# CONTENTS


| | | | |
|---|---|---|---|
| **CHAPTER 1** | **REINFORCED CONCRETE STRUCTURES** | | **1** |
| | 1.1 | Introduction | 1 |
| | 1.2 | Reinforced concrete members | 1 |
| | 1.3 | Reinforced concrete behavior | 4 |
| **CHAPTER 2** | **MATERIALS AND PROPERTIES** | | **5** |
| | 2.1 | Concrete | 5 |
| | 2.2 | Compressive strength | 5 |
| | 2.3 | Tensile strength | 6 |
| | 2.4 | Modulus of elasticity | 7 |
| | 2.5 | Creep and shrinkage | 7 |
| | 2.6 | Steel Reinforcement | 9 |
| **CHAPTER 3** | **DESIGN METHODS AND REQUIREMENTS** | | **12** |
| | 3.1 | ACI Building Code | 12 |
| | 3.2 | Working stress method | 12 |
| | 3.3 | Strength design method | 13 |
| | 3.4 | Safety provisions | 13 |
| | 3.5 | Load factors and strength reduction factors | 14 |
| **CHAPTER 4** | **FLEXURE IN BEAMS** | | **16** |
| | 4.1 | Introduction | 16 |
| | 4.2 | Reinforced concrete beam behavior | 16 |
| | 4.3 | The equivalent rectangular compressive stress distribution (compressive stress block) | 18 |
| | 4.4 | Types of failure and strain limits | 20 |
| | 4.5 | The balanced condition | 22 |
| | 4.6 | Upper (maximum) and lower (minimum) steel percentages | 23 |
| | 4.7 | Spacing limits and concrete protection for reinforcement | 24 |

4.8 Analysis of singly reinforced concrete rectangular sections for flexure 24

4.9 Design of singly reinforced concrete rectangular sections for flexure 25

4.10 Doubly reinforced concrete sections (sections with compression reinforcement) 34

4.10.1 Analysis of doubly reinforced concrete sections 35

4.10.2 Design of doubly reinforced concrete sections 39

4.11 Reinforced concrete flanged sections (T- and L- sections) 42

4.11.1 Effective width 44

4.11.2 Analysis of T-sections 45

4.11.3 Minimum reinforcement of flexural T-section members 47

4.11.4 Analysis of the positive-moment capacity of a T-section 47

4.11.5 Analysis of the negative-moment capacity of a T-section 52

4.11.6 Design of T-section 52

**CHAPTER 5 SHEAR IN BEAMS 57**

5.1 Introduction 57

5.2 Critical sections for shear design 59

5.3 Types of web reinforcement 61

5.4 Design procedure for shear 62

**CHAPTER 6 COLUMNS: COMBINED AXIAL LOAD AND BENDING 71**

6.1 Types of columns 71

6.2 Behavior of axially loaded columns 74

6.3 ACI Code limitations 75

6.4 Design of tied and spiral short concentrically loaded columns 80

6.5 Eccentrically loaded columns: Axial load and bending 85

6.6 Load-Moment (Strength) interaction diagram 87

6.7 Balanced strain condition – Rectangular sections 90

6.8 Nominal strength of a compression-controlled rectangular section ($e < e_b$) 94

6.9 Nominal strength of a rectangular section having ($e > e_b$) 102

6.10 Design for strength. Practical design approach 105

| | | |
|---|---|---|
| 6.11 | Biaxially loaded columns | 143 |
| 6.12 | Square and rectangular columns under biaxial bending. Bresler reciprocal method | 144 |
| **CHAPTER 7** | **SLENDER COLUMNS** | **152** |
| 7.1 | Introduction | 152 |
| 7.2 | Nonsway and sway frames | 153 |
| 7.3 | Buckling of axially loaded elastic columns. The effective column length | 154 |
| 7.4 | Effective length factor $-\ k$ | 157 |
| 7.5 | Limitation of the slenderness ratio $-\ \left(\frac{kl_u}{r}\right)$ | 159 |
| | 7.5.1 Nonsway (braced) frames | 159 |
| | 7.5.2 Sway (unbraced) frames | 161 |
| 7.6 | Moment-magnifier design method | 161 |
| | 7.6.1 Magnified Moments in Nonsway Frames | 161 |
| | 7.6.2 Magnified Moments in Sway Frames | 164 |

# CHAPTER 1 REINFORCED CONCRETE STRUCTURES

## 1.1 INTRODUCTION

Many structures are built of reinforced concrete: bridges, viaducts, buildings, retaining walls, tunnels, tanks, conduits, and others.
Reinforced concrete is a logical union of two materials: plain concrete, which possesses high compressive strength but little tensile strength, and steel bars embedded in the concrete, which can provide the needed strength in tension.
First practical use of reinforced concrete was known in the mid-1800s. In the first decade of the 20th century, progress in reinforced concrete was rapid. Since the mid-1950s, reinforced concrete design practice has made the transition from that based on elastic methods to one based on strength.
Understanding of reinforced concrete behavior is still far from complete; building codes and specifications that give design procedures are continually changing to reflect latest knowledge.

## 1.2 REINFORCED CONCRETE MEMBERS

Every structure is proportioned as to both architecture and engineering to serve a particular function. Form and function go hand in hand, and the beat structural system is the one that fulfills most of the needs of the user while being serviceable, attractive, and economically cost efficient. Although most structures are designed for a life span of 50 years, the durability performance record indicates that properly proportioned concrete structures have generally had longer useful lives.
Reinforced concrete structures consist of a series of “members” (components) that interact to support the loads placed on the structures.
The components can be broadly classified into:

1. **Floor Slabs**

Floor slabs are the main horizontal elements that transmit the moving live loads as well as the stationary dead loads to the vertical framing supports of a structure. They can be:

- Slabs on beams,
- Waffle slabs,
- Slabs without beams (Flat Plates) resting directly on columns,
- Composite slabs on joists.

They can be proportioned such that they act in one direction (one-way slabs) or proportioned so that they act in two perpendicular directions (two-way slabs and flat plates).

2. **Beams**

Beams are the structural elements that transmit the tributary loads from floor slabs to vertical supporting columns. They are normally cast monolithically with the slabs and are

structurally reinforced on one face, the lower tension side, or both the top and bottom faces. As they are cast monolithically with the slab, they form a T-beam section for interior beams or an L beam at the building exterior.
The plan dimensions of a slab panel determine whether the floor slab behaves essentially as a one-way or two-way slab.

**3. Columns**

The vertical elements support the structural floor system. They are compression members subjected in most cases to both bending and axial load and are of major importance in the safety considerations of any structure. If a structural system is also composed of horizontal compression members, such members would be considered as beam-columns.

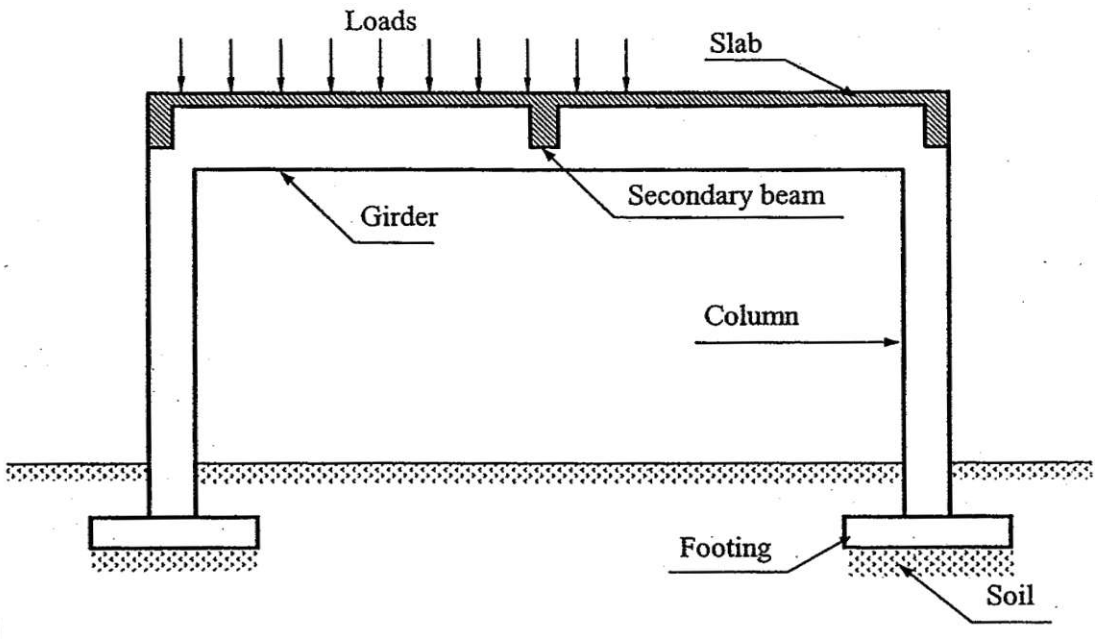
**4. Walls**

Walls are the vertical enclosures for building frames. They are not usually or necessarily made of concrete but of any material that esthetically fulfills the form and functional needs of the structural system. Additionally, structural concrete walls are often necessary as foundation walls, stairwell walls, and shear walls that resist horizontal wind loads and earthquake-induced loads.

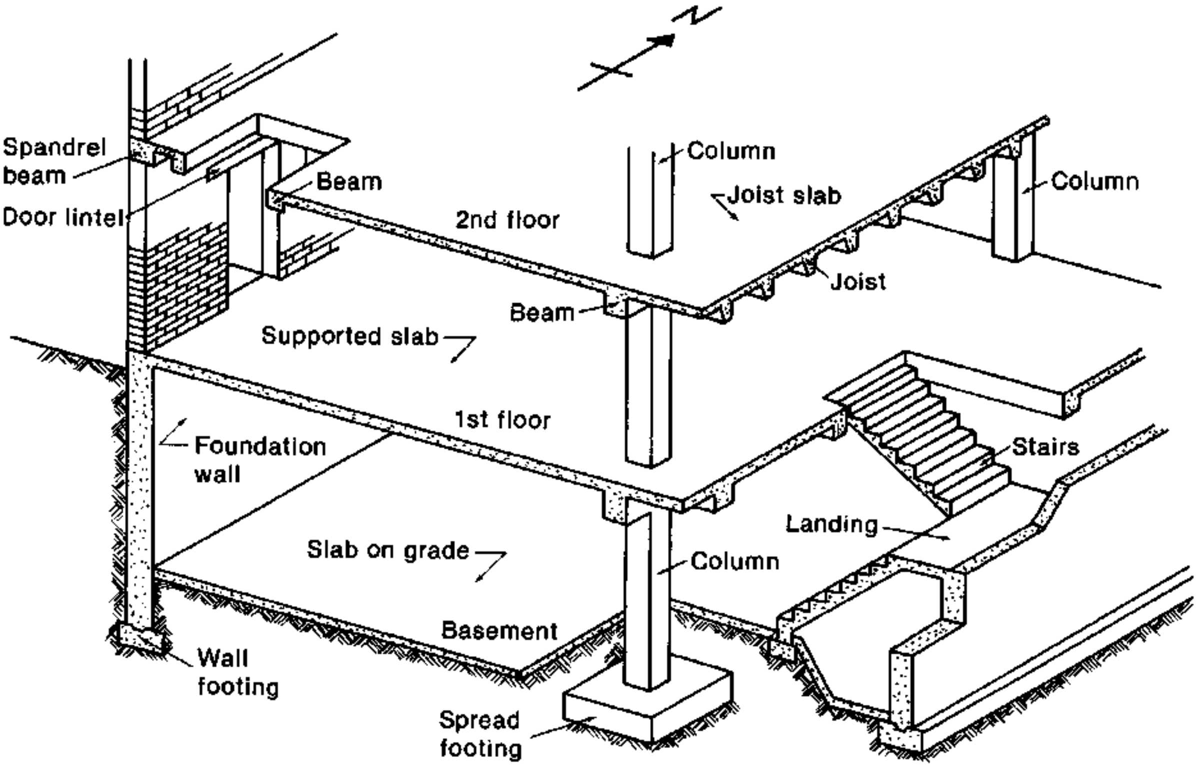
**5. Foundations**

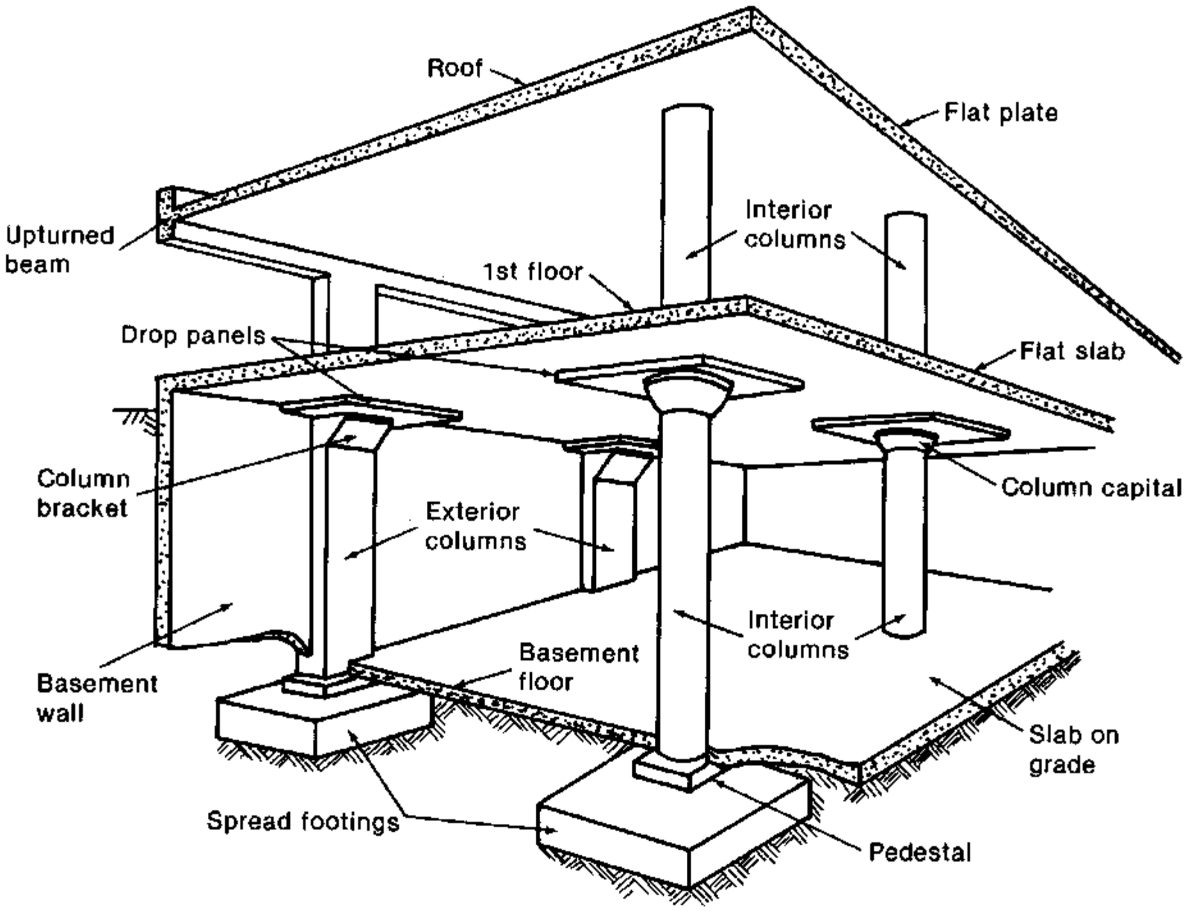
Foundations are the structural concrete elements that transmit the weight of the superstructure to the supporting soil. They could be in many forms:

- Isolated footing - the simplest one. It can be viewed as an inverted slab transmitting a distributed load from the soil to the column.
- Combined footings supporting more than one column.
- Mat foundations, and rafts which are basically inverted slab and beam construction.
- Strip footing or wall footing supporting walls.
- Piles driven to rock.




**Typical reinforced concrete structural framing system**

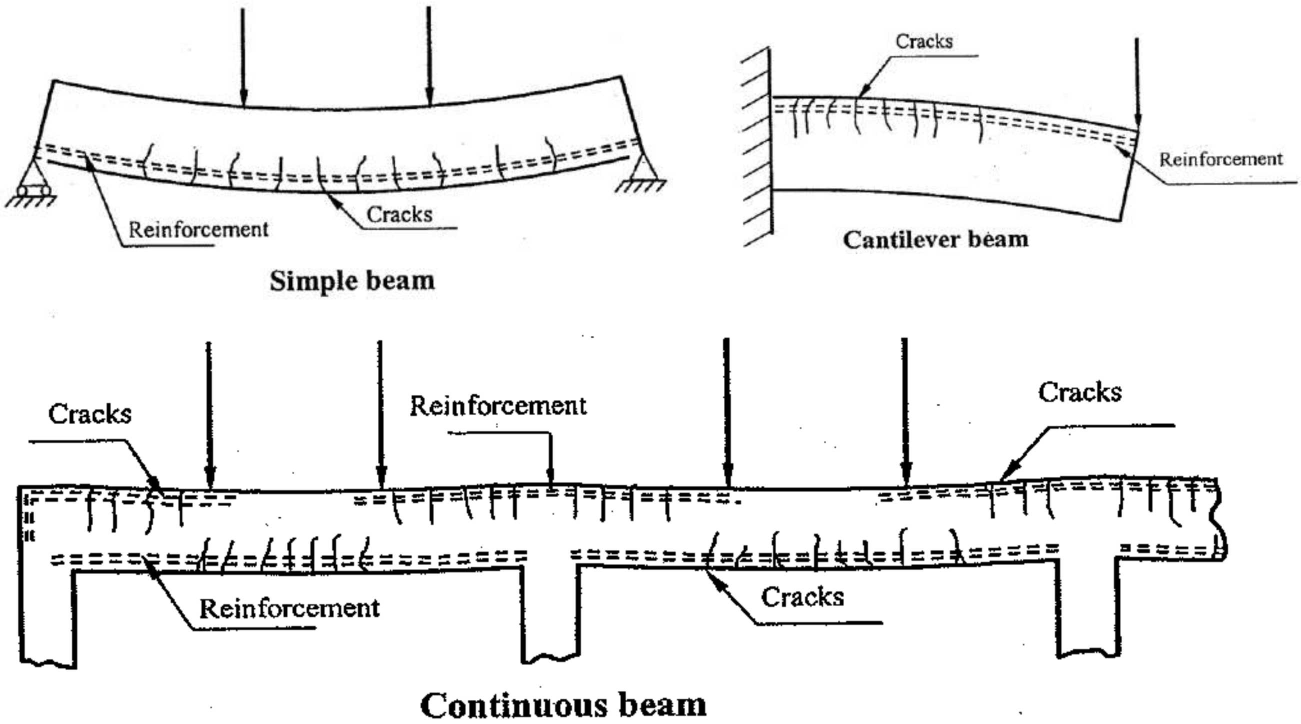



Reinforced concrete building elements

### 1.3 REINFORCED CONCRETE BEHAVIOR

The addition of steel reinforcement that bonds strongly to concrete produces a relatively ductile material capable of transmitting tension and suitable for any structural elements, e.g., slabs, beam, columns. Reinforcement should be placed in the locations of anticipated tensile stresses and cracking areas. For example, the main reinforcement in a simple beam is placed at the bottom fibers where the tensile stresses develop. However, for a cantilever, the main reinforcement is at the top of the beam at the location of the maximum negative moment. Finally for a continuous beam, a part of the main reinforcement should be placed near the bottom fibers where the positive moments exist and the other part is placed at the top fibers where the negative moments exist.




**Reinforcement placement for different types of beams**

# CHAPTER 2 MATERIALS, AND PROPERTIES

## 2.1 CONCRETE

Plain concrete is made by mixing cement, fine aggregate, coarse aggregate, water, and frequently admixtures.

Structural concrete can be classified into:

- Lightweight concrete with a unit weight from about $1350\ to\ 1850\ kg/m^3$ produced from aggregates of expanded shale, clay, slate, and slag.

Other lightweight materials such as pumice, scoria, perlite, vermiculite, and diatomite are used to produce insulating lightweight concretes ranging in density from about $250\ to\ 1450\ kg/m^3$.

- Normal-weight concrete with a unit weight from about $1800\ to\ 2400\ kg/m^3$ produced from the most commonly used aggregates— sand, gravel, crushed stone.
- Heavyweight concrete with a unit weight from about $3200\ to\ 5600\ kg/m^3$ produced from such materials such as barite, limonite, magnetite, ilmenite, hematite, iron, and steel punching or shot. It is used for shielding against radiations in nuclear reactor containers and other structures.

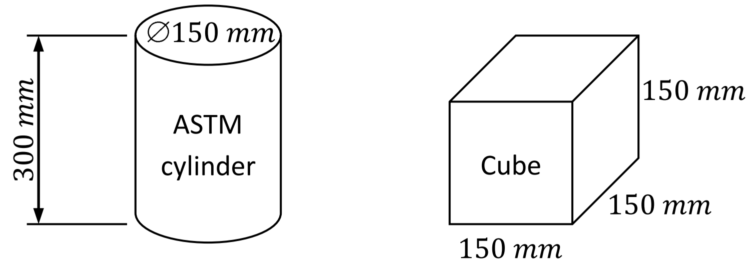
## 2.2 COMPRESSIVE STRENGTH

The strength of concrete is controlled by the proportioning of cement, coarse and fine aggregates, water, and various admixtures. The most important variable is $(w/c)$ ratio.

Concrete strength $(f_c')$ – uniaxial compressive strength measured by a compression test of a standard test cylinder ($150\ mm$ diameter by $300\ mm$ high) on the 28th day–ASTM C31, C39.

In many countries, the standard test unit is the cube ($200 \times 200 \times 200\ mm$).

The concrete strength depends on the size and shape of the test specimen and the manner of testing. For this reason the cylinder ($\varnothing 150\ mm$ by $300\ mm$ high) strength is $80\%$ of the $150-mm$ cube strength and $83\%$ of the $200-mm$ cube strength.




$f_c'$ $f_{cu}$

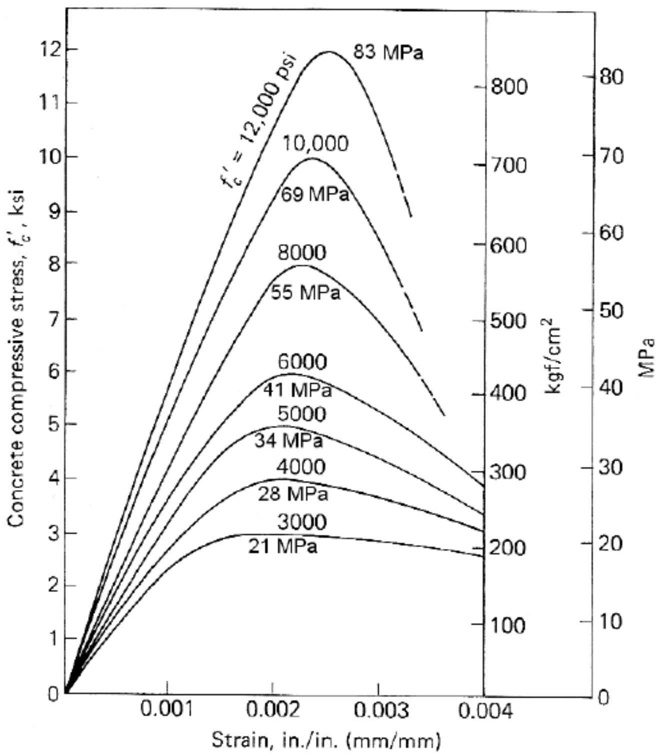
$$\boxed{f_c' \approx 0.80\ f_{cu}}$$

**Stress-strain relationship:** Typical curves for specimens ($150 \times 300\ mm$ cylinders) loaded in compression at 28 days.

