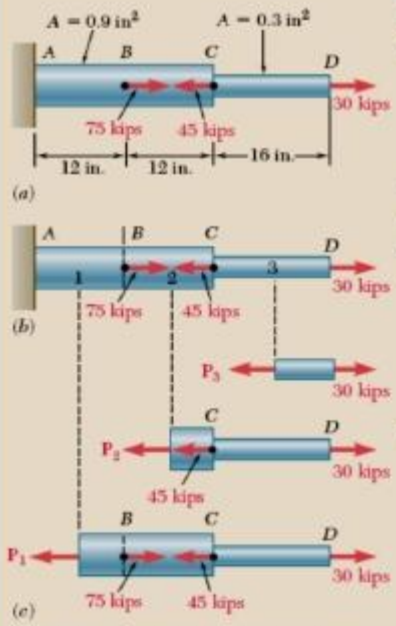


Fig. 2.19 (a) Axially-loaded rod. (b) Rod divided into three sections. (c) Three sectioned free-body diagrams with internal resultant forces P_1, P_2, and P_3.

Determine the deformation of the steel rod shown in Fig. 2.19*a* under the given loads ($E = 29 \times 10^6$ psi).

The rod is divided into three component parts in Fig. 2.19*b*, so

$$L_1 = L_2 = 12\text{ in.} \qquad L_3 = 16\text{ in.}$$
$$A_1 = A_2 = 0.9\text{ in}^2 \qquad A_3 = 0.3\text{ in}^2$$

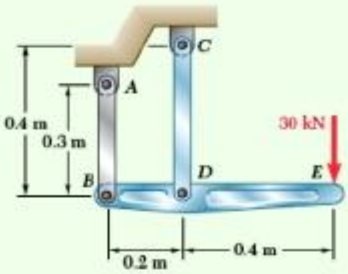
To find the internal forces P_1, P_2, and P_3, pass sections through each of the component parts, drawing each time the free-body diagram of the portion of rod located to the right of the section (Fig. 2.19*c*). Each of the free bodies is in equilibrium; thus

$$P_1 = 60\text{ kips} = 60 \times 10^3\text{ lb}$$
$$P_2 = -15\text{ kips} = -15 \times 10^3\text{ lb}$$
$$P_3 = 30\text{ kips} = 30 \times 10^3\text{ lb}$$

Using Eq. (2.10)

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$
$$= \frac{1}{29 \times 10^6}\left[\frac{(60 \times 10^3)(12)}{0.9} + \frac{(-15 \times 10^3)(12)}{0.9} + \frac{(30 \times 10^3)(16)}{0.3}\right]$$
$$\delta = \frac{2.20 \times 10^6}{29 \times 10^6} = 75.9 \times 10^{-3}\text{ in.}$$

Sample Problem 2.1



The rigid bar *BDE* is supported by two links *AB* and *CD*. Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2. Link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of 600 mm^2. For the 30-kN force shown, determine the deflection (*a*) of *B*, (*b*) of *D*, and (*c*) of *E*.

STRATEGY: Consider the free body of the rigid bar to determine the internal force of each link. Knowing these forces and the properties of the links, their deformations can be evaluated. You can then use simple geometry to determine the deflection of E.

MODELING: Draw the free body diagrams of the rigid bar (Fig. 1) and the two links (Fig. 2 and 3)

ANALYSIS:

Free Body: Bar BDE (Fig. 1)

$+\circlearrowleft \Sigma M_B = 0$: $\quad -(30\text{ kN})(0.6\text{ m}) + F_{CD}(0.2\text{ m}) = 0$

$F_{CD} = +90\text{ kN} \qquad F_{CD} = 90\text{ kN}$ *tension*

$+\circlearrowleft \Sigma M_D = 0$: $\quad -(30\text{ kN})(0.4\text{ m}) - F_{AB}(0.2\text{ m}) = 0$