Lower-strength concrete has greater deformability (ductility) than higher-strength concrete (length of the portion of the curve after the maximum stress is reached at a strain between $0.002$ and $0.0025$).

Ultimate strain at crushing of concrete varies from $0.003$ to as high as $0.008$.

- In usual reinforced concrete design $f_c{}'$ of ($24\ to\ 35\ MPa$) are used for nonprestressed structures.
- $f_c{}'$ of ($35$ to $42\ MPa$) are used for prestressed concrete.
- $f_c{}'$ of ($42$ to $97\ MPa$) are used particularly in columns of tall buildings.




### 2.3 TENSILE STRENGTH

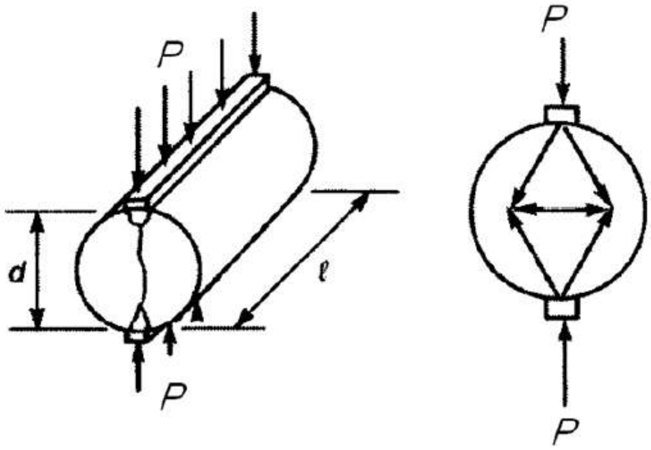
Concrete tensile strength is about $10\ to\ 15\%$ of its compressive strength.

The strength of concrete in tension is an important property that greatly affects that extent and size of cracking in structures.

Tensile strength is usually determined by using:

- Split-cylinder test (ASTM C496). A standard $150 \times 300\ mm$ compression test cylinder is placed on its side and loaded in compression along a diameter. The splitting tensile strength $f_{ct}$ is computed as

$$f_{ct} = \frac{2P}{\pi l d}$$



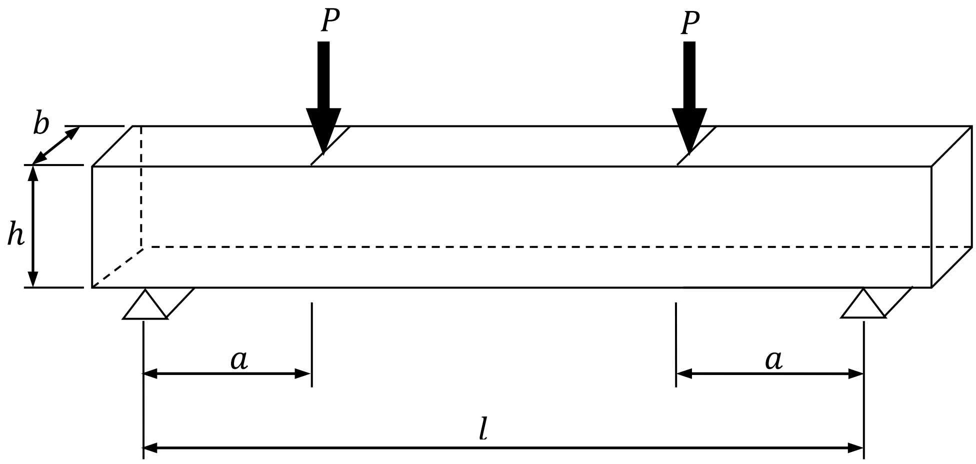

- Tensile strength in flexure (modulus of rupture) (ASTM C78 or C293). A plain concrete beam $150 \times 150\ mm\ \times 750\ mm\ long,$ is loaded in flexure at the third points of 600-mm span until it fails due to cracking on the tension face. Modulus of rupture $f_r$ is computed as

$$f_r = \frac{M}{I}c = \frac{6M}{bh^2} = \frac{6Pa}{bh^2}$$

It is accepted (ACI 9.5.2.3) that an average value for $f_r$ may be taken as

$$f_r = 0.62\,\lambda\sqrt{f_c{}'}\,, \qquad f_c' \quad in\ MPa$$

$$where \qquad \lambda = 1 \quad for\ normalweight\ concrete.$$

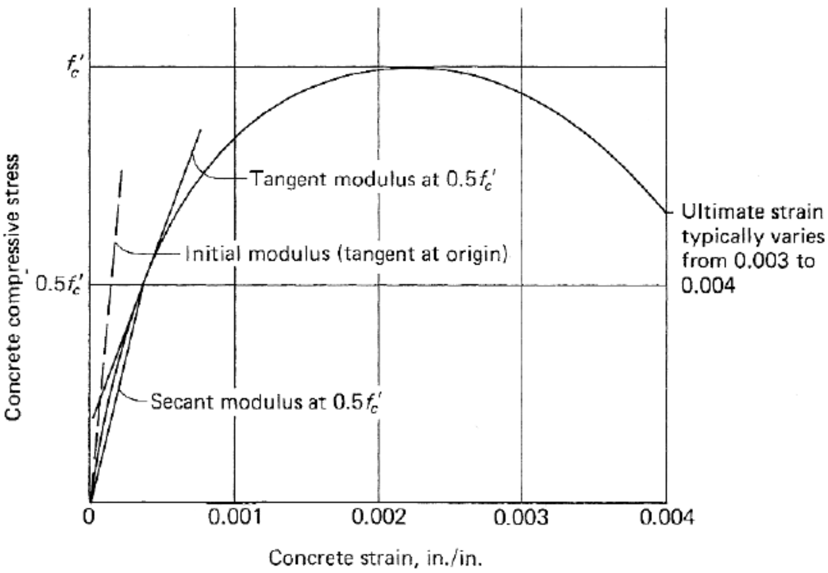



- Direct axial tension test. It is difficult to measure accurately and not in use today.

## 2.4 MODULUS OF ELASTICITY

The modulus of elasticity of concrete varies, unlike that of steel, with strength.

A typical stress-strain curve for concrete in compression is shown. The initial modulus (tangent at origin), the tangent modulus (at $0.5\,f_c{}'$), and the secant modulus are noted. Usually the secant modulus at from $25\ to\ 50\%$ of the compressive strength $f_c{}'$ is considered to be the modulus of elasticity. The empirical formula given by ACI-8.5.1




$$E_c = 0.043 w_c^{1.5}\sqrt{f_c{}'}$$

For normalweight concrete, $E_c$ shall be permitted to be taken as $E_c = 4700\sqrt{f_c{}'}$,

where, $1440 \le w_c \le 2560\ \ kg/m^3 \quad and \qquad f_c'\ in\ MPa$ .

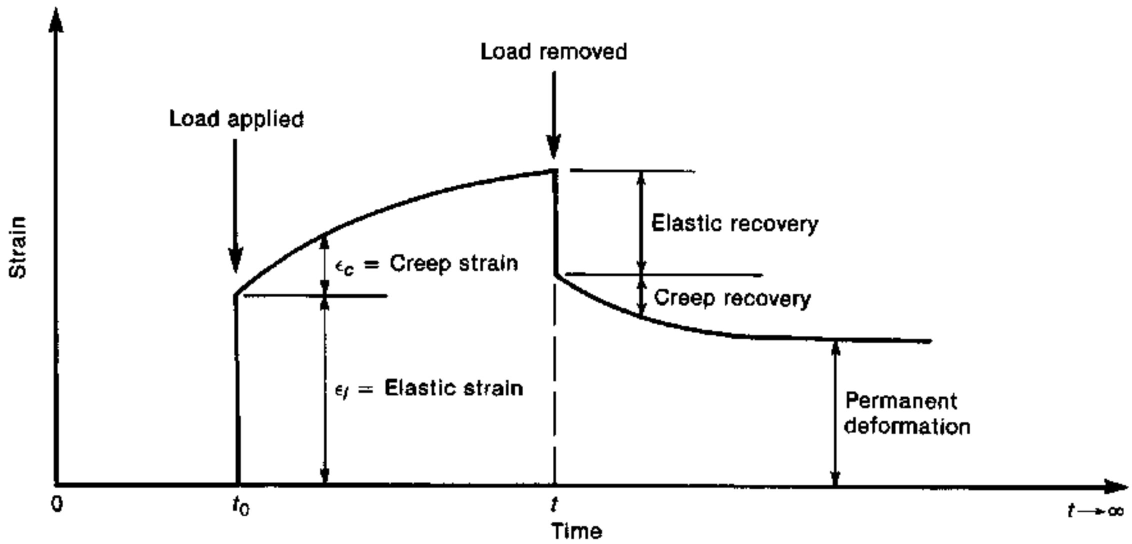
## 2.5 CREEP AND SHRINKAGE

Creep and shrinkage are time-dependent deformations that, along with cracking, provide the greatest concern for the designer because of the inaccuracies and unknowns that surround them. Concrete is elastic only under loads of short duration; and, because of additional deformation with time, the effective behavior is that of an inelastic material. Deflection after a long period of time is therefore difficult to predict, but its control is needed to assure serviceability during the life of the structure.

**Creep (or plastic flow)** is the property of concrete (and other materials) by which it continues to deform with time under sustained loads at unit stresses within the accepted elastic range (say, below $0.5\, f_c'$). This inelastic deformation increases at a decreasing rate during the time of loading, and its total magnitude may be several times as large as the short-time elastic deformation. Frequently creep is associated with shrinkage, since both are occurring simultaneously and often provide the same net effect: increased deformation with time.

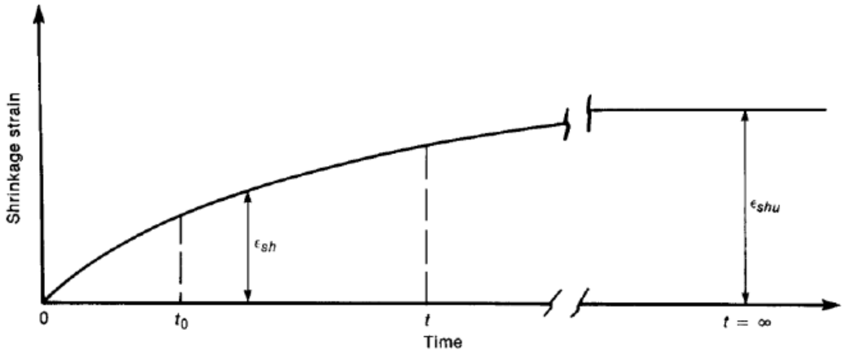
The internal mechanism of creep, or "plastic flow" as it is sometimes called, may be due to any one or a combination of the following: (1) crystalline flow in the aggregate and hardened cement paste; (2) plastic flow of the cement paste surrounding the aggregate; (3) closing of internal voids; and (4) the flow of water out of the cement gel due to external load and drying.

Factors affecting the magnitude of creep are (1) the constituents—such as the composition and fineness of the cement, the admixtures, and the size, grading, and mineral content of the aggregates: (2) proportions such as water content and water-cement ratio; (3) curing temperature and humidity; (4) relative humidity during period of use; (5) age at loading; (6) duration of loading; (7) magnitude of stress; (8) surface-volume ratio of the member; and (9) slump.



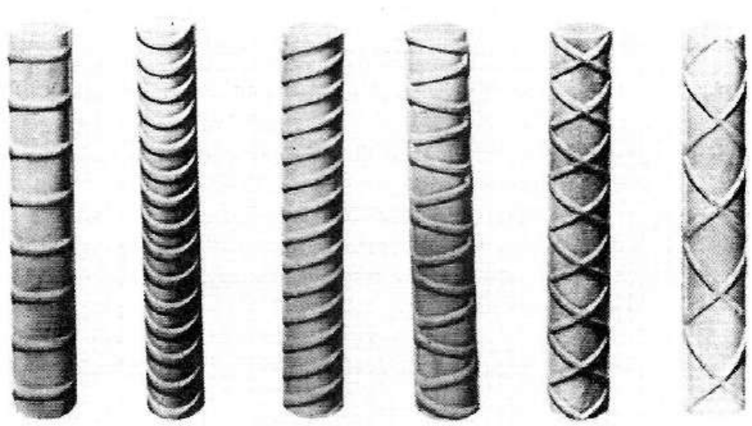

Creep of concrete will often cause an increase in the long-term deflection of members. Unlike concrete, steel is not susceptible to creep. For this reason, steel reinforcement is often provided in the compression zone of beams to reduce their long-term deflection.

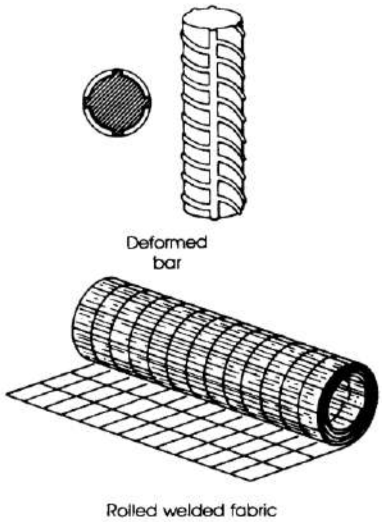
**Shrinkage,** broadly defined, is the volume change during hardening and curing of the concrete. It is unrelated to load application. The main cause of shrinkage is the loss of water as the concrete dries and hardens. It is possible for concrete cured continuously under water to increase in volume; however, the usual concern is with a decrease in volume. In general, the same factors have been found to influence shrinkage strain as those that influence creep—primarily those factors related to moisture loss.




## 2.6 STEEL REINFORCEMENT

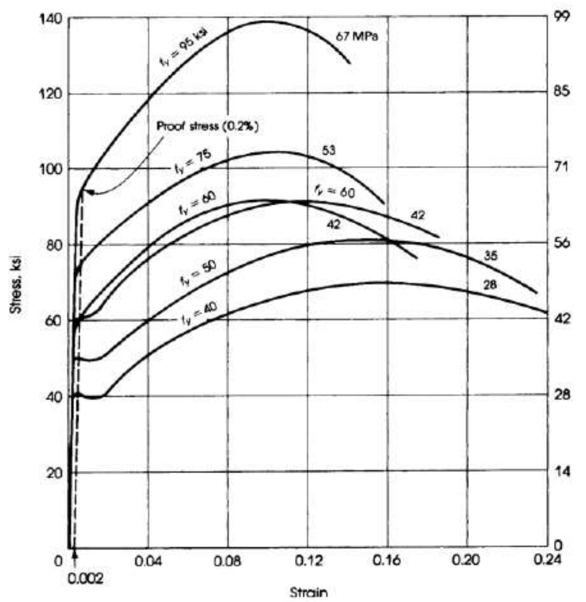
The useful strength of ordinary reinforcing steels in tension as well as compression, the yield strength is about 15 times the compressive strength of common structural concrete and well over 100 times its tensile strength.






Steel reinforcement may consist of :

- Bars (deformed bars, as in picture below) – for usual construction.
- Welded wire fabric – is used in thins slabs, thin shells.
- Wires – are used for prestressed concrete.




The “Grade” of steel is the minimum specified yield stress (point) expressed in:

- $MPa$ for SI reinforcing bar Grades 300, 350, 420, and 520.
- $ksi$ for Inch-Pound reinforcing bar Grades 40, 50, 60, and 75.

The introduction of carbon and alloying additives in steel increases its strength but reduces its ductility. The proportion of carbon used in structural steels varies between 0.2% and 0.3%.

The steel modulus of elasticity $(E_s)$ is constant for all types of steel. The ACI Code has adopted a value of $E_s = 2 \times 10^5\ MPa\ (29 \times 10^6\ psi\ )$.

## Summary of minimum ASTM strength requirements

| Product | ASTM Specification | Designation | Minimum Yield Strength, psi (MPa) | Minimum Tensile Strength, psi (MPa) |
|---|---|---|---|---|
| Reinforcing bars | A615 | Grade 40<br>Grade 60<br>Grade 75 | 40,000 (280)<br>60,000 (420)<br>75,000 (520) | 60,000 (420)<br>90,000 (620)<br>100,000 (690) |
| | A706 | Grade 60 | 60,000 (420)<br>[78,000 (540) maximum] | 80,000 (550)[a] |
| | A996 | Grade 40<br>Grade 50<br>Grade 60 | 40,000 (280)<br>50,000 (350)<br>60,000 (420) | 60,000 (420)<br>80,000 (550)<br>90,000 (620) |
| | A1035 | Grade 100 | 100,000 (690) | 150,000 (1030) |
| Deformed bar mats | A184 | Same as reinforcing bars | | |
| Zinc-coated bars | A767 | Same as reinforcing bars | | |
| Epoxy-coated bars | A775, A934 | Same as reinforcing bars | | |
| Stainless-steel bars[b] | A955 | Same as reinforcing bars | | |
| Wire<br>Plain | A82 | | 70,000 (480) | 80,000 (550) |
| Deformed | A496 | | 75,000 (515) | 85,000 (585) |
| Welded wire reinforcement<br>Plain<br>W1.2 and larger<br>Smaller than W1.2 | A185 | | 65,000 (450)<br>56,000 (385) | 75,000 (515)<br>70,000 (485) |
| Deformed | A497 | | 70,000 (480) | 80,000 (550) |
| Prestressing tendons<br>Seven-wire strand | A416 | Grade 250<br>(stress-relieved) | 212,500 (1465) | 250,000 (1725) |
| | | Grade 250<br>(low-relaxation) | 225,000 (1555) | 250,000 (1725) |
| | | Grade 270<br>(stress-relieved) | 229,500 (1580) | 270,000 (1860) |
| | | Grade 270<br>(low-relaxation) | 243,000 (1675) | 270,000 (1860) |
| Wire | A421 | Stress-relieved | 199,750 (1375) to<br>212,500 (1465)[c] | 235,000 (1620) to<br>250,000 (1725)[c] |
| | | Low-relaxation | 211,500 (1455) to<br>225,000 (1550)[c] | 235,000 (1620) to<br>250,000 (1725)[c] |
| Bars | A722 | Type I (plain)<br>Type II (deformed) | 127,500 (800)<br>120,000 (825) | 150,000 (1035)<br>150,000 (1035) |
| Compacted strand[b] | A779 | Type 245<br>Type 260<br>Type 270 | 241,900 (1480)<br>228,800 (1575)<br>234,900 (1620) | 247,000 (1700)<br>263,000 (1810)<br>270,000 (1860) |

[a] But not less than 1.25 times the actual yield strength.
[b] Not listed in ACI 318.
[c] Minimum strength depends on wire size.

## Cross sectional Areas of standard steel bars for reinforced concrete structures

| Diameter, mm | Area of bars for Number of bars , $cm^2$ | | | | | | | | | Mass, Kg/ m |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| 6 | 0.283 | 0.565 | 0.848 | 1.131 | 1.414 | 1.696 | 1.979 | 2.262 | 2.545 | 0.222 |
| 8 | 0.503 | 1.005 | 1.508 | 2.011 | 2.513 | 3.016 | 3.519 | 4.021 | 4.524 | 0.395 |
| 10 | 0.785 | 1.571 | 2.356 | 3.142 | 3.927 | 4.712 | 5.498 | 6.283 | 7.069 | 0.617 |
| 12 | 1.131 | 2.262 | 3.393 | 4.524 | 5.655 | 6.786 | 7.917 | 9.048 | 10.179 | 0.888 |
| 14 | 1.539 | 3.079 | 4.618 | 6.158 | 7.697 | 9.236 | 10.776 | 12.315 | 13.854 | 1.208 |
| 16 | 2.011 | 4.021 | 6.032 | 8.042 | 10.053 | 12.064 | 14.074 | 16.085 | 18.096 | 1.578 |
| 18 | 2.545 | 5.089 | 7.634 | 10.179 | 12.723 | 15.268 | 17.813 | 20.358 | 22.902 | 1.998 |
| 20 | 3.142 | 6.283 | 9.425 | 12.566 | 15.708 | 18.850 | 21.991 | 25.133 | 28.274 | 2.466 |
| 22 | 3.801 | 7.603 | 11.404 | 15.205 | 19.007 | 22.808 | 26.609 | 30.411 | 34.212 | 2.984 |
| 25 | 4.909 | 9.817 | 14.726 | 19.635 | 24.544 | 29.452 | 34.361 | 39.270 | 44.179 | 3.854 |
| 28 | 6.158 | 12.315 | 18.473 | 24.630 | 30.788 | 36.945 | 43.103 | 49.260 | 55.418 | 4.834 |
| 32 | 8.042 | 16.085 | 24.127 | 32.170 | 40.212 | 48.255 | 56.297 | 64.340 | 72.382 | 6.314 |
| 36 | 10.179 | 20.358 | 30.536 | 40.715 | 50.894 | 61.073 | 71.251 | 81.430 | 91.609 | 7.991 |
| 40 | 12.566 | 25.133 | 37.699 | 50.265 | 62.832 | 75.398 | 87.965 | 100.531 | 113.097 | 9.865 |
| 45 | 15.904 | 31.809 | 47.713 | 63.617 | 79.522 | 95.426 | 111.330 | 127.235 | 143.139 | 12.486 |

# CHAPTER 3 DESIGN METHODS AND REQUIREMENTS

## 3.1 ACI BUILDING CODE

When two different materials, such as steel and concrete, act together, it is understandable that the analysis for strength of a reinforced concrete member has to be partly empirical. These principles and methods are being constantly revised and improved as results of theoretical and experimental research accumulate. The American Concrete Institute (ACI), serving as a clearinghouse for these changes, issues building code requirements, the most recent of which is the Building Code Requirements for Structural Concrete (ACI 318-08), hereafter referred to as the ACI Code.

The ACI Code is a Standard of the American Concrete Institute. In order to achieve legal status, it must be adopted by a governing body as a part of its general building code. The ACI Code is partly a specification-type code, which gives acceptable design and construction methods in detail, and partly a performance code, which states desired results rather than details of how such results are to be obtained. A building code, legally adopted, is intended to prevent people from being harmed; therefore, it specifies minimum requirements to provide adequate safety and serviceability. It is important to realize that a building code is not a recommended practice, nor is it a design handbook, nor is it intended to replace engineering knowledge, judgment, or experience. It does not relieve the designer of the responsibility for having a safe, economical structure.

ACI 318M-08 – Building Code Requirements for Structural Concrete and Commentary.

Two philosophies of design have long been prevalent:

- The working stress method (1900 – 1960).
- The strength design method (1960 till now, with few exceptions).

## 3.2 WORKING STRESS METHOD

In the working stress method, a structural element is so designed that the stresses resulting from the action of service loads (also called working loads) and computed by the mechanics of elastic members do not exceed some predesignated allowable values.

Service load is the load, such as dead, live, snow, wind, and earthquake, which is assumed actually to occur when the structure is in service.

The working stress method may be expressed by the following:

$$f \leq f_{allow}$$

where

$f -$ an elastic stress, such as by using the flexure formula $f = Mc/I$ for a beam, computed under service load.

$f_{allow} -$ a limiting or allowable stress prescribed by a building code as a percentage of the compressive strength $f_c'$ for concrete, or of the yield stress for the steel reinforcing bars.

### 3.3 STRENGTH DESIGN METHOD

In the strength design method (formerly called ultimate strength method), the service loads are increased by factors to obtain the load at which failure is considered to be "imminent". This load is called the factored load or factored service load. The structure or structural element is then proportioned such that the strength is reached when the factored load is acting. The computation of this strength takes into account the nonlinear stress-strain behavior of concrete.

The strength design method may be expressed by the following,

**strength provided ≥ [strength required to carry factored loads]**

where the "strength provided" (such as moment strength) is computed in accordance with the provisions of a building code, and the "strength required" is that obtained by performing a structural analysis using factored loads.

### 3.4 SAFETY PROVISIONS

Structures and structural members must always be designed to carry some reserve load above what is expected under normal use. Such reserve capacity is provided to account for a variety of factors, which may be grouped in two general categories:

- factors relating to overload
- factors relating to understrength (that is, less strength than computed by acceptable calculating procedures).

Overloads may arise from changing the use for which the structure was designed, from underestimation of the effects of loads by oversimplification in calculation procedures, and from effects of construction sequence and methods. Understrength may result from adverse variations in material strength, workmanship, dimensions, control, and degree of supervision, even though individually these items are within required tolerances.

In the strength design method, the member is designed to resist factored loads, which are obtained by multiplying the service loads by load factors. Different factors are used for different loadings. Because dead loads can be estimated quite accurately, their load factors are smaller than those of live loads, which have a high degree of uncertainty. Several load combinations must be considered in the design to compute the maximum and minimum design forces. Reduction factors are used for some combinations of loads to reflect the low probability of their simultaneous occurrences. The ACI Code presents specific values of load factors to be used in the design of concrete structures.

In addition to load factors, the ACI Code specifies another factor to allow an additional reserve in the capacity of the structural member. The nominal strength is generally calculated using accepted analytical procedure based on statistics and equilibrium; however, in order to account for the degree of accuracy within which the nominal strength can be calculated, and for adverse variations in materials and dimensions, a strength reduction factor, $\phi$, should be used in the strength design method.

To summarize the above discussion, the ACI Code has separated the safety provision into an overload or load factor and to an undercapacity (or strength reduction) factor, $\phi$. A safe design is achieved when the structure's strength, obtained by multiplying the nominal strength by the reduction factor, $\phi$, exceeds or equals the strength needed to withstand the factored loadings (service loads times their load factors).
The requirement for strength design may be expressed:

$$\text{Design strength } \geq \text{Factored load (i.e., required strength)}$$
$$\phi P_n \geq P_u$$
$$\phi M_n \geq M_u$$
$$\phi V_n \geq V_u$$

where $P_n$, $M_n$, and $V_n$ are "nominal" strengths in axial compression, bending moment, and shear, respectively, using the subscript n.
$P_u$, $M_u$, and $V_u$ are the factored load effects in axial compression, bending moment, and shear, respectively, using the subscript u.
Given a load factor of 1.2 for dead load and a load factor of 1.6 for live load, the overall safety factor for a structure loaded be a dead load, $D$, and a live load, $L$, is

$$\text{Factor of Safety} = \frac{1.2D + 1.6\,L}{D + L}\left(\frac{1}{\phi}\right)$$

### 3.5 LOAD FACTORS AND STRENGTH REDUCTION FACTORS

**Overload Factors $U$**

The factors $U$ for overload as given by ACI-9.2 are:

$$U = 1.4(D + F)$$
$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$$
$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$$
$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$
$$U = 1.2D + 1.0E + 1.0L + 0.2S$$
$$U = 0.9D + 1.6W + 1.6H$$
$$U = 0.9D + 1.0E + 1.6H$$

where
$D$ - dead load; $L$ - live load; $L_r$- roof live load; $S$ - snow load; $R$ - rain load; $W$ - wind load; $E$ - earthquake load; $F$ - load due to weights and pressures of fluids with well-defined densities and controllable maximum heights; $H$ - load due to weight and pressure of soil, water in soil or other materials; $T$ - the cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage compensating concrete.

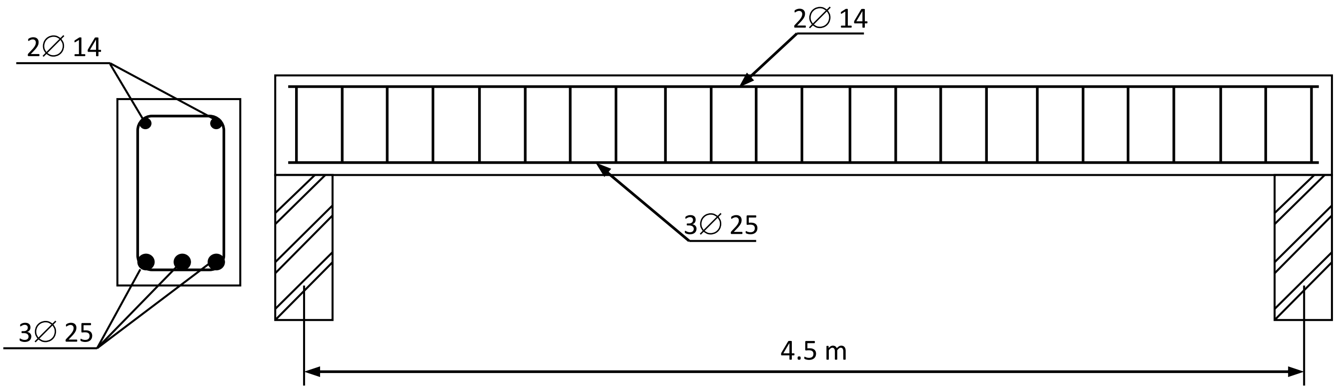
**Strength Reduction Factors $\phi$**

The factors $\phi$ for understrength are called strength reduction factors according to ACI-9.3. and are as follows:

| Strength Condition | $\phi$ Factors |
|---|---|
| 1. Flexure (with or without axial force) | |
| Tension-controlled sections ........ | 0.90 |
| Compression-controlled sections | |
| Spirally reinforced ........ | 0.75 |
| Others ........ | 0.65 |
| 2. Shear and torsion ........ | 0.75 |
| 3. Bearing on concrete ........ | 0.65 |
| 4. Post-tensioned anchorage zones ........ | 0.85 |
| 5. Struts, ties, nodal zones, and bearing areas in strut-and-tie models ..... | 0.75 |

**Example:**

A simple beam is loaded with a dead load of 40 $KN/m$ and a live load of 30 $KN/m$. Check the strength requirement according to ACI code if the nominal bending moment $M_n = 275\ KN.m$




**<u>Solution</u>:**

$$M_n = 275\ KN.m \qquad and \qquad \phi = 0.9$$

$$w_u = 1.2D + 1.6L = 1.2 \cdot 40 + 1.6 \cdot 30 = 96\ \ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{96 \cdot 4.5^2}{8} = 243\ KN \cdot m$$

$$\phi M_n \geq M_u$$

$0.9 \cdot 275 = 247.5\ KN \cdot m > 243\ KN \cdot m$ OK Strength requirement is satisfied

$$\text{Factor of Safety} = \frac{1.2D + 1.6\ L}{D + L}\left(\frac{1}{\phi}\right) = \frac{96}{40 + 30}\left(\frac{1}{0.9}\right) = 1.52$$

# CHAPTER 4 FLEXURE IN BEAMS

## 4.1 INTRODUCTION

Reinforced concrete beams are nonhomogeneous in that they are made of two entirely different materials. The methods used in the analysis of reinforced concrete beams are therefore different from those used in the design or investigation of beams composed entirely of steel, wood, or any other structural material.

Two different types of problems arise in the study of reinforced concrete:

1. Analysis. Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.
2. Design. Given a factored design moment, normally designated as $M_u$. select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.

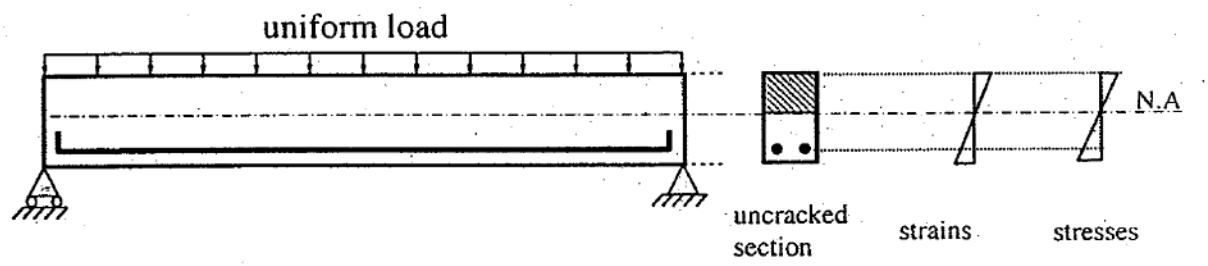
The Strength Design Method requires the conditions of static equilibrium and strain compatibility across the depth of the section to be satisfied.

The following are the assumptions for Strength Design Method:

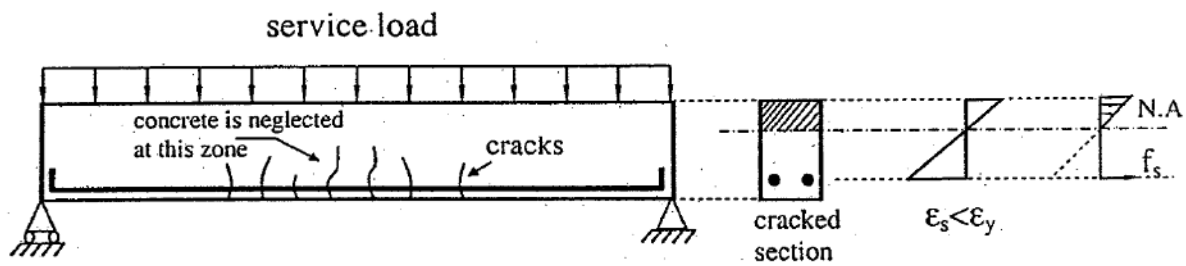
1. Strains in reinforcement and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
2. Concrete sections are considered to have reached their flexural capacities when they develop $0.003$ strain in the extreme compression fiber.
3. Stress in reinforcement varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
4. Tensile strength of concrete is neglected.
5. Compressive stress distribution of concrete can be represented by the corresponding stress-strain relationship of concrete. This stress distribution may be simplified by a rectangular stress distribution as described later.

## 4.2 REINFORCED CONCRETE BEAM BEHAVIOR

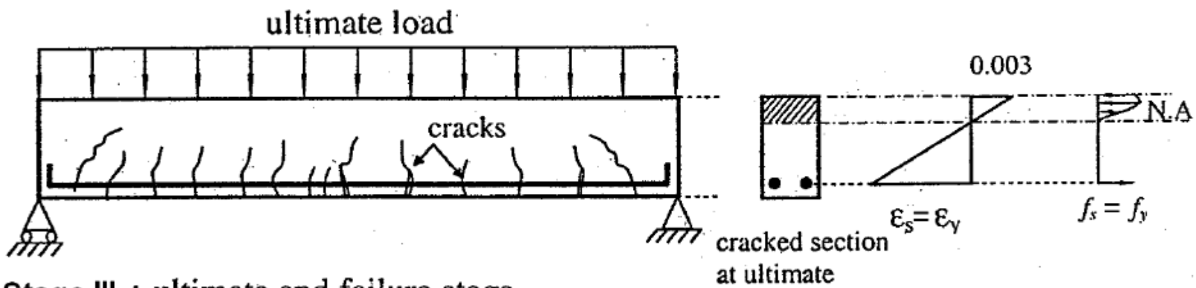
Consider a simply supported and reinforced concrete beam with uniformly distributed load on top. Under such loading and support conditions, flexure-induced stresses will cause compression at the top and tension at the bottom of the beam. Concrete, which is strong in compression, but weak in tension, resists the force in the compression zone, while steel reinforcing bars are placed in the bottom of the beam to resist the tension force. As the applied load is gradually increased from zero to failure of the beam (ultimate condition), the beam may be expected to behave in the following manner:



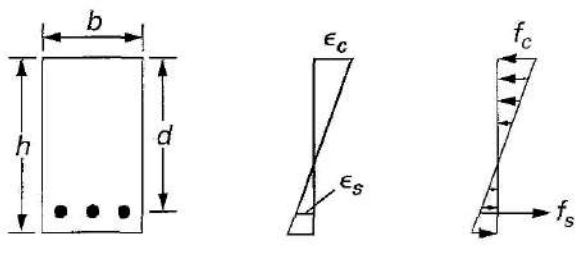

**Stage I** : before cracking



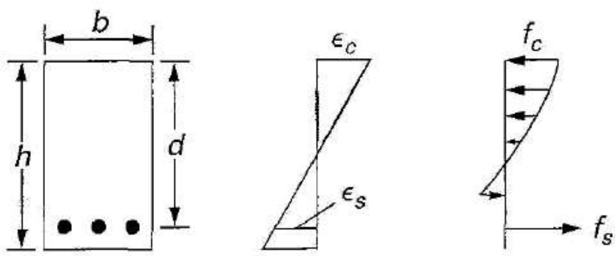

**Stage II** : cracking stage, before yield, working load




**Stage III** : ultimate and failure stage

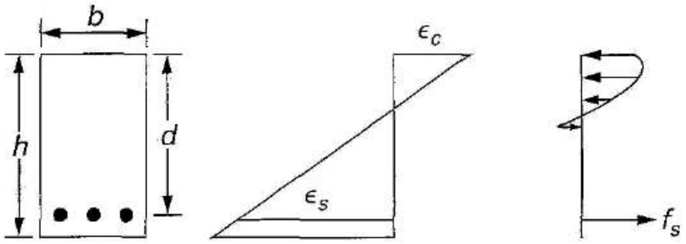



**Stage I**: when the applied load is low, the stress distribution is essentially linear over the depth of the section. The tensile stresses in the concrete are low enough so that the entire cross-section remains uncracked and the stress distribution is as shown in (a). In the compression zone, the concrete stresses are low enough (less than about $0.5\ f_c'$) so that their distribution is approximately linear.



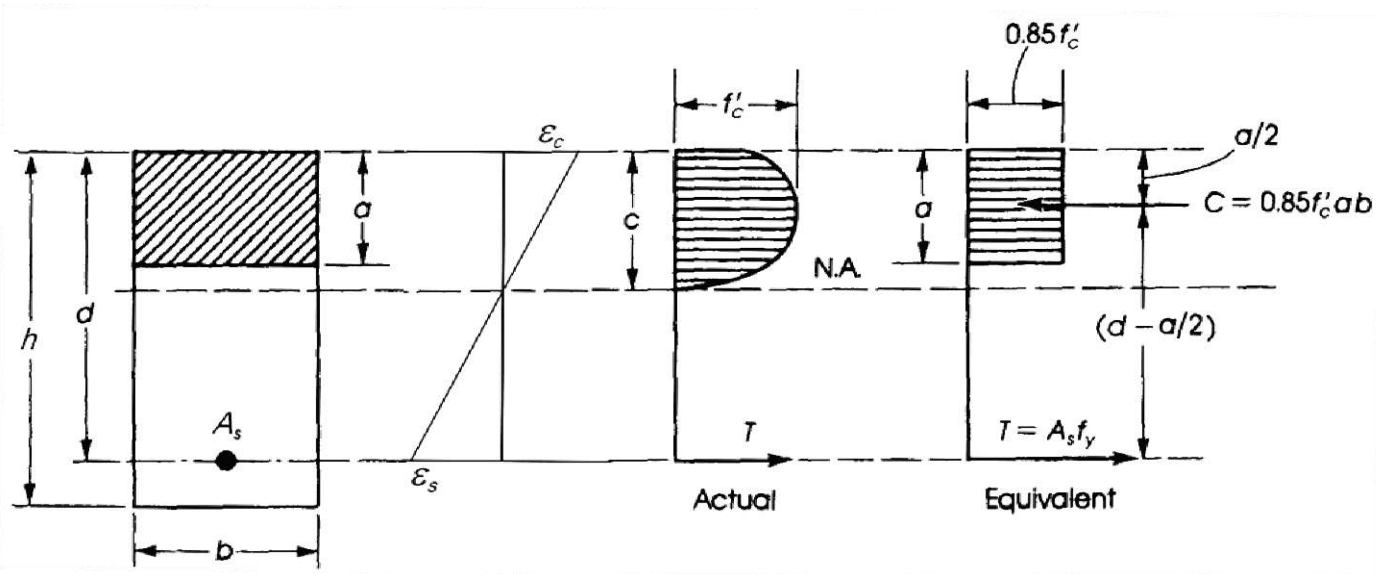

**Stage II:** On increasing the applied load, the tensile stresses at the bottom of the beam become high enough to exceed the tensile strength at which the concrete cracks. After cracking, the tensile force is resisted mainly by the steel reinforcement. Immediately below the neutral axis, a small portion of the beam remains uncracked. These tensile stresses in the concrete offer, however, only a small contribution to the flexural strength. The concrete stress distribution in the compression zone becomes nonlinear.

**Stage III:** at nominal (so,-called ultimate) strength, the neutral axis has moved farther up-ward as flexural cracks penetrate more and more toward the compression face. The steel reinforcement has yielded and the concrete stress distribution in the compression zone becomes more nonlinear. Below the neutral axis, the concrete is cracked except for a very small zone.



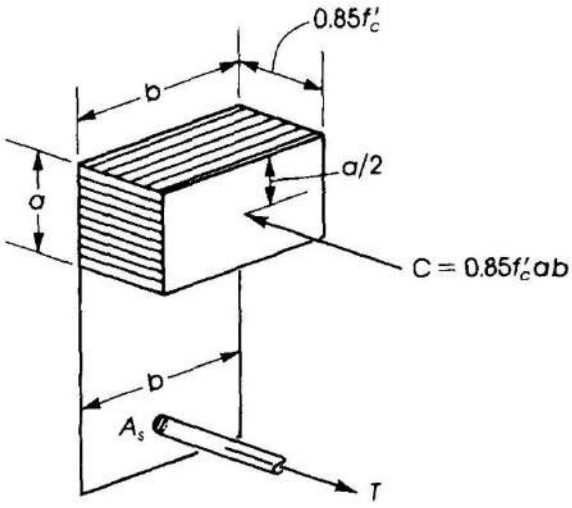

At the ultimate stage, two types of failure can be noticed. If the beam is reinforced with a small amount of steel, ductile failure will occur. In this type of failure, the steel yields and the concrete crushes after experiencing large deflections and lots of cracks. On the other hand, if the beam is reinforced with a large amount of steel, brittle failure will occur. The failure in this case is sudden and occurs due to the crushing of concrete in the compression zone without yielding of the steel and under relatively small deflections and cracks. This is not a preferred mode of failure because it does not give enough warning before final collapse.

## 4.3 THE EQUIVALENT RECTANGULAR COMPRESSIVE STRESS DISTRIBUTION (COMPRESSIVE STRESS BLOCK)




The actual distribution of the compressive stress in a section has the form of rising parabola. It is time consuming to evaluate the volume of compressive stress block. An equivalent rectangular stress block can be used without loss of accuracy.

The flexural strength $M_n$, using the equivalent rectangular, is obtained as follows:




$$C = 0.85\, f_c' ab$$
$$T = A_s f_y$$

$\sum F_x = 0$ gives $T = C$

$$A_s f_y = 0.85\, f_c' ab$$

or

$$a = \frac{A_s f_y}{0.85\, f_c' b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' ab\left(d - \frac{a}{2}\right)$$

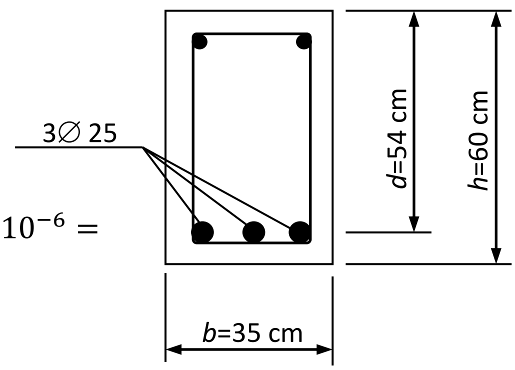
**Notation:**

$a$ – depth of rectangular compressive stress block,
$b$ – width of the beam at the compression side,
$c$ – depth of the neutral axis measured from the extreme compression fibers,
$d$ – effective depth of the beam, measured from the extreme compression fibers to the centroid of the steel area,
$h$ – total depth of the beam,
$\varepsilon_c$ – strain in extreme compression fibers,
$\varepsilon_s$ – strain at tension steel,
$f_c'$ – compressive strength of concrete,
$f_y$ – yield stress of steel,
$A_s$ – area of the tension steel,
$C$ – resultant compression force in concrete,
$T$ – resultant tension force in steel,
$M_n$ – nominal moment strength of the section.

**Example:**

Determine the nominal moment strength of the beam section. Take $f_c' = 20\, MPa$, $f_y = 400\, MPa$.

**Solution:**




$A_s(3\varnothing\, 25) = 14.72\, cm^2$

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{14.72 \cdot 100 \cdot 400}{0.85 \cdot 20 \cdot 350} = 98.96\, mm$$

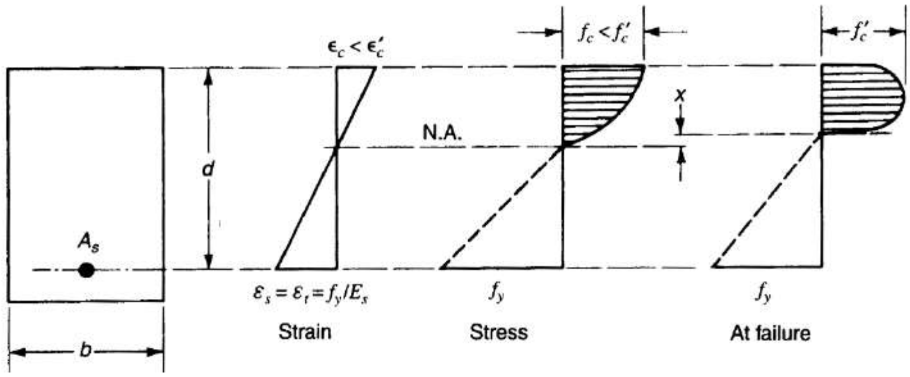
$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 14.72 \cdot 100 \cdot 400\left(540 - \frac{98.96}{2}\right) \cdot 10^{-6} =$$
$$= 288.82\, KN \cdot m$$

### 4.4 TYPES OF FAILURE AND STRAIN LIMITS

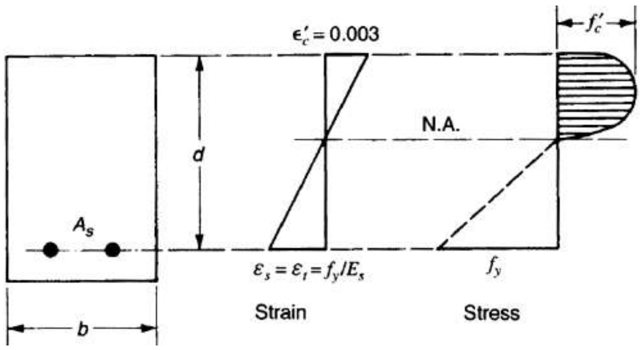
**Types of failure**

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

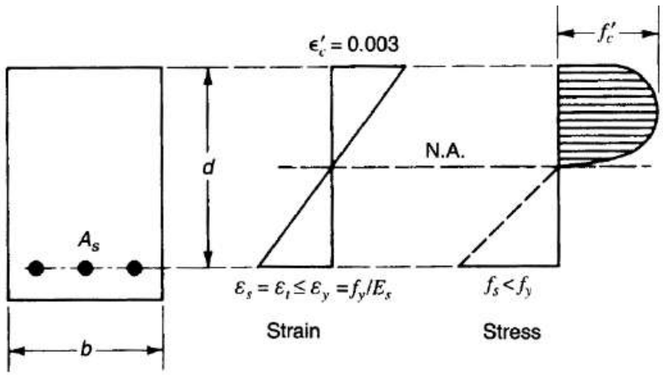
1. Steel may reach its yield strength before the concrete reaches its maximum strength, In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than $0.005$. The section contains a relatively small amount of steel and is called a **tension-controlled section**.




2. Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a **balanced section**.




3. Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of $0.003$ are reached, but the steel stress is less than the yield strength, that is, $f_s$ is less than $f_y$. The strain in the steel is equal to or less than $0.002$. This section is called a **compression-controlled section**.




The ACI Code assumes that concrete fails in compression when the concrete strain reaches $0.003$.

In beams designed as tension-controlled sections, steel yields before the crushing of concrete. Cracks widen extensively, giving warning before the concrete crushes and the structure collapses. The ACI Code adopts this type of design. In beams designed as balanced or compression-controlled sections, the concrete fails suddenly, and the beam collapses immediately without warning. The ACI Code **does not allow** this type of design.

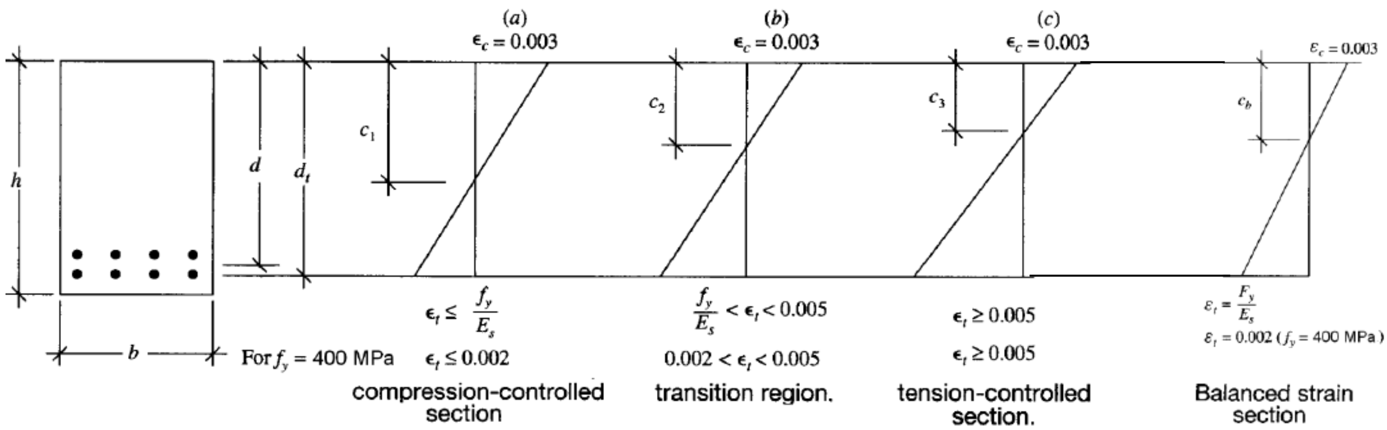
**Strain Limits for Tension and Tension-Controlled Sections**

The ACI Code, Section 10.3. defines the concept of tension or compression-controlled sections in terms of net tensile strain $\varepsilon_t$ (net tensile strain in the reinforcement closest to the tension face). Moreover, two other conditions may develop: (1) the balanced strain condition and (2) the transition region condition.

These four conditions are defined as follows:

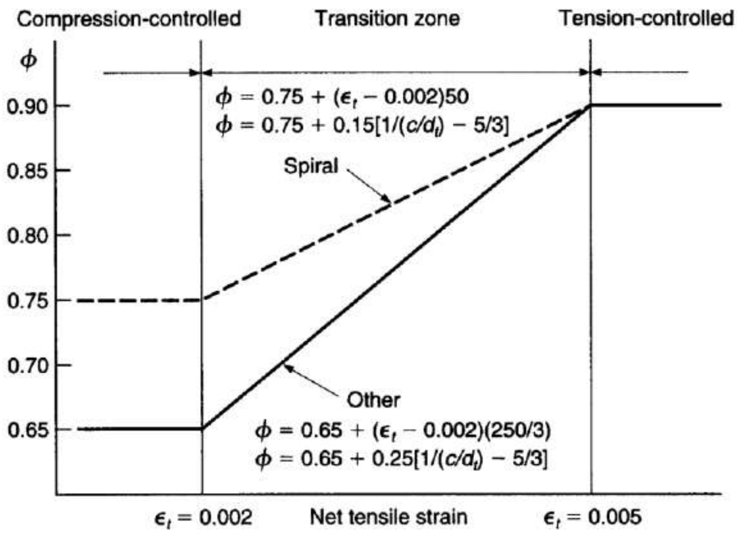
1. Compression-controlled sections are those sections in which $\varepsilon_t$ at nominal strength is equal to or less than the compression-controlled strain limit (the compression-controlled strain limit may be taken as a net strain of $\varepsilon_y = 0.002 -$ for $f_y = 400\ MPa$ ) at the time when concrete in compression reaches its assumed strain limit of $0.003$, ($\varepsilon_c = 0.003$). This case occurs mainly in columns subjected to axial forces and moments.
2. Tension-controlled sections are those sections in which the $\varepsilon_t$ is equal to or greater than $0.005$ just as the concrete in the compression reaches its assumed strain limit of $0.003$
3. Sections in which the $\varepsilon_t$ lies between the compression-controlled strain limit of $0.002$ (for $f_y = 400\ MPa$) and the tension-controlled strain limit of $0.005$ constitute the transition region.
4. The balanced strain condition develops in the section when the tension steel, with the first yield, reaches a strain corresponding to its yield strength, $f_y$ or $\varepsilon_s = \frac{f_y}{E_s}$, just as the maximum strain in concrete at the extreme compression fibers reaches $0.003$.

**In addition** to the above four conditions, Section 10.3.5 of the ACI Code indicates that the net tensile strain, $\varepsilon_t$, at nominal strength, within the transition region, shall not be less than $0.004$ for reinforced concrete flexural members without or with an axial load less than $0.10\ f_c' A_g$, where $A_g =$ gross area of the concrete section.




Strain limit distribution

Note that in cases where strain is less than $0.005$ namely, the section is in the transition zone, a value of the reduction $\phi$ lower than $0.9$ for flexural has to be used for final design moment, with a strain not less than $0.004$ as a limit.



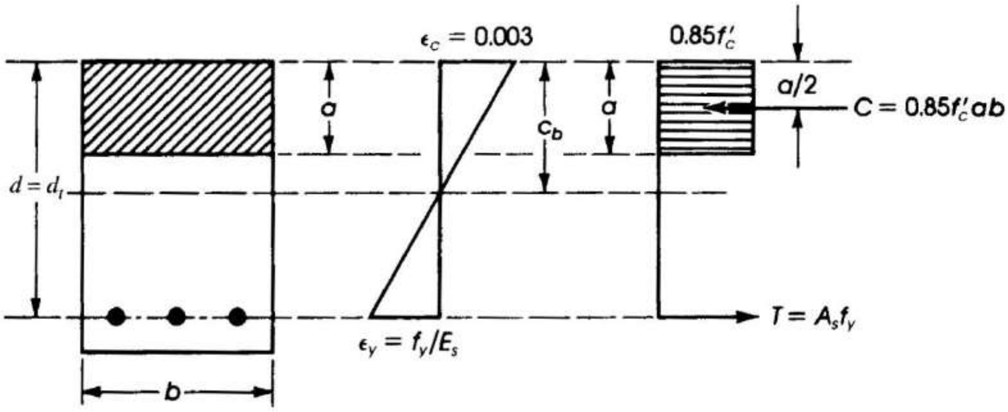

For transition region $\phi$ may be determined by linear interpolation:

$$\phi = 0.75 + (\varepsilon_t - 0.002)50 - \ for\ spiral\ members$$

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) - \ for\ other\ members$$

### 4.5 THE BALANCED CONDITION

Let us consider the case of balanced section, which implies that at ultimate load the strain in concrete equals 0.003 and that of steel equals $\varepsilon_t = \frac{f_y}{E_s}$ (at distance $d_t$).




$$\frac{c_b}{0.003} = \frac{d}{0.003 + \frac{f_y}{E_s}}, \qquad or \qquad c_b = \frac{d}{0.003 + \frac{f_y}{E_s}} 0.003$$

Substituting $E_s = 200\ 000\ MPa$

$$c_b = \frac{600}{600 + f_y} d$$

From equation of equilibrium $\sum F_x = 0$

$$T = C \qquad \Rightarrow \qquad A_s f_y = 0.85\, f_c' ab$$

$a -$ the depth of compressive block and equal $a = \beta_1 c$.
For balanced condition, $a_b = \beta_1 c_b$.
where $\beta_1$ as defined in ACI 10.2.7.3 equal:

$$\beta_1 = 0.85 - 0.007(f_c' - 28) \qquad 0.65 \le \beta_1 \le 0.85$$

The reinforcement ratio for tension steel

$$\rho = \frac{A_s}{bd} \qquad and\ balanced\ reinforcement\ ratio\ \rho_b = \frac{(A_s)_b}{bd}$$

$$\frac{(A_s)_b}{bd} = 0.85\, \frac{f_c'}{f_y} \beta_1 \frac{600}{600 + f_y}$$

$$\rho_b = 0.85\, \frac{f_c'}{f_y} \beta_1 \left(\frac{600}{600 + f_y}\right)$$

### 4.6 UPPER AND LOWER (MINIMUM) STEEL PERCENTAGES.

The maximum reinforcement ratio $\rho_{max}$ that ensures a minimum net tensile steel strain of 0.004.

$$\rho\ (\varepsilon_t = 0.004) = \frac{0.003 + \varepsilon_y}{0.003 + 0.004} \rho_b = \frac{0.003 + \varepsilon_y}{0.007} \rho_b = \rho_{max}$$

For Grade 420 reinforcing bars $\varepsilon_y = 0.002$, then

$$\rho_{max} = \frac{0.003 + 0.002}{0.007} \rho_b = \frac{0.005}{0.007} \rho_b = 0.724 \rho_b$$

If the factored moment applied on a beam is very small and the dimensions of the section are specified (as is sometimes required architecturally) and are larger than needed to resist the factored moment, the calculation may show that very small or no steel reinforcement is required. The ACI Code, 10.5, specifies a minimum steel area, $A_{s,min}$

$$A_{s,min} = 0.25 \frac{\sqrt{f_c'}}{f_y} b_w d$$

and not less than

$$A_{s,min} = \frac{1.4}{f_y} b_w d$$

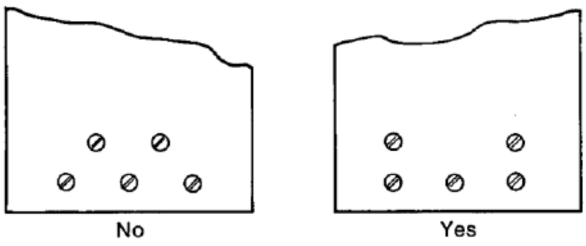
The above requirements of $A_{s,min}$ need not be applied if, at every section, $A_s$ provided is at least one-third greater than that required by analysis ($A_{s,provided} \ge 1.33 A_{s,required}$). This exception provides sufficient additional reinforcement in large members where the amount required by the above equations would be excessive.

$b_w -$ width of section, width of web for T-section, $mm$.

### 4.7 SPACING LIMITS AND CONCRETE PROTECTION FOR REINFORCEMENT.

The minimum limits were originally established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that may cause shear or shrinkage cracking.

According to ACI 7.6. The minimum clear spacing between parallel bars in a layer shall be $d_b$, but not less than $25\,mm$. Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than $25\,mm$.



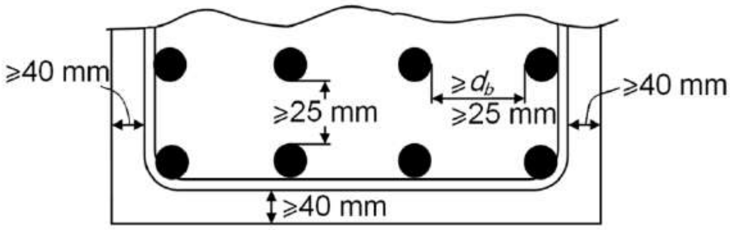
Arrangement of bars in two layers (ACI Section 7.6.2).

In addition, the nominal maximum size of coarse aggregate shall be not larger than:

(a) $1/5$ the narrowest dimension between sides of forms, nor

(b) $1/3$ the depth of slabs, nor

(c) $3/4$ the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, individual tendons, bundled tendons, or ducts.




Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies. Where concrete cover is prescribed for a class of structural members, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars. According to ACI, 7.7, minimum clear cover in cast-in-place concrete beams and columns should not be less than $40\,mm$.

To limit the widths of flexural cracks in beams and slabs, ACI Code Section 10.6.4 defines upper limit on the center-to-center spacing between bars in the layer of reinforcement closest to the tension face of a member. In some cases, this requirement could force a designer to select a larger number of smaller bars in the extreme layer of tension reinforcement. The spacing limit is:

$$s = 380\left(\frac{280}{f_s}\right) - 2.5C_c \qquad \text{but} \qquad s \le 300\left(\frac{280}{f_s}\right)$$

where $C_c$ is the least distance from surface of reinforcement to the tension face. It shall be permitted to take $f_s$ as $\frac{2}{3}f_y$.

### 4.8 ANALYSIS OF SINGLY REINFORCED CONCRETE RECTANGULAR SECTIONS FOR FLEXURE.

**Given**: section dimensions $b, h$; reinforcement $A_s$; material strength $f_c', f_y$.

**Required**: $M_n$ − Nominal moment strength.

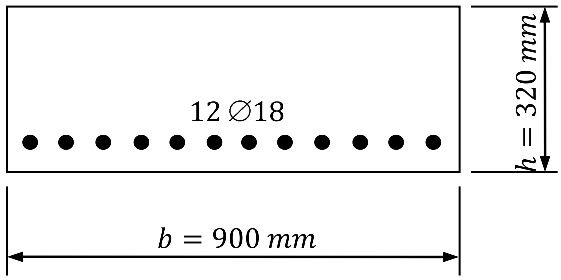
$$T = C \quad \Longrightarrow \quad A_s f_y = 0.85\, f_c' ab \quad \Longrightarrow \quad a = \frac{A_s f_y}{0.85\, f_c' b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' ab\left(d - \frac{a}{2}\right)$$

**Example:**

Determine the nominal moment strength of the beam section. Take $f_c' = 30\ MPa$, $f_y = 420\ MPa$.




**Solution:**

$A_s(12\varnothing\ 18) = 30.536\ cm^2 = 3053.6\ mm^2$

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{3053.6\ \cdot 420}{0.85 \cdot 30 \cdot 900} = 55.88\ mm$$

$$d = 320 - 40 - 10 - \frac{18}{2} = 261\ mm$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 3053.6\ \cdot 420\left(261 - \frac{55.88}{2}\right) \cdot 10^{-6} = 298.9\ KN \cdot m$$

Check for strain:

$$\varepsilon_s = 0.003\left(\frac{d - c}{c}\right)$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$$c = \frac{55.88}{0.836} = 66.84\ mm$$

$$\varepsilon_s = 0.003\left(\frac{261 - 66.84}{66.84}\right) = 0.00871 > 0.005$$

Take $\phi = 0.9\ for\ flexure$

$$\phi M_n = 0.9 \cdot 298.9 = 269.01\ KN \cdot m$$

## 4.9 DESIGN OF SINGLY REINFORCED CONCRETE RECTANGULAR SECTIONS FOR FLEXURE.

**Given**: $M_u$ − factored moment ($M_u \le \phi M_n$); material strength $f_c', f_y$.

**Required**: section dimensions $b, h$; reinforcement $A_s$.

The two conditions of equilibrium are

$$T = C \tag{1}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right) \qquad (2)$$

Reinforcement ratio

$$\rho = \frac{A_s}{bd} \qquad or \qquad A_s = \rho bd$$

Substituting into (1)

$$\rho bdf_y = 0.85\, f_c' ab$$

$$a = \rho\left(\frac{f_y}{0.85 f_c'}\right) d \qquad (3)$$

Substituting (3) into (2)

$$M_n = \rho bdf_y\left[d - \frac{\rho}{2}\left(\frac{f_y}{0.85 f_c'}\right) d\right] \qquad (4)$$

A strength coefficient of resistance $R_n$ is obtained by dividing (4) by $(bd^2)$ and letting

$$m = \left(\frac{f_y}{0.85 f_c'}\right)$$

Thus

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{\rho m}{2}\right) \qquad (5)$$

From which $\rho$ may be determined

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) \qquad (6)$$

**Design Procedure:**

1. Set $M_u = \phi M_n = \phi R_n\, bd^2$
2. For ductile behavior such that beam is well into the tension controlled zone, a reinforcement percentage $\rho$ should be chosen in the range of $(40 - 60)\%$ of $\rho_b$. Assume $\rho = (0.4 - 0.6)\rho_b$.

$$\rho_b = 0.85\, \frac{f_c'}{f_y} \beta_1 \left(\frac{600}{600 + f_y}\right).$$

3. Find the flexural resistance factor $R_n$

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right), \qquad\qquad m = \left(\frac{f_y}{0.85 f_c'}\right)$$

4. Determine the required dimensions $b, d$

$$bd^2 = \frac{M_n}{R_n} = \frac{M_u}{\phi R_n}$$

5. Determine the required steel area for the chosen $b, d$

$$A_s = \rho bd$$

Where

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right), \qquad R_n = \frac{M_n}{bd^2}, \qquad m = \left(\frac{f_y}{0.85f_c'}\right)$$

6. Check for minimum steel reinforcement area

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

Or

$$\rho_{min} = 0.25\frac{\sqrt{f_c'}}{f_y} \geq \frac{1.4}{f_y}$$

$$If \quad A_{s,provided} \geq \frac{4}{3}A_{s,required} \quad - \quad NO\ need\ to\ use\ A_{s,min}$$

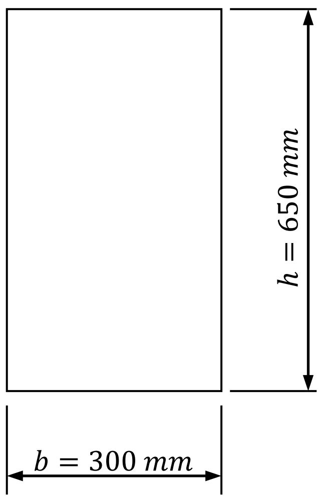
7. Check for strain ($\varepsilon_s \geq 0.005$) – tension-controlled section.
8. Check for steel bars arrangement in section.

**Example:**

Calculate the area of steel reinforcement required for the beam. $M_u = 360\ KN \cdot m$

Take $f_c' = 30\ MPa,\ f_y = 400\ MPa.$

Assume ∅ 25 with one layer arrangement.




**Solution:**

$$d = h - cover - \varnothing\text{stirrups} - \frac{\varnothing\text{bar}}{2} = 650 - 40 - 10 - \frac{25}{2} = 587.5\ mm$$

Take $\phi = 0.9\ for\ flexure$

$$R_n = \frac{M_n}{bd^2} = \frac{M_u}{\phi bd^2} = \frac{360 \cdot 10^6}{0.9 \cdot 300 \cdot 587.5^2} = 3.86\ MPa$$

$$m = \left(\frac{f_y}{0.85f_c'}\right) = \frac{400}{0.85 \cdot 30} = 15.69$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{15.69}\left(1 - \sqrt{1 - \frac{2 \cdot 15.69 \cdot 3.86}{400}}\right) = 0.0105$$

$$A_s = \rho bd = 0.0105 \cdot 300 \cdot 587.5 = 1850.625\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

$$A_{s,min} = 0.25\frac{\sqrt{30}}{400}300 \cdot 587.5 = 603.35\ mm^2$$

$$A_{s,min} = \frac{1.4}{400} 300 \cdot 587.5 = 617\ mm^2 \quad -control$$

$$A_s = 1850.625\ mm^2 > A_{s,min} = 617\ mm^2 \quad -OK$$

Use $4 \varnothing 25$ with $A_s(4 \varnothing 25) = 19.634\ cm^2 > A_{s,req} = 18.5\ cm^2 \quad -OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{1963.4\ \cdot 400}{0.85 \cdot 30 \cdot 300} = 102.66\ mm$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$$c = \frac{102.66}{0.836} = 122.8\ mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{587.5 - 122.8}{122.8}\right) = 0.01135 > 0.005 \qquad OK$$

Check for bar placement:

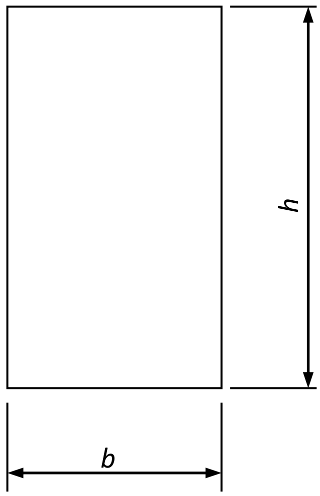
$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 4 \times 25}{3} = 33.33\ mm > d_b = 25\ mm, \quad > 25\ mm \qquad OK$$

**Example:**

Select an economical rectangular beam sizes and select bars using ACI strength method. The beam is a simply supported span of a 12 $m$ and it is to carry a live load of 20 $KN/m$ and a dead load of 25 $KN/m$ including beam weight.

Take $f_c' = 28\ MPa$, $f_y = 400\ MPa$.

Assume $d \approx 2b$




**Solution:**

$$w_u = 1.2DL + 1.6LL = 1.2 \cdot 25 + 1.6 \cdot 20 = 62\ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{62 \cdot 12^2}{8} = 1116\ KN \cdot m$$

Take $\phi = 0.9$ for flexure as tension-controlled section

Assume $\rho = 0.4\rho_b$.