$F_{AB} = -60\text{ kN} \qquad F_{AB} = 60\text{ kN}$ *compression*

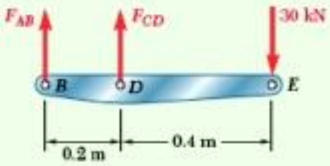


Fig. 1 Free-body diagram of rigid bar *BDE*.

a. Deflection of B. Since the internal force in link *AB* is compressive (Fig. 2), $P = -60$ kN and

$$\delta_B = \frac{PL}{AE} = \frac{(-60 \times 10^3\text{ N})(0.3\text{ m})}{(500 \times 10^{-6}\text{ m}^2)(70 \times 10^9\text{ Pa})} = -514 \times 10^{-6}\text{ m}$$

The negative sign indicates a contraction of member *AB*. Thus, the deflection of end *B* is upward:

$$\delta_B = 0.514\text{ mm}\uparrow$$

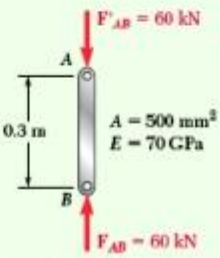


Fig. 2 Free-body diagram of two-force member *AB*.

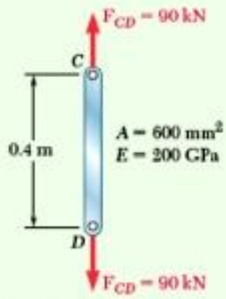


Fig. 3 Free-body diagram of two-force member *CD*.

(continued)

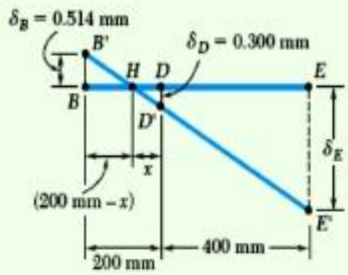


Fig. 4 Deflections at *B* and *D* of rigid bar are used to find δ_E.

b. Deflection of D. Since in rod *CD* (Fig. 3), $P = 90$ kN, write

$$\delta_D = \frac{PL}{AE} = \frac{(90 \times 10^3\ \text{N})(0.4\ \text{m})}{(600 \times 10^{-6}\ \text{m}^2)(200 \times 10^9\ \text{Pa})}$$

$$= 300 \times 10^{-6}\ \text{m} \qquad \delta_D = 0.300\ \text{mm} \downarrow$$

c. Deflection of E. Referring to Fig. 4, we denote by *B′* and *D′* the displaced positions of points *B* and *D*. Since the bar *BDE* is rigid, points *B′*, *D′*, and *E′* lie in a straight line. Therefore,

$$\frac{BB'}{DD'} = \frac{BH}{HD} \qquad \frac{0.514\ \text{mm}}{0.300\ \text{mm}} = \frac{(200\ \text{mm}) - x}{x} \qquad x = 73.7\ \text{mm}$$

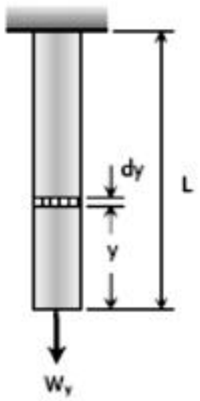
$$\frac{EE'}{DD'} = \frac{HE}{HD} \qquad \frac{\delta_E}{0.300\ \text{mm}} = \frac{(400\ \text{mm}) + (73.7\ \text{mm})}{73.7\ \text{mm}}$$

$$\delta_E = 1.928\ \text{mm} \downarrow$$

REFLECT and THINK: Comparing the relative magnitude and direction of the resulting deflections, you can see that the answers obtained are consistent with the loading and the deflection diagram of Fig. 4.

Example 201: A uniform bar of length L, cross-sectional area A, and unit mass ρ is suspended vertically from one end. Show that its total elongation is $\delta = \rho g L^2 / 2E$. If the total mass of the bar is M, show also that $\delta = MgL/2AE$.

Solution 201



$$\delta = \frac{PL}{AE}$$

From the figure:
$\delta = d\delta$
$P = Wy = (\rho Ay)g$
$L = dy$

$$d\delta = \frac{(\rho Ay)g\,dy}{AE}$$

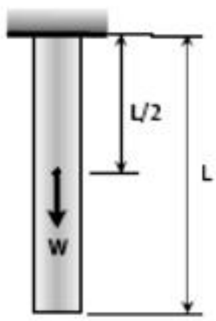
$$\delta = \frac{\rho g}{E}\int_0^L y\,dy = \frac{\rho g}{E}\left[\frac{y^2}{2}\right]_0^L$$

$$\delta = \frac{\rho g}{2E}\left[L^2 - 0^2\right] = \rho g L^2/2E \qquad \textit{ok!}$$

Given the total mass M:
$\rho = M/V = M/AL$

$\delta = \rho g L^2/2E = (M/AL)(gL^2/2E)$
$\delta = MgL/2AE$ *ok!*

Another Solution:



The weight will act at the center of gravity of the bar:

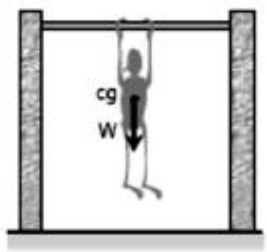
$$\delta = \frac{PL}{AE}$$

Where: $P = W = (\rho AL)g$
$L = L/2$

$$\delta = \frac{[(\rho AL)g](L/2)}{AE}$$

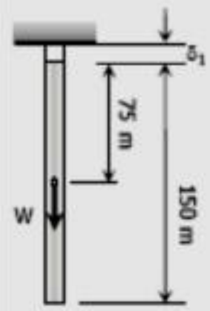
$$\delta = \frac{\rho g L^2}{2E} \qquad \textit{ok!}$$

For you to feel the situation, position yourself in pull-up exercise with your hands on the bar and your body hang freely above the ground. Notice that your arms suffer all your weight and your lower body fells no stress (center of weight is approximately just below the chest). If your body is the bar, the elongation will occur at the upper half of it.



Example 201A : A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and $E = 200 \times 10^3\ MN/m^2$, find the total elongation of the rod.

Solution 201A



Let δ = total elongation

δ_1 = elongation due to its own weight

δ_2 = elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: $P = W = 7850(1/1000)^3(9.81)[300(150)(1000)]$

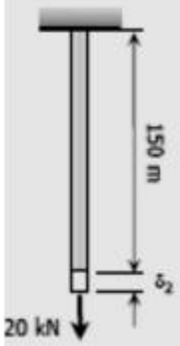
$P = 3465.3825$ N

$L = 75(1000) = 75\ 000$ mm

$A = 300\ mm^2$

$E = 200\ 000$ MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\ 000)} = 4.33 \text{ mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: $P = 20$ kN $= 20\ 000$ N

$L = 150$ m $= 150\ 000$ mm

$A = 300\ mm^2$

$E = 200\ 000$ MPa

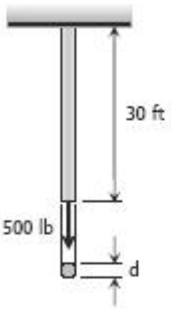
$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

Total elongation:

$$\delta = 4.33 + 50 = 54.33 \text{ mm}$$

Example 202 :A steel wire 30 ft long, hanging vertically, supports a load of 500 lb. Neglecting the weight of the wire, determine the required diameter if the stress is not to exceed 20 ksi and the total elongation is not to exceed 0.20 in. Assume E = 29 × 10^6 psi.