Take $\beta_1 = 0.85$ ($f_c' = 28\ MPa$)

$$\rho_b = 0.85\ \frac{f_c'}{f_y}\beta_1\left(\frac{600}{600 + f_y}\right) = 0.85\ \frac{28}{400} 0.85\left(\frac{600}{600 + 400}\right) = 0.030345$$

$$\rho = 0.4\rho_b = 0.4 \cdot 0.030345 = 0.012138$$

$$m = \left(\frac{f_y}{0.85 f_c'}\right) = \left(\frac{400}{0.85 \cdot 28}\right) = 16.807$$

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right) = 0.012138 \cdot 400\left(1 - \frac{0.012138 \cdot 16.807}{2}\right) = 4.36\ MPa$$

$$bd^2 = \frac{M_u}{\phi R_n} = \frac{1116 \cdot 10^6}{0.9 \cdot 4.36} = 4b^3 \qquad \rightarrow \qquad b = \sqrt[3]{\frac{1116 \cdot 10^6}{4 \cdot 0.9 \cdot 4.36}} = 414.28\, mm$$

Take $b = 400\, mm$ and $d = 2b = 2 \cdot 400 = 800\, mm$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{1116 \cdot 10^6}{0.9 \cdot 400 \cdot 800^2} = 4.84\, MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{16.807}\left(1 - \sqrt{1 - \frac{2 \cdot 16.807 \cdot 4.84}{400}}\right) = 0.01367$$

$$A_s = \rho b d = 0.01367 \cdot 400 \cdot 800 = 4374.54\, mm^2$$

$$A_{s,min} = 0.25 \frac{\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

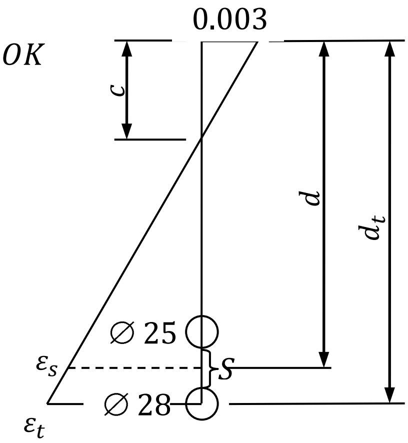
$$A_{s,min} = 0.25 \frac{\sqrt{28}}{400} 400 \cdot 800 = 1058.3\, mm^2$$

$$A_{s,min} = \frac{1.4}{400} 400 \cdot 800 = 1120\, mm^2 \quad -control$$

$$A_s = 4374.54\, mm^2 > A_{s,min} = 1120\, mm^2 \quad -OK$$

Take $4 \varnothing 28 + 4 \varnothing 25$ in two layers with

$$A_s = 24.63 + 19.635 = 44.265\, cm^2 > A_{s,req} = 43.74\, cm^2 \quad -OK$$




Check for strain:

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{4426.5 \cdot 400}{0.85 \cdot 28 \cdot 400} = 185.99\, mm$$

$$c = \frac{a}{\beta_1} = \frac{185.99}{0.85} = 218.81\, mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 800 + \frac{25}{2} + \frac{28}{2} = 826.5\, mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{826.5 - 218.81}{218.81}\right) = 0.00833 > 0.005 \qquad OK$$

Check for bar placement:

$$S_b = \frac{400 - 40 \times 2 - 10 \times 2 - 4 \times 28}{3} = 62.67\, mm > d_b = 28\, mm, \quad > 25\, mm \qquad OK$$

$$h = d_t + \frac{d_b}{2} + \varnothing\text{stirrups} + cover = 826.5 + \frac{28}{2} + 10 + 40 = 890.5\, mm$$

Take $b = 400\, mm$ and $h = 900\, mm.$

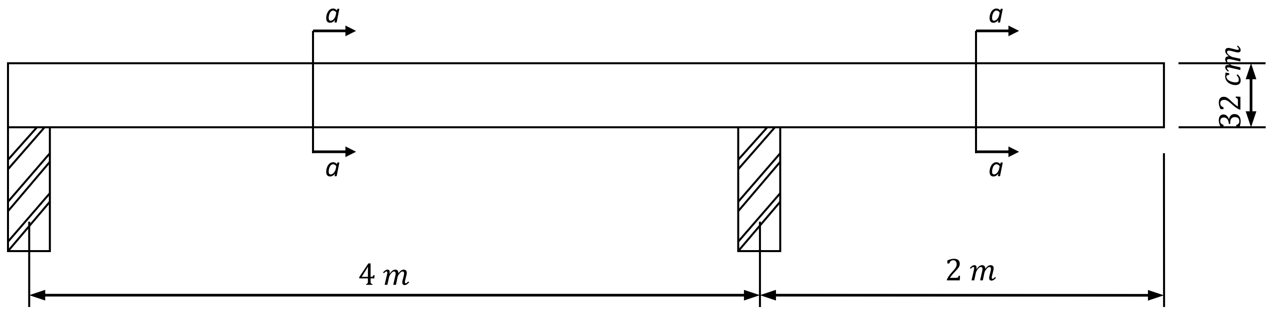
**Example:**

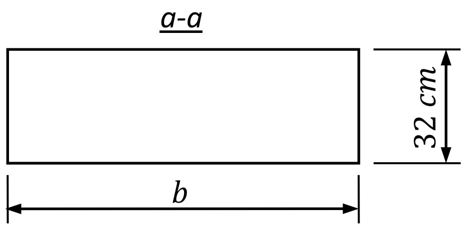
The beam shown below is loaded by service (unfactored) dead load of 45 $KN/m$ and service live load of 25 $KN/m$. Design the beam for flexure given the following information:

$f_c' = 24\ MPa, \qquad f_y = 420\ MPa.$

Assume the depth of the beam $h = 32\ cm$

Use bars $\varnothing\, 16$



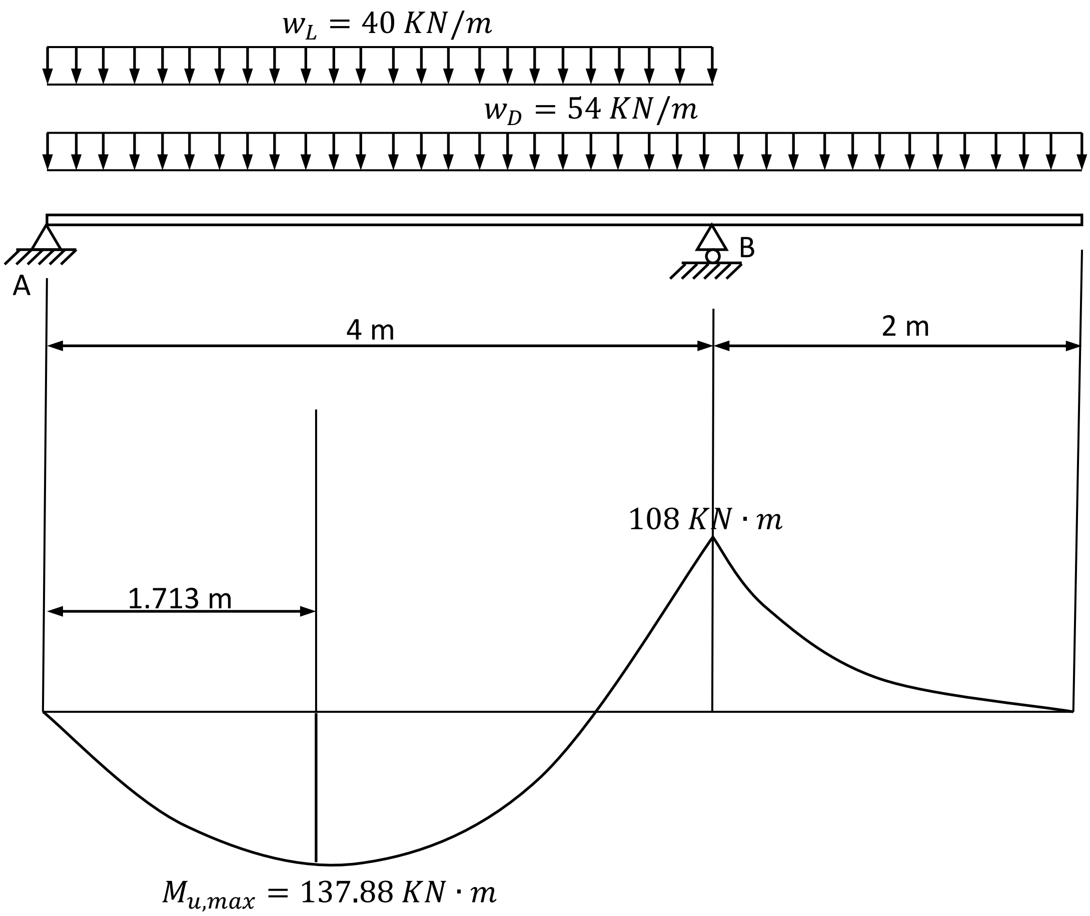

**Solution:**

$w_D = 1.2 \cdot 45 = 54\ KN/m$

$w_L = 1.6 \cdot 25 = 40\ KN/m$

**Determination the maximum positive and negative bending moments for the beam:**

- Maximum positive bending moment.



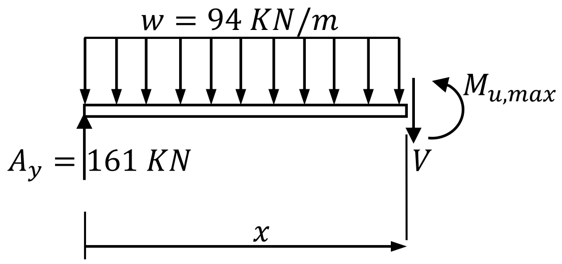

$\curvearrowright + \sum M_B = 0, \qquad A_y \cdot 4 - 94 \cdot 4 \cdot 2 + 54 \cdot 2 \cdot 1 = 0 \qquad\qquad A_y = 161\ KN$

Location of Maximum positive moment at distance $x$ from support A from condition of zero shear force.

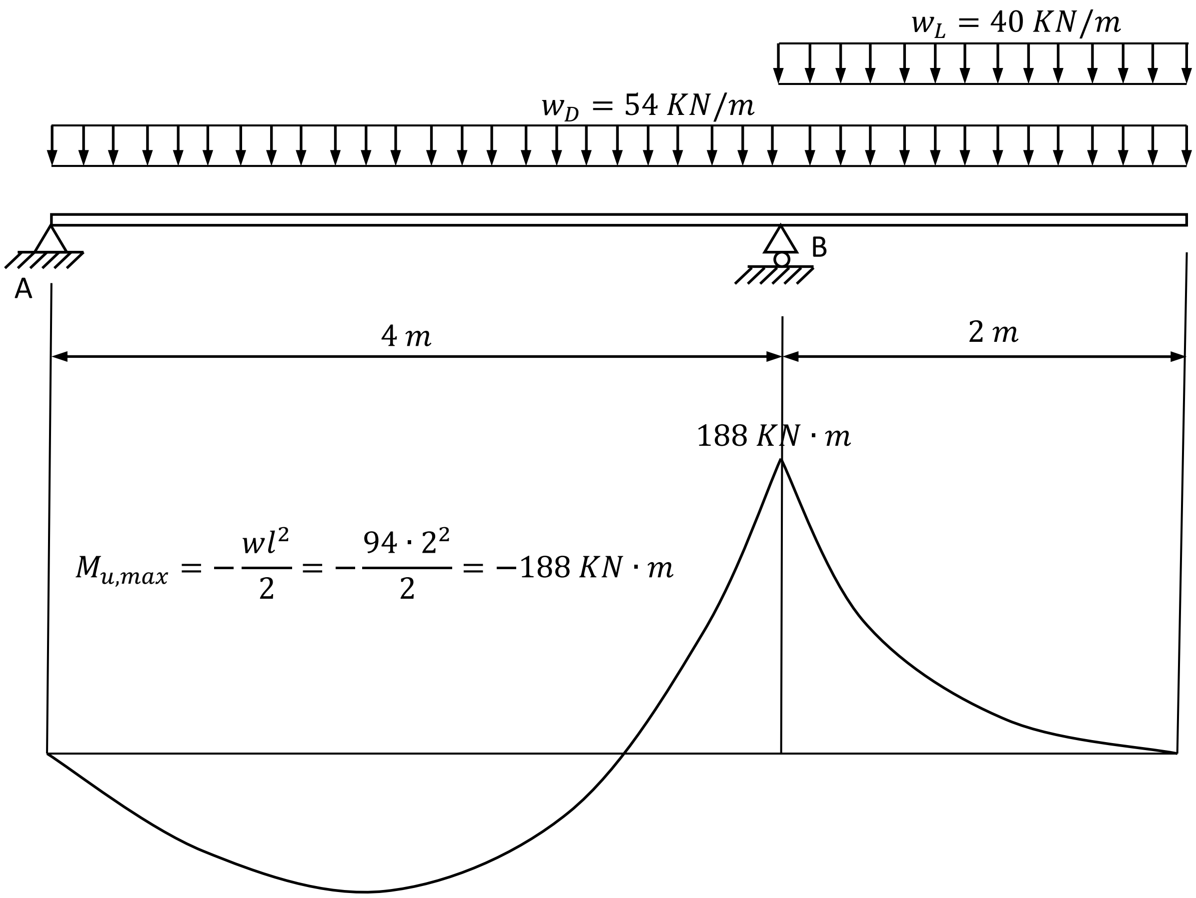
$V(x) = 0, \qquad 161 - 94 \cdot x = 0 \qquad x = 1.713\ m$

$$M_{u,max} = 161 \cdot 1.713 - 94 \cdot \frac{1.713^2}{2} = 137.88\ KN \cdot m$$

$$M_B = -54 \cdot \frac{2^2}{2} = -108\ KN \cdot m$$



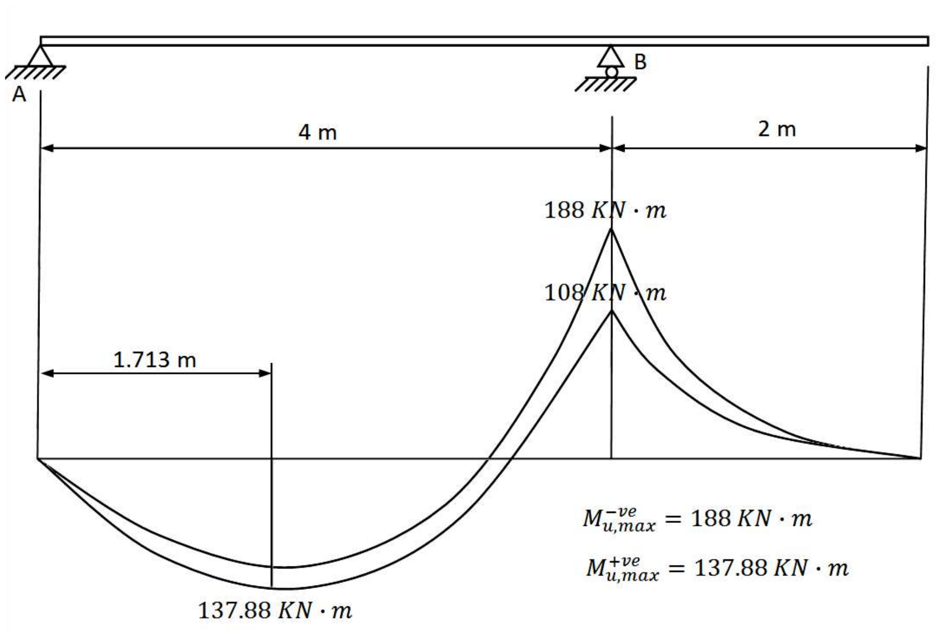

- Maximum negative bending moment.




The maximum moments from all cases (envelope):

$M_{u,max}^{+ve} = 137.88\ KN \cdot m$

$M_{u,max}^{-ve} = 188\ KN \cdot m$



**Determination of the beam width $b$ and Design for negative moment $M_u = 188\ KN \cdot m$**

Take $\phi = 0.9$ for flexure as tension-controlled section

Assume $\rho = 0.4\rho_b$.

Take $\beta_1 = 0.85$ $(f_c' = 24\ MPa)$

$$\rho_b = 0.85\ \frac{f_c'}{f_y}\beta_1\left(\frac{600}{600+f_y}\right) = 0.85\ \frac{24}{420}0.85\left(\frac{600}{600+420}\right) = 0.02429$$

$$\rho = 0.4\rho_b = 0.4 \cdot 0.02429 = 0.01$$

$$m = \left(\frac{f_y}{0.85 f_c'}\right) = \left(\frac{420}{0.85 \cdot 24}\right) = 20.6$$

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right) = 0.01 \cdot 420\left(1 - \frac{0.01 \cdot 20.6}{2}\right) = 3.767\ MPa$$

$$d = h - cover - \varnothing\text{stirrups} - \frac{\varnothing\text{bar}}{2} = 320 - 40 - 10 - \frac{16}{2} = 262\ mm$$

$$bd^2 = \frac{M_u}{\phi R_n} = \frac{188 \cdot 10^6}{0.9 \cdot 3.767} = b \cdot 262^2 \qquad \rightarrow \qquad b = \frac{188 \cdot 10^6}{0.9 \cdot 3.767 \cdot 262^2} = 807.8\ mm$$

Take $b = 900\ mm$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{188 \cdot 10^6}{0.9 \cdot 900 \cdot 262^2} = 3.38\ MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{20.6}\left(1 - \sqrt{1 - \frac{2 \cdot 20.6 \cdot 3.38}{420}}\right) = 0.0089$$

$$A_s = \rho b d = 0.0089 \cdot 900 \cdot 262 = 2099\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

$$A_{s,min} = 0.25\frac{\sqrt{24}}{420}900 \cdot 262 = 688\ mm^2$$

$$A_{s,min} = \frac{1.4}{420}900 \cdot 262 = 786\ mm^2 \quad - control$$

$$A_s = 2099\ mm^2 > A_{s,min} = 786\ mm^2 \quad - OK$$

Take 11 $\varnothing$ 16 in one layer with $A_s = 22.11\ cm^2 > A_{s,req} = 20.99\ cm^2$ $\quad - OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{2211 \cdot 420}{0.85 \cdot 24 \cdot 900} = 50.6\ mm$$

$$c = \frac{a}{\beta_1} = \frac{50.6}{0.85} = 59.5\ mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{262 - 59.5}{59.5}\right) = 0.01 > 0.005 \qquad OK$$

Check for bar placement:

$$S_b = \frac{900 - 40 \times 2 - 10 \times 2 - 11 \times 16}{10} = 62.4\,mm > 25\,mm \qquad OK$$

**Design for positive moment $M_u = 137.88\,KN \cdot m$**

$$R_n = \frac{M_u}{\phi b d^2} = \frac{137.88 \cdot 10^6}{0.9 \cdot 900 \cdot 262^2} = 2.48\,MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{20.6}\left(1 - \sqrt{1 - \frac{2 \cdot 20.6 \cdot 2.48}{420}}\right) = 0.0063$$

$$A_s = \rho b d = 0.0063 \cdot 900 \cdot 262 = 1486\,mm^2$$

$$A_{s,min} = 786\,mm^2$$

$$A_s = 1486\,mm^2 > A_{s,min} = 786\,mm^2 \quad - OK$$

Take 8 $\varnothing$ 16 in one layer with $A_s = 16.08\,cm^2 > A_{s,req} = 14.86\,cm^2 \quad - OK$

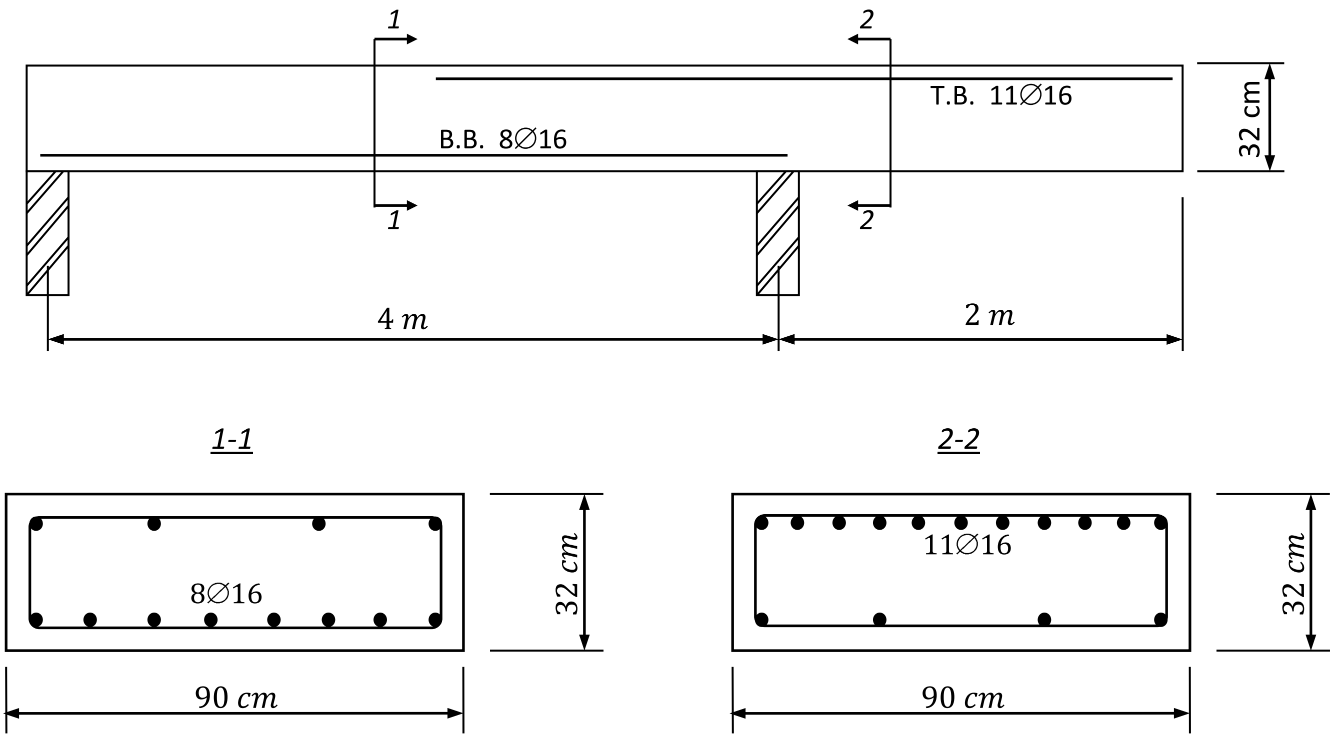
Check for strain:

$$a = \frac{A_s f_y}{0.85\,f_c' b} = \frac{1608 \cdot 420}{0.85 \cdot 24 \cdot 900} = 36.78\,mm$$

$$c = \frac{a}{\beta_1} = \frac{37}{0.85} = 43.28\,mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{262 - 43.28}{43.28}\right) = 0.0152 > 0.005 \qquad OK$$

Check for bar placement: $S_b > 25\,mm \quad OK$

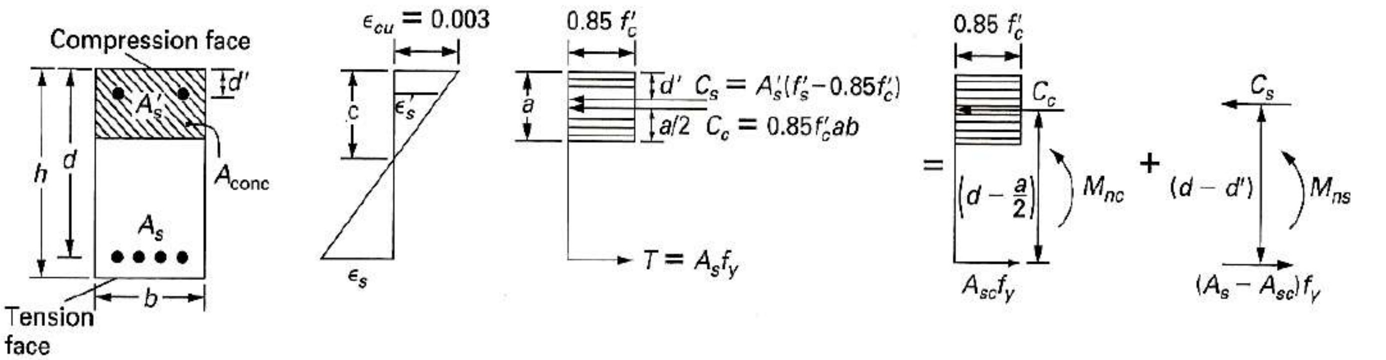


## 4.10 DOUBLY REINFORCED CONCRETE SECTIONS (SECTIONS WITH COMPRESSION REINFORCEMENT).

Flexural members are designed for tension reinforcement. Any additional moment capacity required in the section is usually provided by increasing the section size or the amount of tension reinforcement.

However, the cross-sectional dimensions in some applications may be limited by architectural or functional requirements (architectural limitations restrict the beam web depth at midspan, or the midspan section dimensions are not adequate to carry the support negative moment even when tensile steel at the support is sufficiently increased), and the extra moment capacity may have to be provided by additional tension and compression reinforcement. The extra steel generates an internal force couple, adding to the sectional moment capacity without changing the ductility of the section. In such cases, the total moment capacity consists of two components:

1. moment due to the tension reinforcement that balances the compression concrete, $M_{nc}$, and
2. moment generated by the internal steel force couple consisting of compression reinforcement and equal amount of additional tension reinforcement, $M_{ns}$ as illustrated in figure below.



(a) Cross section (b) Strain diagram (c) Stress diagram (d) Moment carried as a singly reinforced beam, $M_{nc}$ (e) Moment carried by the compression steel, $M_{ns}$

**Notation:**

$\varepsilon'_s$ – strain in compression steel.