Solution 202



Based on maximum allowable stress:

$$\sigma = \frac{P}{A}$$

$$20\,000 = \frac{500}{\frac{1}{4}\pi d^2}$$

$$d = 0.0318 \text{ in}$$

Based on maximum allowable deformation:

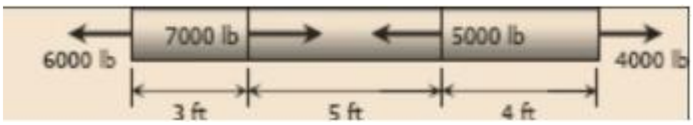
$$\delta = \frac{PL}{AE}$$

$$0.20 = \frac{500(30\times12)}{\frac{1}{4}\pi d^2(29\times10^6)}$$

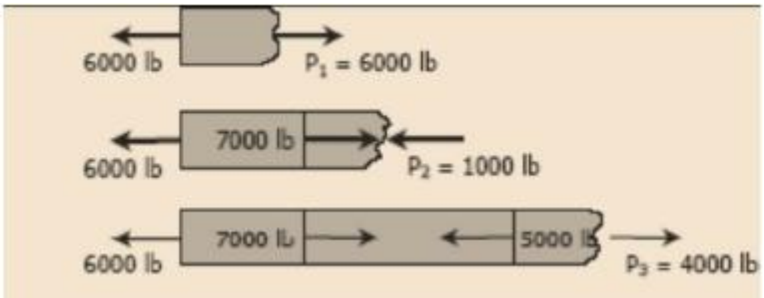
$$d = 0.0395 \text{ in}$$

Use the bigger diameter, $d = \mathbf{0.0395 \text{ in}}$

Example 203 : An aluminum bar having a cross-sectional area of 0.5 in^2 carries the axial loads applied at the positions shown in Fig. P-209. Compute the total change in length of the bar if E = 10 × 10^6 psi. Assume the bar is suitably braced to prevent lateral buckling.



Solution 203



$P_1 = 6000$ lb tension
$P_2 = 1000$ lb compression
$P_3 = 4000$ lb tension

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

$$\delta = \frac{6000(3\times12)}{0.5(10\times10^6)} - \frac{1000(5\times12)}{0.5(10\times10^6)} + \frac{4000(4\times12)}{0.5(10\times10^6)}$$

$$\delta = \mathbf{0.0696\ in\ (lengthening)}$$

Example 204: Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.

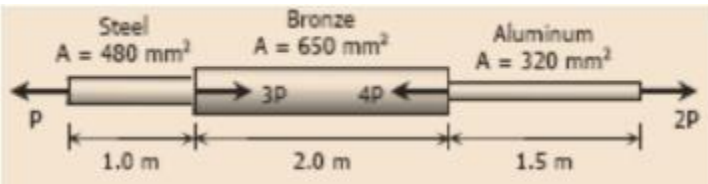
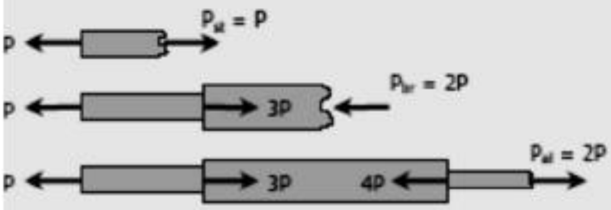


Figure P -211

Solution 204



Based on allowable stresses:

Steel:

$P_{st} = \sigma_{st}A_{st}$

$P = 140(480) = 67\ 200$ N

$P = 67.2$ kN

Bronze:

$P_{br} = \sigma_{br}A_{br}$

$2P = 120(650) = 78\ 000$

$P = 39\ 000$ N $= 39$ kN

Aluminum:

$P_{al} = \sigma_{al}A_{al}$

$2P = 80(320) = 25\ 600$ N

$P = 12\ 800$ N $= 12.8$ kN

Based on allowable deformation:
(steel and aluminum lengthens, bronze shortens)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$

$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70000)} + \frac{2P(1500)}{320(83000)}$$