$f'_s = E_s\varepsilon'_s \leq f_y$ – compression steel stress

$A'_s$ – area fo compression steel

$d'$ – distance from extreme compression fiber to centroid of compression steel

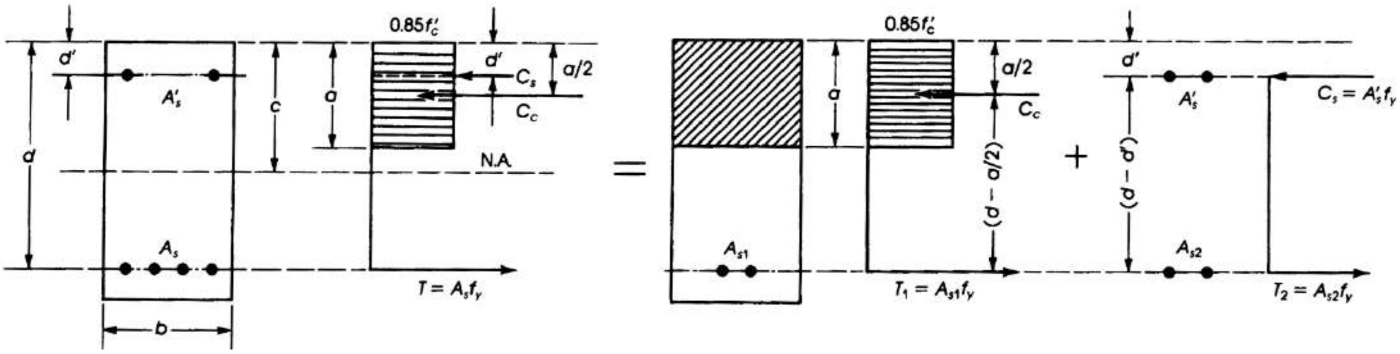
$\rho' = \dfrac{A'_s}{bd}$ – compression steel reinforcement ratio.

$A_{sc}$ – part of the tension steel that match $C_c$.

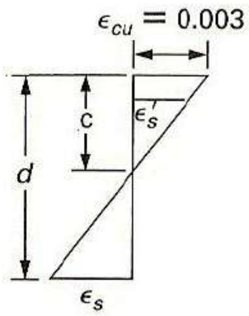
$C_c$ – concrete compression resultant for a beam without compression reinforcement.

$C_s$ – compression steel resultant as if $A'_s$ were stressed at $(f'_s - 0.85f'_c)$.

### 4.10.1 Analysis of doubly reinforced concrete sections




- **<u>Compression steel is yielded</u>**




Compression steel is yielded when $\varepsilon_s' \geq \varepsilon_y = \dfrac{f_y}{E_s}$

$\varepsilon_s' = 0.003\left(\dfrac{c-d'}{c}\right)$, $\quad c = \dfrac{a}{\beta_1}$

$A_{s1} = A_s - A_{s2}$, $\quad A_{s2} = A_s'$

$T_1 = A_{s1}f_y = C_c \quad and \quad M_{nc} = A_{s1}f_y\left(d-\dfrac{a}{2}\right) \quad or \quad M_{nc} = (A_s - A_s')f_y\left(d-\dfrac{a}{2}\right)$

where $\quad a = \dfrac{A_{s1}f_y}{0.85\, f_c' b} = \dfrac{(A_s - A_s')f_y}{0.85\, f_c' b} \qquad \rho = \dfrac{A_s}{bd} \qquad \rho' = \dfrac{A_s'}{bd}$

substituting "$a$" into $\quad c = \dfrac{a}{\beta_1} = \dfrac{(A_s - A_s')f_y}{\beta_1 \cdot 0.85\, f_c' b} = \dfrac{(\rho - \rho')f_y d}{\beta_1 \cdot 0.85\, f_c'}$

substituting "$c$" into $\quad \varepsilon_s' = 0.003\left(1 - \dfrac{d'}{c}\right) = 0.003\left[1 - \dfrac{0.85\beta_1 f_c' d'}{(\rho - \rho')d f_y}\right]$

Compression steel is yielded when

$$\varepsilon_s' \geq \varepsilon_y = \frac{f_y}{E_s}$$

$$0.003\left[1 - \frac{0.85\beta_1 f_c' d'}{(\rho - \rho')d f_y}\right] \geq \frac{f_y}{E_s = 200000\ MPa}$$

or in the form

$$\rho - \rho' \geq \frac{0.85 f_c' d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right)$$

$$\rho \geq \bar{\rho}_{cy}$$

where $$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right) + \rho' \qquad (*)$$

$\bar{\rho}_{cy}$ − minimum tensile reinforcement ratio that will ensure yielding of compression steel at failure.

In the previous equation of $\bar{\rho}_{cy}$ was ignored that part of the compression zone is occupied by the compression reinforcement, the value of ignored compressive force is $A_s'(0.85f_c{}')$. So the depth of stress block can be expressed

$$a = \frac{A_s f_y - A_s'\left(f_y - 0.85f_c{}'\right)}{0.85f_c{}'b},$$

and

$$\bar{\rho}_{cy} = \frac{0.85f_c'd'}{df_y}\beta_1\left(\frac{600}{600-f_y}\right) + \rho'(1 - \frac{0.85f_c'}{f_y})$$

In all calculations, the equation $(*)$ for $\bar{\rho}_{cy}$ will be used.

$T = A_s f_y = C_c + C_s = T_1 + T_2$

$C_c = 0.85f_c'ab, \qquad C_s = A_s'\left(f_y - 0.85f_c'\right)$

$$A_s f_y = 0.85f_c'ab + A_s'\left(f_y - 0.85f_c'\right) \qquad from\ where \quad a = \frac{A_s f_y - A_s'\left(f_y - 0.85f_c{}'\right)}{0.85f_c{}'b}$$

The nominal moment strength for rectangular section with tension and **compression steel is yielded**

$$M_n = \left(A_s f_y - A_s'\left(f_y - 0.85f_c{}'\right)\right)\left(d - \frac{a}{2}\right) + A_s'\left(f_y - 0.85f_c{}'\right)(d - d'),$$

$$or \qquad M_n = 0.85f_c'ab\left(d - \frac{a}{2}\right) + A_s'\left(f_y - 0.85f_c{}'\right)(d - d').$$

For simplicity, $A_s'(0.85f_c{}')$ can be ignored and then:

$$T = A_s f_y = C_c + C_s = T_1 + T_2, \qquad C_c = 0.85f_c'ab, \qquad C_s = A_s' f_y \qquad a = \frac{(A_s - A_s')f_y}{0.85f_c{}'b}$$

$$M_n = (A_s - A_s')f_y\left(d - \frac{a}{2}\right) + A_s' f_y (d - d') = 0.85f_c'ab\left(d - \frac{a}{2}\right) + A_s' f_y (d - d')$$

- **<u>Compression steel is NOT yielded</u>**

Compression steel is NOT yielded when

$$\varepsilon_s' < \varepsilon_y = \frac{f_y}{E_s} \qquad \text{or} \qquad f_s' = \varepsilon_s' E_s < f_y \qquad \text{or} \qquad \rho < \bar{\rho}_{cy}$$

$$f_s' = \varepsilon_s' E_s = 0.003\left(\frac{c - d'}{c}\right)200\,000 = 600\left(\frac{c - d'}{c}\right)$$

$T = A_s f_y = C_c + C_s = T_1 + T_2$

$C_c = 0.85f_c'ab, \qquad C_s = A_s'(f_s' - 0.85f_c')$

$$A_s f_y = 0.85 f_c' ab + A_s'(f_s' - 0.85 f_c') \quad from\ where \quad a = \frac{A_s f_y - A_s'(f_s' - 0.85 f_c')}{0.85 f_c' b} = \beta_1 c.$$

Note that in the above equation two unknowns "$c$" and "$f_s'$". Substitiuting $f_s' = 600\left(\frac{c-d\prime}{c}\right)$ in "$a$" we get an quadratic equation in "$c$", the only unknown, which is easily solved for "$c$".

The nominal moment strength for rectangular section with tension and compression **steel is NOT yielded**

$$M_n = \left(A_s f_y - A_s'(f_s' - 0.85 f_c')\right)\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c')(d - d'),$$

$$or \qquad M_n = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c')(d - d').$$

For simplicity, $A_s'(0.85 f_c')$ can be ignored and then:

$$T = A_s f_y = C_c + C_s = T_1 + T_2, \qquad C_c = 0.85 f_c' ab, \qquad C_s = A_s' f_s' \qquad a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b}$$

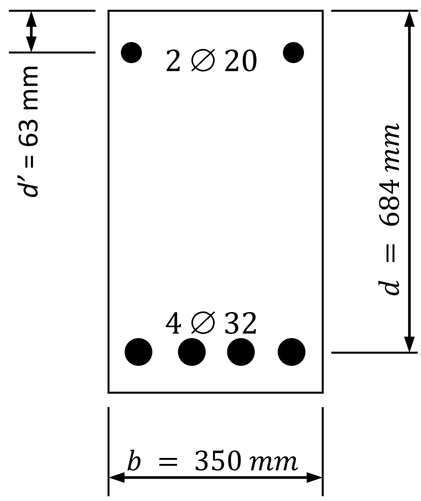
$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d') = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d')$$

For both cases (compression steel is yielded and is NOT yielded) $\varepsilon_s \geq 0.005$ (tension-controlled section).

**Example:**

Determine the nominal positive moment strength of the section of rectangular cross sectional beam. The beam is reinforced with 4 ∅ 32 in the tension zone and 2 ∅ 20 in the compression zone.

Take $f_c' = 20\ MPa, \quad f_y = 400\ MPa.$




**Solution:**

$A_s(4 \varnothing 32) = 32.17\ cm^2$

$A_s'(2 \varnothing 20) = 6.28\ cm^2$

$$\rho = \frac{A_s}{bd} = \frac{3217}{350 \cdot 684} = 0.0134$$

$$\rho' = \frac{A_s'}{bd} = \frac{628}{350 \cdot 684} = 0.0026, \qquad \beta_1 = 0.85,$$

$$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right) + \rho' = \frac{0.85 \cdot 20 \cdot 63}{684 \cdot 400} 0.85\left(\frac{600}{600 - 400}\right) + 0.0026 = 0.01258$$

$\rho = 0.0134 > \bar{\rho}_{cy} = 0.01258$ compression steel is yielded ($\varepsilon_s' \geq \varepsilon_y$)

$T = A_s f_y = C_c + C_s$

$A_s f_y = 0.85 f_c' ab + A_s'(f_y - 0.85 f_c')$ from where

$$a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c')}{0.85 f_c' b} = \frac{3217 \cdot 400 - 628 \cdot (400 - 0.85 \cdot 20)}{0.85 \cdot 20 \cdot 350} = 175.84\ mm,$$

$$c = \frac{a}{\beta_1} = \frac{175.84}{0.85} = 206.88\ mm,$$

$$M_n = 0.85 f_c' ab \left(d - \frac{a}{2}\right) + A_s'(f_y - 0.85 f_c')(d - d') =$$

$$= \left[0.85 \cdot 20 \cdot 175.84 \cdot 350 \left(684 - \frac{175.84}{2}\right) + 628(400 - 0.85 \cdot 20)(684 - 63)\right] \times 10^{-6} =$$

$$= 773.01\ KN \cdot m$$

Check for $\varepsilon_s \geq 0.005$:

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{684 - 206.88}{206.88}\right) = 0.00691 > 0.005 \qquad OK$$

Take $\phi = 0.9$ for flexure as tension-controlled section.

$$\phi M_n = 0.9 \cdot 773.01 = 695.71\ KN \cdot m$$

**Example:**

Repeat the previous example using $f_c' = 30\ MPa$.

**Solution:**

$$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y} \beta_1 \left(\frac{600}{600 - f_y}\right) + \rho' = \frac{0.85 \cdot 30 \cdot 63}{684 \cdot 400} 0.836 \left(\frac{600}{600 - 400}\right) + 0.0026 = 0.0173$$

$\rho = 0.0134 < \bar{\rho}_{cy} = 0.0173$ compression steel is NOT yielded ($\varepsilon_s' < \varepsilon_y$)

$$T = A_s f_y = C_c + C_s$$

$$f_s' = 600 \left(\frac{c - d'}{c}\right), \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$A_s f_y = 0.85 f_c' ab + A_s'(f_s' - 0.85 f_c')$ from where

$$a = \frac{A_s f_y - A_s'(f_s' - 0.85 f_c')}{0.85 f_c' b} = \beta_1 c$$

$$\frac{3217 \cdot 400 - 628 \cdot \left(600 \left(\frac{c - 63}{c}\right) - 0.85 \cdot 30\right)}{0.85 \cdot 30 \cdot 350} = 0.836\ c$$

$$103.755 + \frac{2659.764}{c} = 0.836\ c, \qquad \Rightarrow \ 0.836\ c^2 - 103.755\ c - 2659.764 = 0,$$

solution of quadratic equation $\quad x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$c^2 - 124.109\ c - 3181.536 = 0,$$

$$c_{1,2} = \frac{124.109 \pm \sqrt{124.109^2 - 4 \cdot 1 \cdot (-3181.536)}}{2} = \frac{124.109 \pm 167.718}{2}$$

Choose only $c > 0$, $c = 145.91\ mm$

$a = \beta_1 c = 0.836 \cdot 145.91 = 121.98\,mm,$

$$f_s' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{145.91-63}{145.91}\right) = 340.94\,MPa \quad < f_y = 400\,MPa,$$

$$M_n = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c')(d - d') =$$

$$= \left[0.85 \cdot 30 \cdot 121.98 \cdot 350\left(684 - \frac{121.98}{2}\right) + 628(340.94 - 0.85 \cdot 30)(684 - 63)\right] \times 10^{-6} =$$

$$= 801.27\,KN \cdot m$$

Check for $\varepsilon_s \geq 0.005$:

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{684-145.91}{145.91}\right) = 0.011 > 0.005 \qquad OK$$

Take $\phi = 0.9$ for flexure as tension-controlled section.

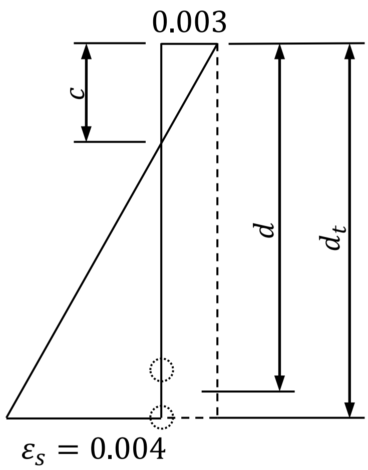
$\phi M_n = 0.9 \cdot 801.27 = 721.14\,KN \cdot m$

### 4.10.2 Design of doubly reinforced concrete sections.

When the factored moment $M_u$ is greater than the design strength $\phi M_n$ of the beam when it is reinforced with the maximum permissible amount of tension reinforcement, compression reinforcement becomes necessary.

The logical procedure for designning a doubly reinforced sections is to determine first whether compression steel is needed for strength. This may be done by comparing the required moment strength with the moment strength of a singly reinforced section with the maximum permissible amount of tension steel $\rho_{max}$.

For example, for steel Grade 420 $\rho_{max} = 0.724\rho_b$ which defined from strain conditon $\varepsilon_t = 0.004$ for beams.




$$\frac{c}{0.003} = \frac{d_t}{0.003 + 0.004} \quad \Longrightarrow \quad c = \frac{3}{7}d_t, \qquad a = \beta_1 c$$

The maximum moment strength as a singly reinforced section

$$M_{n,max} = 0.85 f_c' ab\left(d - \frac{a}{2}\right),$$

If $M_u > \phi M_{n,max}$ Design the section as doubly reinforced section,

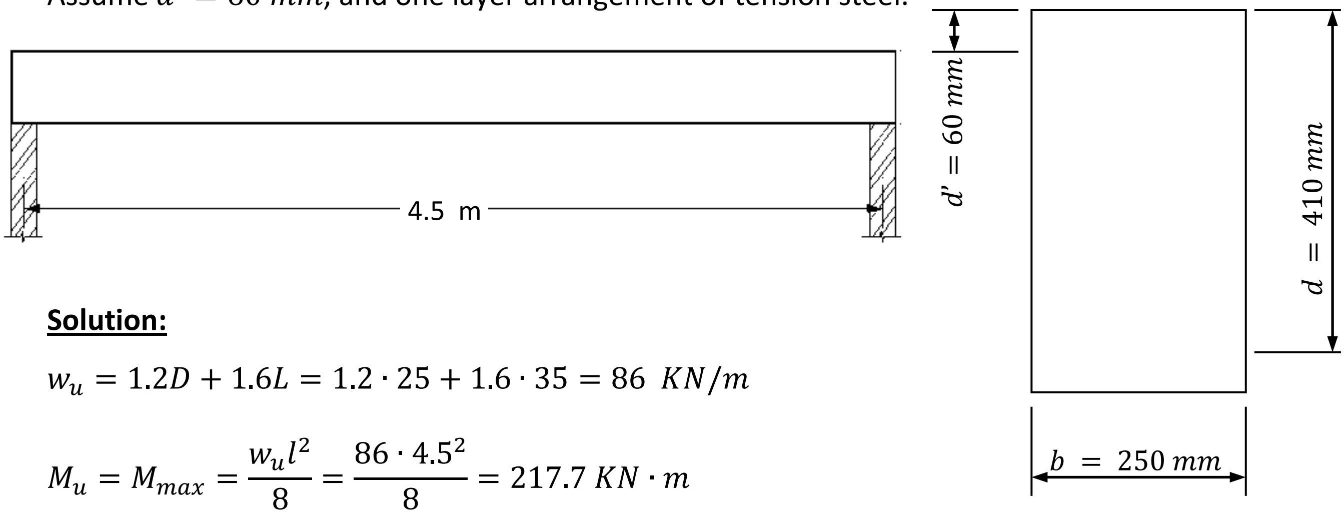
where $\phi = 0.65 + (0.004 - 0.002)\dfrac{250}{3} = 0.817 \approx 0.82$

**Example:**

The beam is loaded by a uniform service $DL = 25\ KN/m$ and a uniform service $LL = 35\ KN/m$. Compute the area of steel reinforcement for the section.

Take $f_c' = 20\ MPa,\quad f_y = 400\ MPa.$

Assume $d' = 60\ mm$, and one layer arrangement of tension steel.




**Solution:**

$$w_u = 1.2D + 1.6L = 1.2 \cdot 25 + 1.6 \cdot 35 = 86\ \ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{86 \cdot 4.5^2}{8} = 217.7\ KN \cdot m$$

Maximum nominal moment strength from strain condition $\varepsilon_s = 0.004$

$$c = \frac{3}{7} d = \frac{3}{7} 410 = 175.7\ mm, \qquad \beta_1 = 0.85$$

$$a = \beta_1 c = 0.85 \cdot\ 175.7 = 149.4\ mm$$

$$M_{n,max} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) = 0.85 \cdot 20 \cdot 149.4 \cdot 250\left(410 - \frac{149.4}{2}\right) \times 10^{-6} = 212.9\ KN \cdot m$$

$$\phi = 0.82$$

$$M_u = 217.7\ KN \cdot m > \phi M_n = 0.82 \cdot 212.9 = 174.6\ \ KN \cdot m$$

Design the section as doubly reinforced concrete section.

$$M_{ns} = \frac{M_u}{\phi} - M_{nc} = \frac{217.7}{0.82} - \ 212.9 = 52.59\ KN \cdot m$$

$$M_{ns} = C_s(d - d') = A_s'(f_s' - 0.85 f_c')(d - d') \quad \Rightarrow \quad A_s' = \frac{M_{ns}}{(f_s' - 0.85 f_c')(d - d')}$$

$$f_s' = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{175.7 - 60}{175.7}\right) = 395.1\ MPa\ < f_y = 400\ MPa,$$

Compression steel does NOT yield

$$A_s' = \frac{M_{ns}}{(f_s' - 0.85 f_c')(d - d')} = \frac{52.59 \cdot 10^6}{(395.1\ - 0.85 \cdot 20)(410 - 60)} = 397.4\ mm^2$$

$$T = C_c + C_s = 0.85 f_c' ab + A_s'(f_s' - 0.85 f_c') =$$

$$= [0.85 \cdot 20 \cdot 149.4 \cdot 250 + 397.4(395.1 - 0.85 \cdot 20)] \times 10^{-3} = 785.21\ KN$$

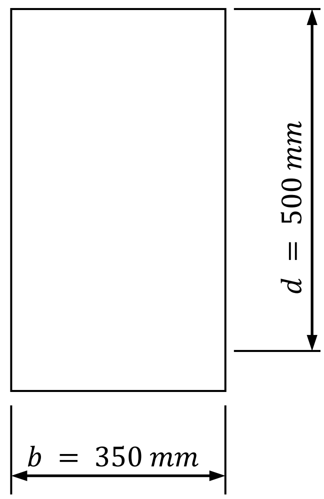
$$A_s = \frac{T}{f_y} = \frac{785.21 \cdot 10^3}{400} = 1963.02\ mm^2$$

**Example:**

The beam section shown below is loaded by a factored bending moment $M_u = 520\ KN \cdot m$. Design the beam for flexure given the following information:

$f_c' = 24\ MPa, \qquad f_y = 400\ MPa.$

Use bars $\varnothing\, 25\ mm$, and assume one layer arrangement of tension steel.




**Solution:**

Check whether the section will be designed as singly or doubly:

Maximum nominal moment strength from strain condition $\varepsilon_s = 0.004$

$$c = \frac{3}{7}d = \frac{3}{7}500 = 214.29\ mm, \qquad \beta_1 = 0.85$$

$$a = \beta_1 c = 0.85 \cdot\ 214.29 = 182.14\ mm$$

$$M_{n,max} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) = 0.85 \cdot 24 \cdot 182.14 \cdot 350\left(500 - \frac{182.14}{2}\right) \times 10^{-6} = 531.81\ KN \cdot m$$

$$\phi = 0.82$$

$$M_u = 520\ KN \cdot m > \phi M_n = 0.82 \cdot 531.81 = 436.1\ \ KN \cdot m$$

Design the section as doubly reinforced concrete section.

$$M_{ns} = \frac{M_u}{\phi} - M_{nc} = \frac{520}{0.82} -\ 531.81 = 102.34\ KN \cdot m$$

$$M_{ns} = C_s(d - d') = A_s'(f_s{}' - 0.85 f_c{}')(d - d') \quad \Rightarrow \qquad A_s' = \frac{M_{ns}}{(f_s{}' - 0.85 f_c{}')(d - d')}$$

$$d' = cover + \varnothing stirrups + \frac{\varnothing bar}{2} = 40 + 10 + \frac{25}{2} = 62.5\ mm$$

$$f_s' = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{214.29 - 62.5}{214.29}\right) = 425\ MPa > f_y = 400\ MPa,$$

Compression steel is yielded. Take $\quad f_s' = f_y = 400\ MPa$

$$A_s' = \frac{M_{ns}}{(f_y - 0.85 f_c{}')(d - d')} = \frac{102.34 \cdot 10^6}{(400\ - 0.85 \cdot 24)(500 - 62.5)} = 616.23\ mm^2$$

$$T = C_c + C_s = 0.85 f_c' ab + A_s'(f_y - 0.85 f_c{}') =$$

$$= [0.85 \cdot 24 \cdot 182.14 \cdot 350 + 616.23(400 - 0.85 \cdot 24)] \times 10^{-3} = 1534.4\ KN$$

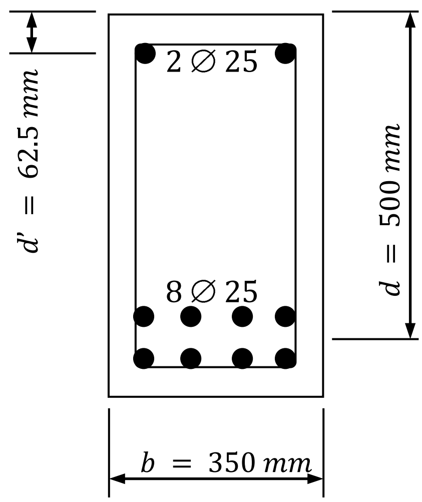
$$A_s = \frac{T}{f_y} = \frac{1534.4\ \cdot 10^3}{400} = 3836\ mm^2$$

Take 8 $\varnothing$ 25 in two layers with $A_s = 39.27\ cm^2 >\ A_{s,req} = 38.36\ cm^2 \quad - OK$

Take 2 $\varnothing$ 25 in one layer with $A_s' = 9.817\ cm^2 >\ A_{s,req}' = 6.16\ cm^2 \quad - OK$

Now it's an analysis problem of doubly reinforced concrete section.