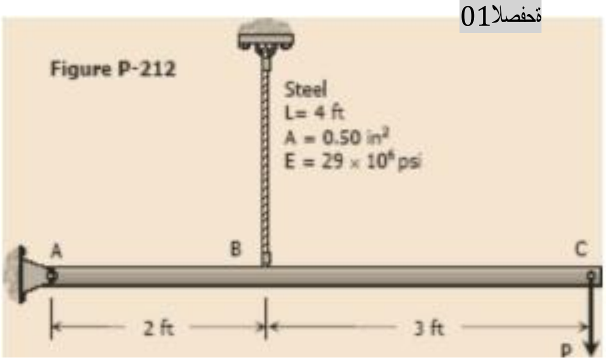
$$3 = \left(\frac{1}{96000} - \frac{1}{11375} + \frac{3}{26560}\right)P$$

$$P = 84\ 610.99 \text{ N} = 84.61 \text{ kN}$$

Use the smallest value of P, $P = \mathbf{12.8\ kN}$

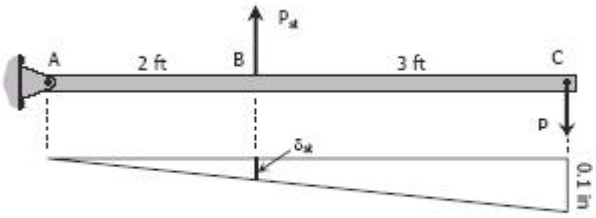
Example 205 : The rigid bar ABC shown in Fig. P-212 is hinged at A and supported by a steel rod at B. Determine the largest load P that can be applied at C if the stress in the steel rod is limited to 30 ksi and the vertical movement of end C must not exceed 0.10 in.

الصفحة01



Solution 205

Free body and deformation diagrams:



Based on maximum stress of steel rod:

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4\sigma_{st}A_{st}$

$P = 0.4[30(0.50)]$

$P = 6$ kips

Based on movement at C:

$$\frac{\delta_{st}}{2} = \frac{0.1}{5}$$

$\delta_{st} = 0.04$ in

$$\frac{P_{st}L}{AE} = 0.04$$

$$\frac{P_{st}(4\times12)}{0.50(29\times10^6)} = 0.04$$

$P_{st} = 12\ 083.33$ lb

$\Sigma M_A = 0$

$5P - 2P_{st}$

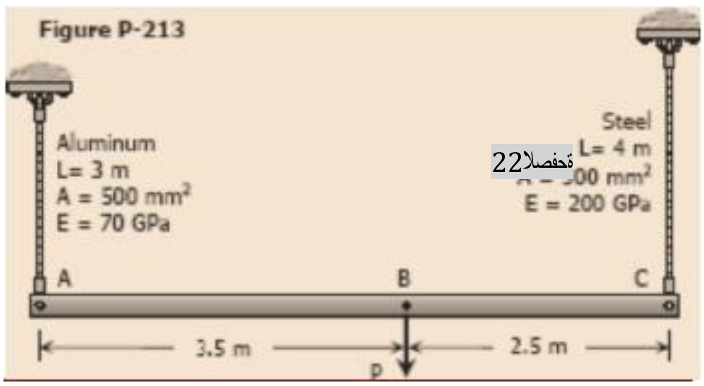
$P = 0.4P_{st}$

$P = 0.4(12\ 083.33)$

$P = 4833.33$ lb $= 4.83$ kips

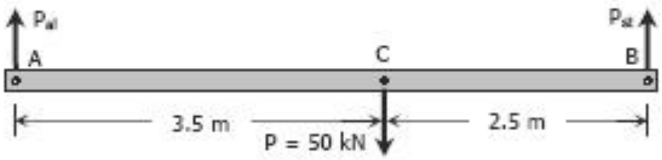
Use the smaller value, $P =$ **4.83 kips**

Example 206 : The rigid bar AB, attached to two vertical rods as shown in Fig. P-213, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.



Solution 206

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\quad 6P_{al} = 2.5(50)$

$P_{al} = 20.83$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{al} \qquad \delta_{al} = \frac{20.83(3)1000^2}{500(70000)}$$

$\delta_{al} = 1.78$ mm

For steel:

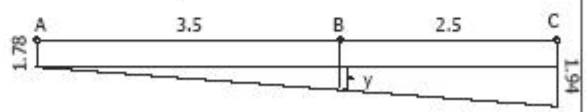
$[\Sigma M_A = 0]$ $\quad 6P_{st} = 3.5(50)$

$P_{st} = 29.17$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{st} \qquad \delta_{st} = \frac{29.17(4)1000^2}{300(200000)}$$

$\delta_{st} = 1.94$ mm

Movement diagram:



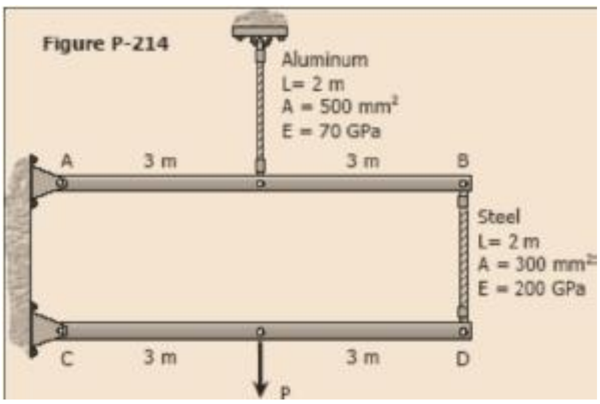
$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$

$y = 0.09$ mm

δ_B = vertical movement of P

$\delta_B = 1.78 + y = 1.78 + 0.09$

$\delta_B = \mathbf{1.87}$ **mm**

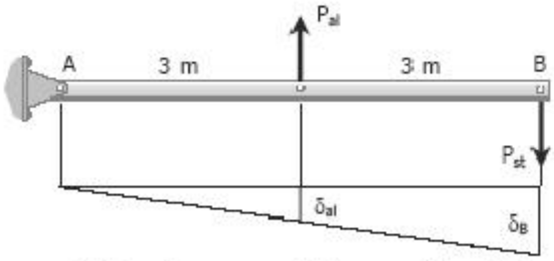


Example 207: The rigid bars AB and CD shown in Figure are supported by pins at A and C and the two rods. Determine the maximum force P that can be applied as shown if its vertical movement is limited to 5 mm. Neglect the weights of all members.

$[\Sigma M_A = 0]$ $\quad 3P_{al} = 6P_{st}$

$P_{al} = 2P_{st}$

Solution 207



FBD and movement diagram of bar AB

By ratio and proportion:

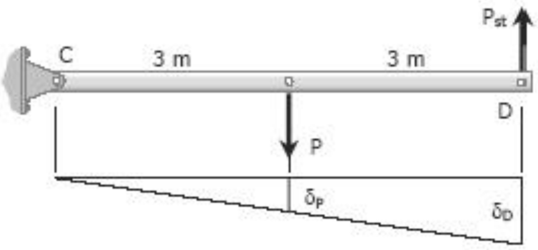
$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$

$$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$$

$$\delta_B = 2\left[\frac{P_{al}(2000)}{500(70\,000)}\right]$$

$$\delta_B = \tfrac{1}{8750}P_{al} = \tfrac{1}{8750}(2P_{st})$$

$$\delta_B = \tfrac{1}{4375}P_{st} \rightarrow \text{movement of B}$$



FBD and movement diagram of bar CD

Movement of D:

$$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \tfrac{1}{4375}P_{st}$$

$$\delta_D = \frac{P_{st}(2000)}{300(200\,000)} + \tfrac{1}{4375}P_{st}$$

$$\delta_D = \tfrac{11}{42\,000}P_{st}$$

$[\Sigma M_C = 0]$ $\quad 6P_{st} = 3P$

$P_{st} = \tfrac{1}{2}P$

By ratio and proportion:

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$

$$\delta_P = \tfrac{1}{2}\delta_D = \tfrac{1}{2}\left(\tfrac{11}{42000}P_{st}\right)$$

$$\delta_P = \tfrac{11}{84000}P_{st}$$

$$5 = \tfrac{11}{84000}\left(\tfrac{1}{2}P\right)$$

$$P = 76\,363.64 \text{ N} = \mathbf{76.4 \text{ kN}}$$