Check whether compression steel has yielded:




$$\rho = \frac{A_s}{bd} = \frac{3927}{350 \cdot 500} = 0.02244$$

$$\rho' = \frac{A_s'}{bd} = \frac{981.7}{350 \cdot 500} = 0.00561, \qquad \beta_1 = 0.85,$$

$$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y} \beta_1 \left( \frac{600}{600 - f_y} \right) + \rho' =$$

$$= \frac{0.85 \cdot 24 \cdot 62.5}{500 \cdot 400} 0.85 \left( \frac{600}{600 - 400} \right) + 0.00561 = 0.02187$$

$$\rho = 0.02244 > \bar{\rho}_{cy} = 0.02187$$

compression steel is yielded ($\varepsilon_s' \geq \varepsilon_y$)

Check for $\varepsilon_t \geq 0.005$:

$T = A_s f_y = C_c + C_s \quad \Rightarrow \quad A_s f_y = 0.85 f_c' ab + A_s'(f_y - 0.85 f_c')$ from where

$$a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c')}{0.85 f_c' b} = \frac{3927 \cdot 400 - 981.7 \cdot (400 - 0.85 \cdot 24)}{0.85 \cdot 24 \cdot 350} = 167.81\, mm,$$

$$c = \frac{a}{\beta_1} = \frac{167.81}{0.85} = 197.42\, mm, \qquad d_t = d + \frac{S}{2} + \frac{d_b}{2} = 500 + \frac{25}{2} + \frac{25}{2} = 525\, mm$$

$$\varepsilon_t = 0.003 \left( \frac{d_t - c}{c} \right) = 0.003 \left( \frac{525 - 197.42}{197.42} \right) = 0.00498 < 0.005$$

When $0.004 < \varepsilon_t < 0.005$ (in transition zone between compression-controlled section and tension-controlled section), it is obvious here that the nominal moment strength of the section will satisfy the strength condition $\phi M_n \geq M_u, \ where \quad 0.82 < \phi < 0.9$ .

This step is a proof of the above statement.

$$\phi = 0.65 + (0.00498 - 0.002) \frac{250}{3} = 0.8983 > 0.82 - as\ was\ used$$

$$M_n = 0.85 f_c' ab \left( d - \frac{a}{2} \right) + A_s'(f_s' - 0.85 f_c')(d - d') =$$

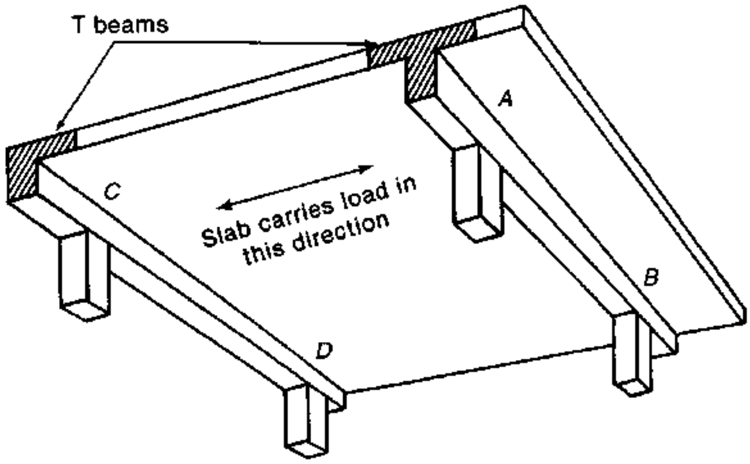
$$= \left[ 0.85 \cdot 24 \cdot 167.81 \cdot 350 \left( 500 - \frac{167.81}{2} \right) + 981.7(400 - 0.85 \cdot 24)(500 - 62.5) \right] \times 10^{-6} =$$

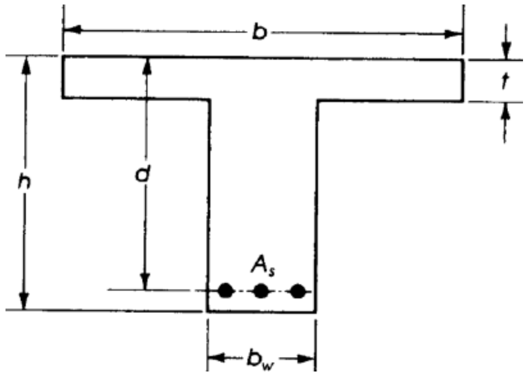
$$= 661.59\, KN \cdot m$$

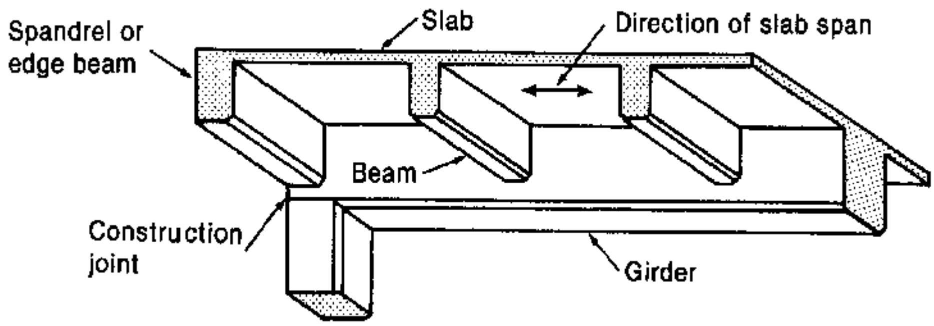
$$\phi M_n = 0.8983 \cdot 661.59 = 594.31\, KN \cdot m > \ M_u = 520 \quad KN \cdot m$$

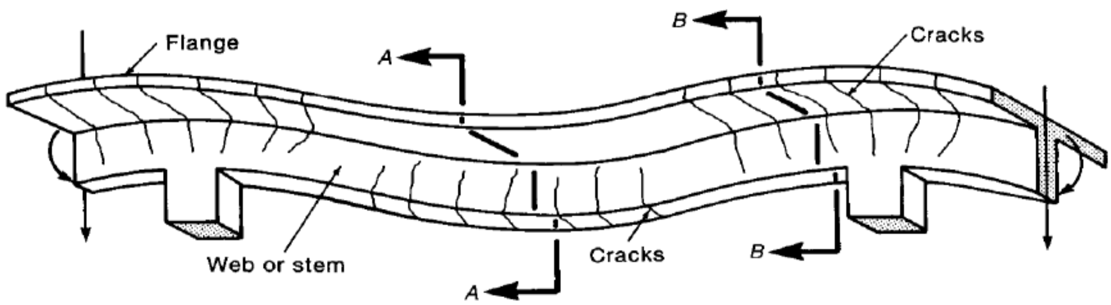
## 4.1 REINFORCED CONCRETE FLANGED SECTIONS (T- AND L- SECTIONS).

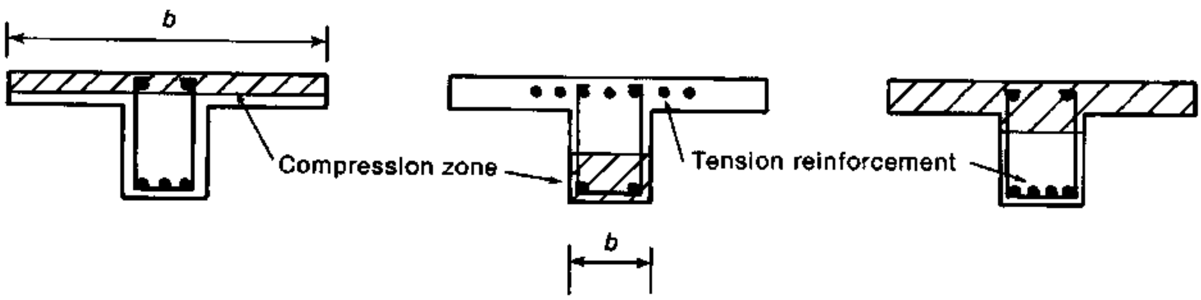
It is normal to cast concrete slabs and beams together, producing a monolithic structure. Slabs have smaller thicknesses than beams. Under bending stresses, those parts of the slab on either side of the beam will be subjected to compressive stresses, depending on the position of these parts relative to the top fibers and relative to their distances from the beam. The part of the slab acting with the beam is called the flange. The rest of the section is called the stem, or web.



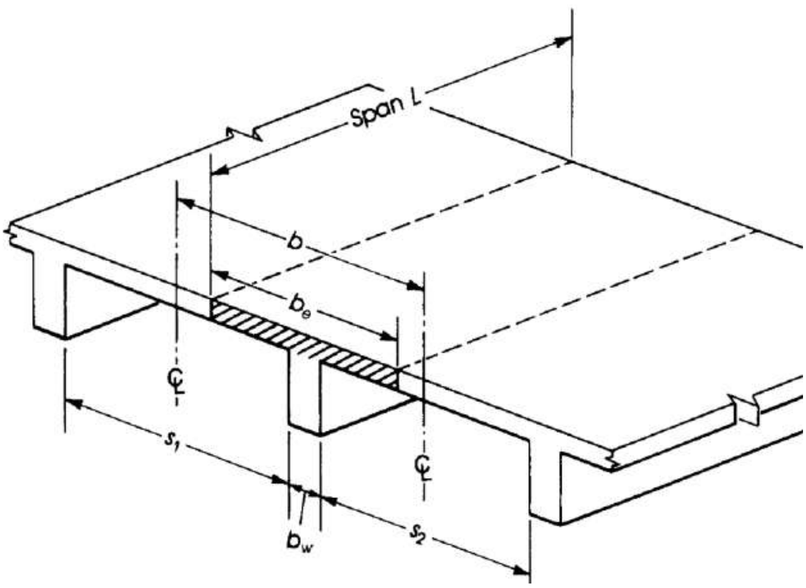

(b) Section $A$-$A$ (rectangular compression zone).

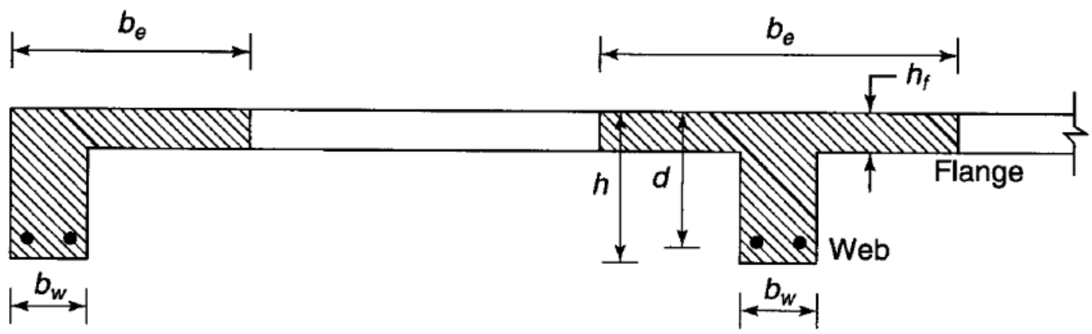
(c) Section $B$-$B$ (negative moment).

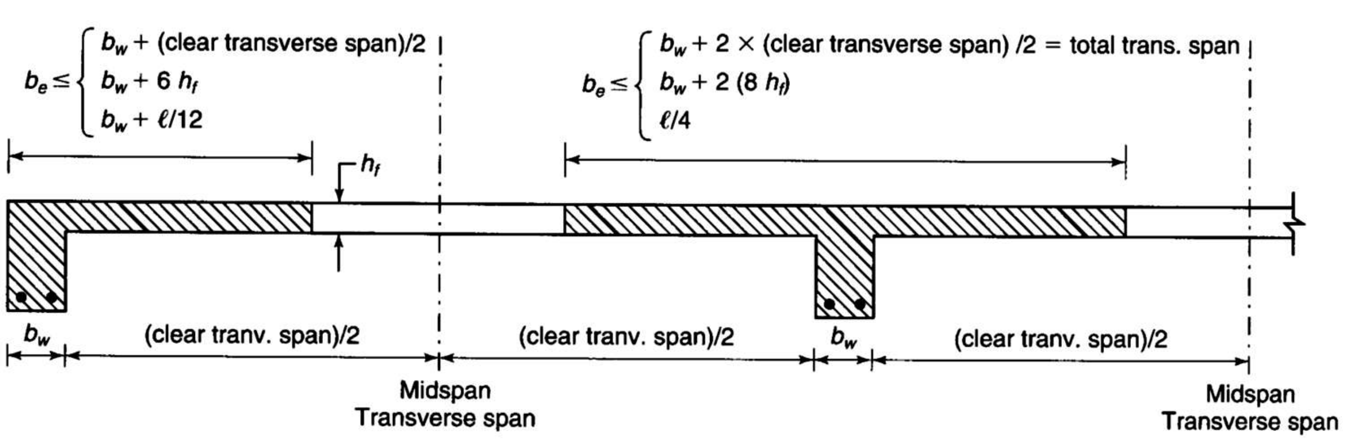
(d) Section $A$-$A$ (T-shaped compression zone).

### 4.11.1 Effective width.

The ACI Code definitions for the effective compression flange width for T- and inverted L-shapes in continuous floor systems are illustrated in figure below.




$\ell$ = length of beam span (longitudinal span)

For T-shapes, the total effective compression flange width, $b_e$, is limited to one-quarter of the span length of the beam ($L$), and the effective overhanging portions of the compression flange on each side of the web are limited to

(a) eight times the thickness of the flange (slab), and
(b) one-half the clear distance to the next beam web.

The ACI Code, 8.12.2, prescribes a limit on the effective flange width, $b_e$ , of interior T-section to the smallest of the following:

$$(a) \quad b_e \leq \frac{L}{4}$$

$$(b) \quad b_e \leq b_w + 16h_f$$

$$(c) \quad b_e \leq b_w + \frac{1}{2} \text{ the clear distance to the next beam web from both sides}$$

For symmetrical T-section (the clear distance to the next beam web from both sides is the same ) the previous $(c)$ will be

$$(c) \quad b_e \leq \text{Center to Center spacing between adjacent beams}$$

For inverted L-shapes, the following three limits are given for the effective width of the overhanging portion of the compression flange:

(a) one-twelfth of the span length of the beam,
(b) six times the thickness of the flange (slab), and
(c) one-half the clear transverse distance to the next beam web.

The ACI Code, 8.12.3, prescribes a limit on the effective flange width, $b_e$ , of exterior T-section (L-shape) to the smallest of the following:

$$(a) \quad b_e \leq b_w + \frac{L}{12}$$

$$(b) \quad b_e \leq b_w + 6h_f$$

$$(c) \quad b_e \leq b_w + \frac{1}{2} \text{ the clear distance to the next beam web.}$$

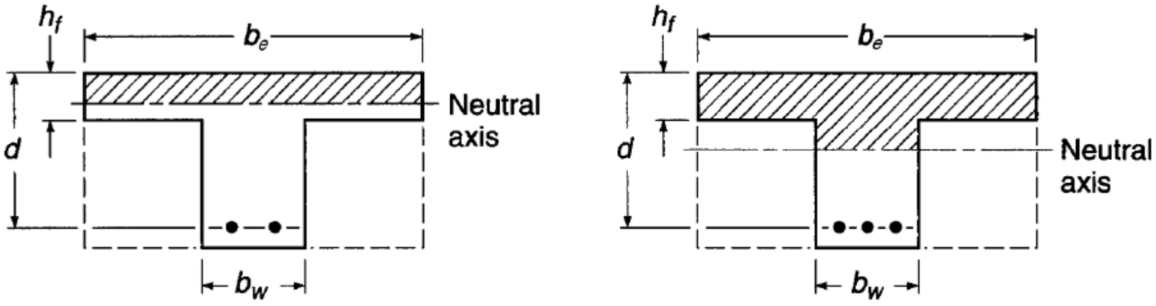
Isolated beams, in which the T-shape is used to provide a flange for additional compression area, shall have a flange thickness (ACI 8.12.4)

$$(a) \quad b_e \leq 4b_w$$

$$(b) \quad t \geq \frac{1}{2} b_w$$

### 4.11.2 Analysis of T-sections.

The neutral axis of a T-section beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strength of the materials.




**Procedure of analysis**:

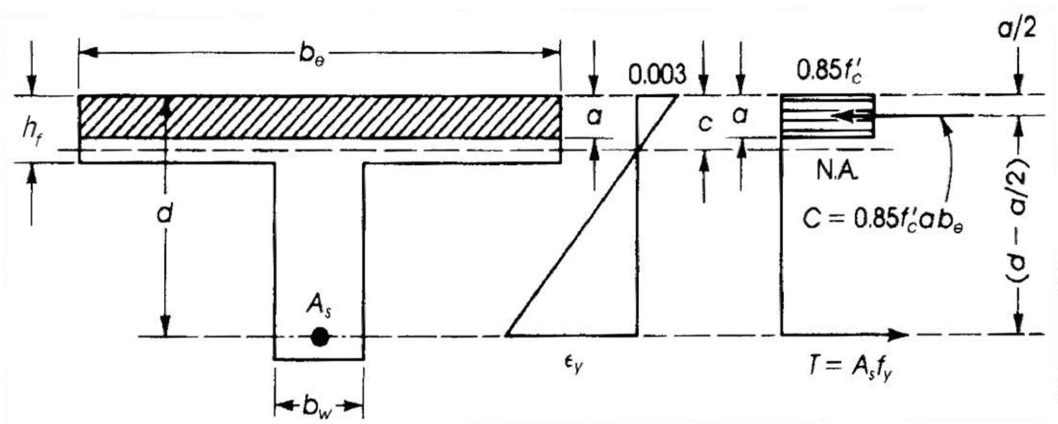
1. Assume that T-section is a rectangular section with total $b_e$ width.

$$T = C \qquad \Longrightarrow \qquad A_s f_y = 0.85\, f_c' a b_e \qquad \Longrightarrow \qquad a = \frac{A_s f_y}{0.85\, f_c' b_e}$$

2. Compare $a$ with $h_f -$ the thickness of flange.

Here may be TWO CASES:

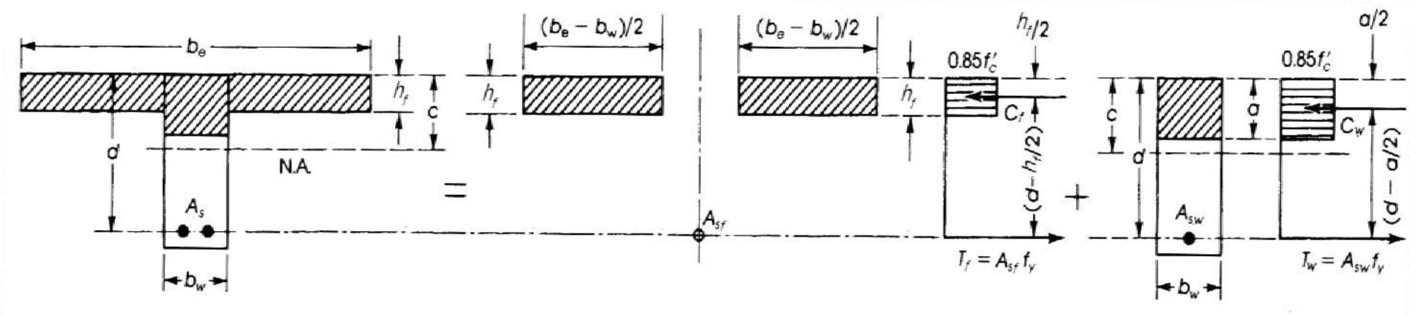
**Case I:** $a \le h_f$ analyze as rectangular section.




$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' a b_e \left(d - \frac{a}{2}\right)$$

$$A_s = \frac{0.85\, f_c' a b_e}{f_y}$$

**Case II:** $a > h_f$ analyze as T-section.




1. $M_n = M_{nf} + M_{nw}$

where $M_n -$ Moment capacity of the T-section,

$M_{nf}$ – Moment capacity of the flange,
$M_{nw}$ – Moment capacity of the web.

2. $M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 0.85\, f_c'(b_e - b_w)h_f \left(d - \frac{h_f}{2}\right)$

$$T_f = C_f \quad \Longrightarrow \quad A_{sf} f_y = 0.85\, f_c'(b_e - b_w)h_f \quad \Longrightarrow \quad A_{sf} = \frac{0.85\, f_c'(b_e - b_w)h_f}{f_y}$$

3. $M_{nw} = A_{sw} f_y \left(d - \frac{a}{2}\right) = 0.85\, f_c' b_w a \left(d - \frac{a}{2}\right),$ $\quad A_{sw} = A_s - A_{sf}$

$$T_w = C_w \quad \Longrightarrow \quad A_{sw} f_y = 0.85\, f_c' b_w a \quad \Longrightarrow \quad a = \frac{A_{sw} f_y}{0.85\, f_c' b_w}$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right)$$

*or*

$$M_n = 0.85\, f_c'(b_e - b_w)h_f \left(d - \frac{h_f}{2}\right) + 0.85\, f_c' b_w a \left(d - \frac{a}{2}\right)$$

4. Check for strain $\varepsilon_s \geq 0.005$.

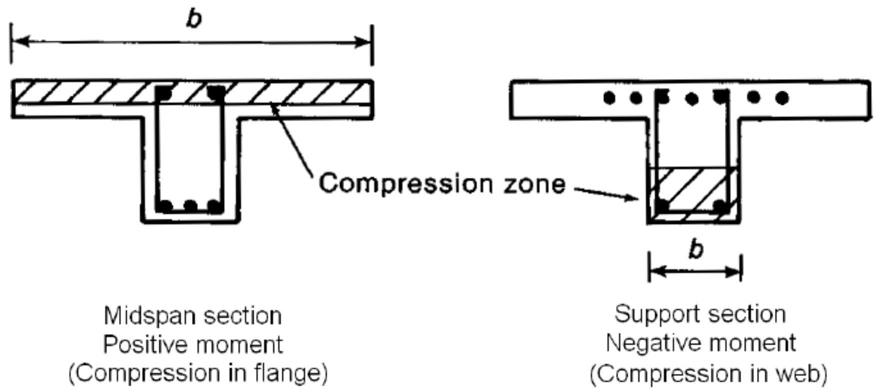
### 4.11.3 Minimum reinforcement of flexural T-section members.

$A_{s,min}$ for T-sections is as in 4.6 (page 23).
For statically determinate members with a flange in tension, ACI Code, 10.5.2., as in the case of cantilever beams, $A_{s,min}$ shall not be less than the value given by equations in section 4.6 (see page 23), except that $b_w$ is replaced by either $2b_w$ or the width of the flange, whichever is smaller.

$$A_{s,min} = \frac{0.5\sqrt{f_c'}}{f_y}\, b_w d, \qquad A_{s,min} = \frac{0.25\sqrt{f_c'}}{f_y}\, bd.$$

According to ACI code, 10.6.6, where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in 8.12, or a width equal to one-tenth the span, whichever is smaller. If the effective flange width exceeds one-tenth the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

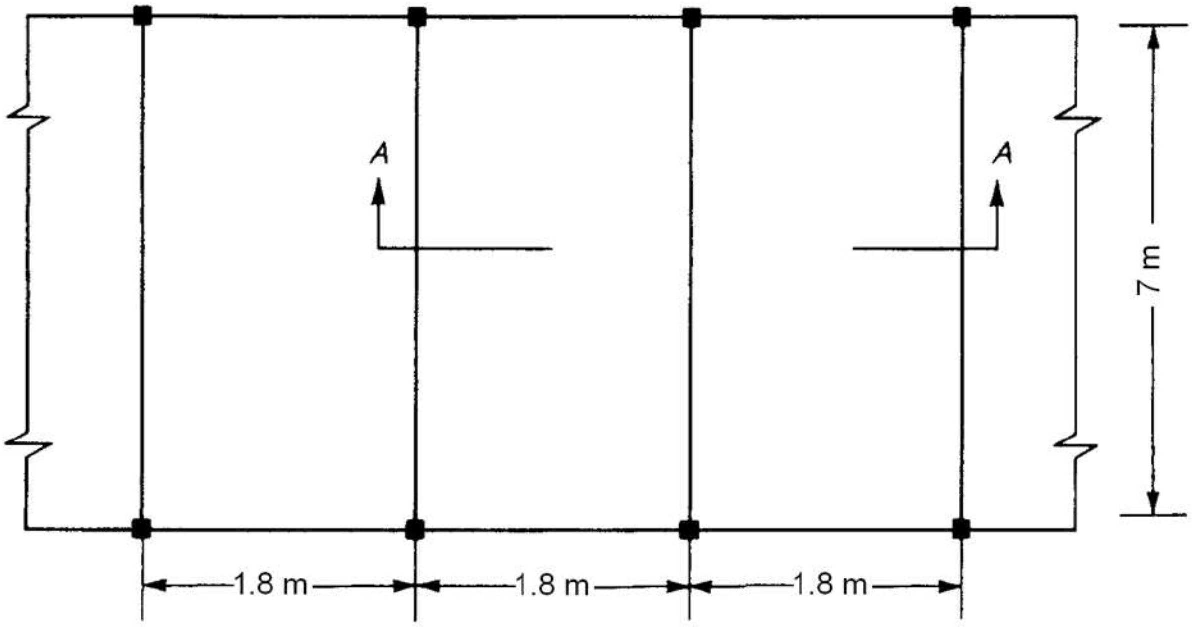
### 4.11.4 Analysis of the positive-moment capacity of a T-section.



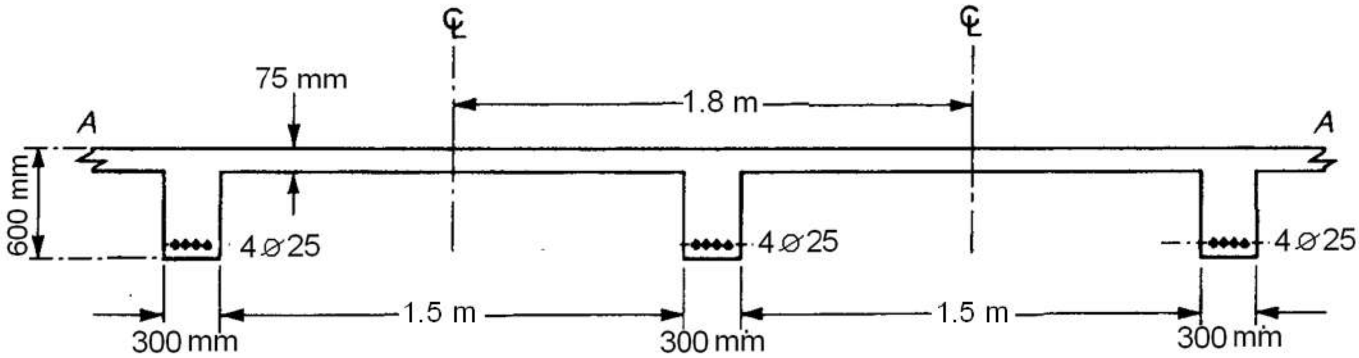
**Example:**

Calculate the design strength $\phi M_n$ for one of the T beams in the positive moment region. The beam has a clear span of 7 m (face to face).

$f_c' = 28\ MPa, \qquad f_y = 420\ MPa.$




(a)




(b)

**Solution:**

From the Geometry of T-section:

$b_w = 300\ mm, \qquad h = 600\ mm, \qquad t = h_f = 75\ mm$

$A_s(4\varnothing\ 25) = 1963.5\ mm^2$

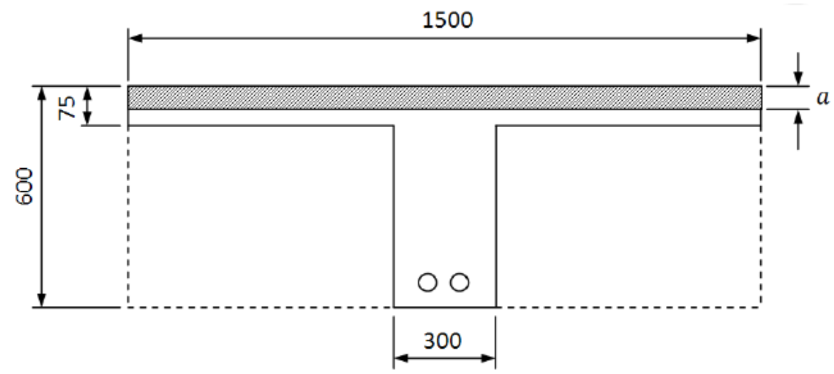
$b_e$ is the smallest of:

$$(a)\ \ b_e \le \frac{L}{4} = \frac{7000}{4} = 1750\ mm,$$

$$(b)\ \ b_e \le b_w + 16h_f = 300 + 16 \cdot 75 = 1500\ mm, \qquad -control$$

$$(c)\ \ b_e \le \text{Center to Center spacing between adjacent beams} = 1800\ mm.$$

Take $b_e = 1500\ mm.$




$$a = \frac{A_s f_y}{0.85\ f_c' b_e} = \frac{1963.5\ \cdot 420}{0.85\ \cdot 28 \cdot 1500} = 23.1\ mm\ \ < h_f = 75\ mm$$

The beam section will be considered as rectangular with $b = b_e = 1500\ mm.$

$$d = 600 - 40 - 10 - \frac{25}{2} = 537.5\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1963.5 \cdot 420 \left(537.5 - \frac{23.1}{2}\right) \times 10^{-6} = 433.74\ KN \cdot m$$

Check for strain $\varepsilon_s \geq 0.005$

$c = \frac{a}{\beta_1} = \frac{23.1}{0.85} = 27.18\ mm,$ $\beta_1 = 0.85$

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{537.5 - 27.18}{27.18}\right) = 0.0565 > 0.005 \qquad OK$$

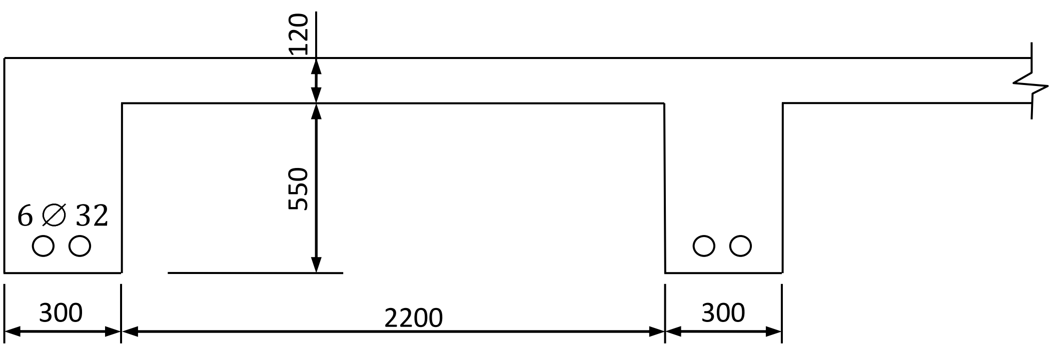
Take $\phi = 0.9$ for flexure as tension-controlled section.

$$M_u = \phi M_n = 0.9 \cdot 433.74 = 390.37\ KN \cdot m$$

**Example:**

Determine the positive moment capacity of the edge L-section beam. The beam has a clear span of 6 m (face to face).

$f_c' = 20\ MPa,$ $f_y = 400\ MPa.$




**Solution:**

From the Geometry of T-section:

$b_w = 300\ mm,$ $h = 670\ mm,$ $t = h_f = 120\ mm$

$A_s(6\varnothing\ 32) = 4825.5\ mm^2$

$b_e$ is the smallest of:

$$(a)\ \ b_e \le b_w + \frac{L}{12} = 300 + \frac{6000}{12} = 800\ mm, \qquad -control$$
$$(b)\ \ b_e \le b_w + 6h_f = 300 + 6 \cdot 120 = 1020\ mm,$$
$$(c)\ \ b_e \le b_w + \frac{1}{2}\text{ the clear distance to the next beam web} = 300 + \frac{2200}{2} = 1400\ mm.$$

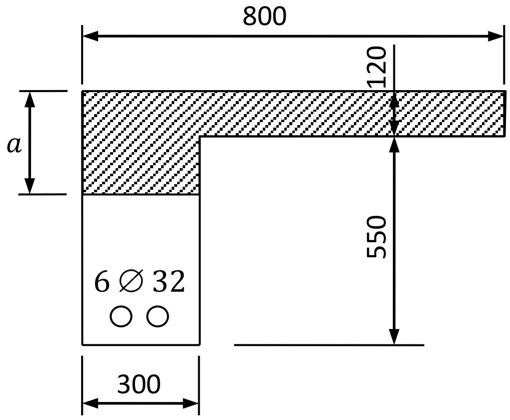
Take $b_e = 800\ mm.$

Check if $a > h_f$

$$a = \frac{A_s f_y}{0.85\ f_c' b_e} = \frac{4825.5\ \cdot 400}{0.85\ \cdot 20 \cdot 800} = 141.93\ mm\ > h_f = 120\ mm$$

The beam section will be considered as L-section with

$b_e = 800\ mm.$



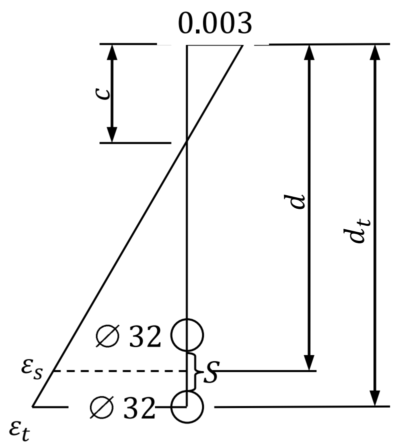

$$A_{sf} = \frac{0.85\ f_c'(b_e - b_w)h_f}{f_y} =$$
$$= \frac{0.85\ \cdot 20(800 - 300)120}{400} = 2550\ mm^2$$
$$A_{sw} = A_s - A_{sf} = 4825.5 - 2550 = 2275.5\ mm^2$$
$$a = \frac{A_{sw} f_y}{0.85\ f_c' b_w} = \frac{2275.5 \cdot 400}{0.85\ \cdot 20 \cdot 300} = 178.47\ mm$$

$A_s\ (6\varnothing\ 32)$ are arranged in two layers

$$d = 670 - 40 - 10 - 32 - \frac{25}{2} = 575.5\ mm$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) =$$
$$= \left[2550 \cdot 400\left(575.5 - \frac{120}{2}\right) + 2275.5 \cdot 400\left(575.5 - \frac{178.47}{2}\right)\right] \times 10^{-6} = 968.4\ KN \cdot m$$

Check for strain $\varepsilon_s \ge 0.005$




$$c = \frac{a}{\beta_1} = \frac{178.47}{0.85} = 209.96\ mm, \qquad \beta_1 = 0.85$$
$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 575.5 + \frac{25}{2} + \frac{32}{2} = 604\ mm$$
$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{604 - 209.96}{209.96}\right) =$$
$$= 0.00563 > 0.005 \qquad OK$$

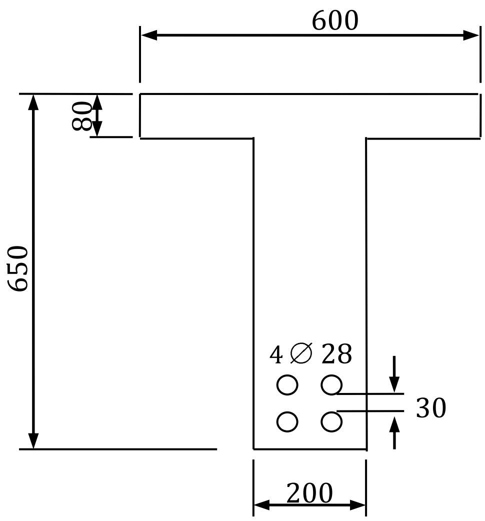
Take $\phi = 0.9$ for flexure as tension-controlled section.

$$M_u = \phi M_n = 0.9 \cdot 961.65 = 865.49\ KN \cdot m$$

**Example:**

Compute the positive design moment capacity of the T-section beam.

$f_c' = 20\ MPa, \qquad f_y = 420\ MPa.$




**Solution:**

From the Geometry of T-section:

$b_w = 200\ mm, \quad h = 650\ mm, \quad t = h_f = 80\ mm$

$A_s(4\varnothing\ 28) = 2463\ mm^2$

Check if $a > h_f$

$$a = \frac{A_s f_y}{0.85\ f_c' b_e} = \frac{2463\ \cdot 420}{0.85\ \cdot 20 \cdot 600} = 101.42\ mm$$

$a = 101.42\ mm > h_f = 80\ mm$ . The beam section will be considered as T-section.

$$A_{sf} = \frac{0.85\ f_c'(b_e - b_w)h_f}{f_y} = \frac{0.85\ \cdot 20(600 - 200)80}{420} = 1295.2\ mm^2$$

$$A_{sw} = A_s - A_{sf} = 2463 - 1295.2 = 1167.76\ mm^2$$

$$a = \frac{A_{sw} f_y}{0.85\ f_c' b_w} = \frac{1167.76\ \cdot 420}{0.85\ \cdot 20 \cdot 200} = 144.25\ mm$$

$A_s\ (4\varnothing\ 28)$ are arranged in two layers

$$d = 650 - 40 - 10 - 28 - \frac{30}{2} = 557\ mm$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) =$$

$$= \left[1295.2\ \cdot 420 \left(557 - \frac{80}{2}\right) + 1167.76 \cdot 420 \left(557 - \frac{144.25}{2}\right)\right] \times 10^{-6} = 519.05\ KN \cdot m$$

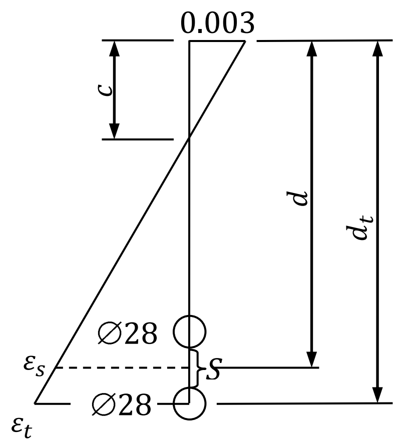
Check for strain $\varepsilon_s \geq 0.005$

$$c = \frac{a}{\beta_1} = \frac{144.25}{0.85} = 169.7\ mm, \qquad \beta_1 = 0.85$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 557 + \frac{30}{2} + \frac{28}{2} = 586\ mm$$

$$\varepsilon_t = 0.003 \left(\frac{d_t - c}{c}\right) = 0.003 \left(\frac{586 - 169.7}{169.7}\right) =$$

$$= 0.00736 > 0.005 \qquad OK$$




Take $\phi = 0.9$ for flexure as tension-controlled section.

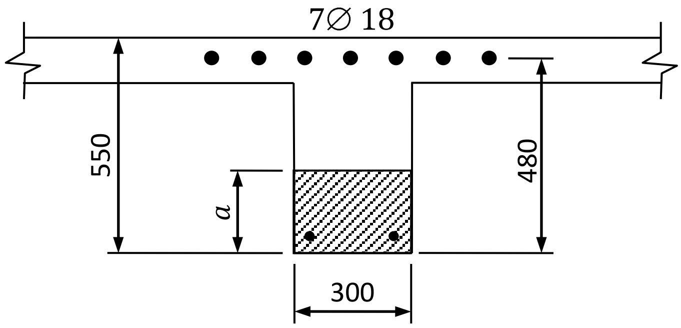
$$M_u = \phi M_n = 0.9 \cdot 519.05 \ = 467.15 \ KN \cdot m$$

#### 4.11.5 Analysis of the negative-moment capacity of a T-section.

**Example:**

Compute the negative design moment capacity of the T-section beam.

$f_c' = 20 \ MPa, \qquad f_y = 400 \ MPa.$




**Solution:**

Analyze as rectangular section because that the compression zone is within the web depth.

$A_s(7\varnothing\,18) = 1781.3 \ mm^2$

$$a = \frac{A_s f_y}{0.85 \, f_c' b_w} = \frac{1781.3 \ \cdot 400}{0.85 \ \cdot 20 \cdot 300} = 139.71 \ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1781.3 \cdot 400 \ \left(480 - \frac{139.71}{2}\right) \times 10^{-6} = 292.23 \ KN \cdot m$$

Check for strain $\varepsilon_s \geq 0.005$

$$c = \frac{a}{\beta_1} = \frac{139.71}{0.85} = 164.36 \ mm, \qquad \beta_1 = 0.85$$

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{480 - 164.36}{164.36}\right) = 0.00576 > 0.005 \qquad OK$$

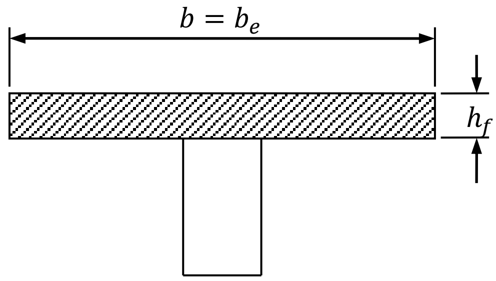
Take $\phi = 0.9$ for flexure as tension-controlled section.

$$M_u = \phi M_n = 0.9 \cdot 292.23 \ = 263.01 \ KN \cdot m$$

#### 4.11.6 Design of T-section.

The design of a T-section beam involves the choice of the cross section and the reinforcement required. The flange thickness and width are usually established during the design of the floor slab. The size of the beam stem is influenced by the same factores that affect the size of a rectangular beam. In the case of a continuous T-beam, the concrete compressive stresses are most critical in the negative-moment regions, where the compression zone is in the beam stem (web).

**Design Procedure:**




1. Check if the depth of the compression block within the thickness of the flange.

Let $a = h_f$, then compute $\bar{M}_{nf} -$ the total moment capacity of the flange.

$$\bar{M}_{nf} = 0.85 f_c' b h_f \left( d - \frac{h_f}{2} \right)$$

**Case I:**

$$a \le h_f \qquad or \qquad \bar{M}_{nf} \ge \frac{M_u}{\phi},$$

Design as rectangular section.

**Case II:**

$$a > h_f \qquad or \qquad \bar{M}_{nf} < \frac{M_u}{\phi},$$

Design as T-section. **GO to step 2.**

2. $M_n = M_{nf} + M_{nw},$ $\qquad A_s = A_{sf} + A_{sw},$

$T_f = C_f \quad \Longrightarrow \quad A_{sf} f_y = 0.85 f_c' (b - b_w) h_f, \quad from\ where$

$$A_{sf} = \frac{0.85 f_c' (b - b_w) h_f}{f_y}, \qquad and$$

$$M_{nf} = A_{sf} f_y \left( d - \frac{h_f}{2} \right) = 0.85 f_c' (b - b_w) h_f \left( d - \frac{h_f}{2} \right)$$

3. Design the web as rectangular section with $b = b_w$, where

$$M_{nw} = M_n - M_{nf} = \frac{M_u}{\phi} - M_{nf}$$

$$A_{sw} = \rho_w b_w d, \qquad \rho_w = \frac{1}{m}\left( 1 - \sqrt{1 - \frac{2 R_{nw} m}{f_y}} \right), \qquad R_{nw} = \frac{M_{nw}}{b_w d^2}$$

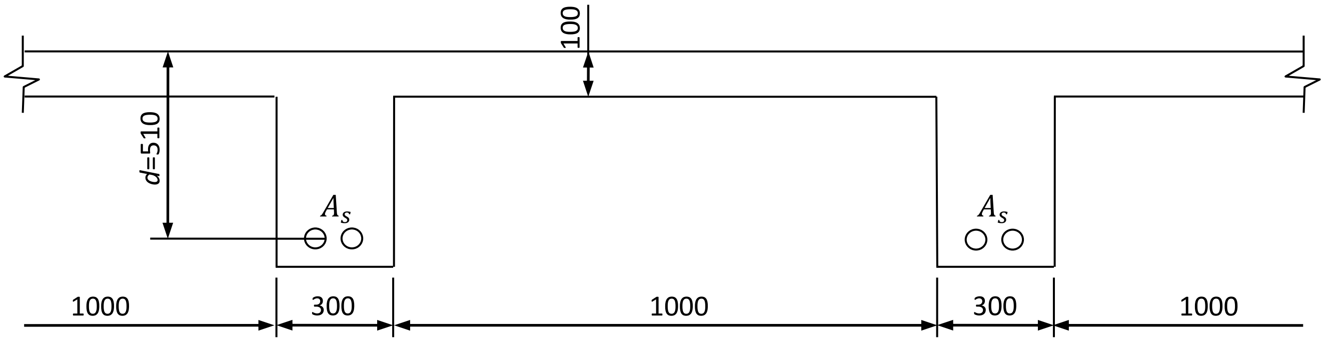
The index $XX_f$ and $XX_w$ in the previous notation refers to $f - flange,\quad w - web.$

**Example:**

Compute the area of steel reinforcement for the interior beam shown below. The beam has a clear span of 6 m (face to face).

Ultimate factored moment $M_u = 720\ KN \cdot m$

$f_c' = 20\ MPa, \qquad f_y = 400\ MPa.$




**Solution:**

From the Geometry of T-section:

$b_w = 300\ mm, \quad d = 510\ mm, \quad t = h_f = 100\ mm$

$b_e$ is the smallest of:

$$(a)\ \ b_e \leq \frac{L}{4} = \frac{6000}{4} = 1500\ mm,$$

$$(b)\ \ b_e \leq b_w + 16h_f = 300 + 16 \cdot 100 = 1900\ mm,$$

$(c)$ $b_e \leq$ Center to Center spacing between adjacent beams

$$b_e = 1300\ mm. -control$$

Take $b = b_e = 1300\ mm.$

$$\bar{M}_{nf} = 0.85 f_c' b h_f \left(d - \frac{h_f}{2}\right) = 0.85 \cdot 20 \cdot 1300 \cdot 100 \left(510 - \frac{100}{2}\right) \times 10^{-6} = 1016.6\ KN \cdot m$$

$$\bar{M}_{nf} = 1016.6\ KN \cdot m > \frac{M_u}{\phi} = \frac{720}{0.9} = 800\ KN \cdot m \quad \Longrightarrow \quad a < h_f$$

The section will be designed as rectangular section with $b = 1300\ mm.$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{720}{0.9 \cdot 1300 \cdot 510^2} = 2.366\ \ MPa, \qquad m = \frac{f_y}{0.85 f_c'} = \frac{400}{0.85 \cdot 20} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2R_n m}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \cdot 2.366 \cdot 23.53}{400}}\right) = 0.0064,$$

$A_s = \rho b d = 0.0064 \cdot 1300 \cdot 510 = 4243.2\ mm^2$

Check for $A_{s,min}$

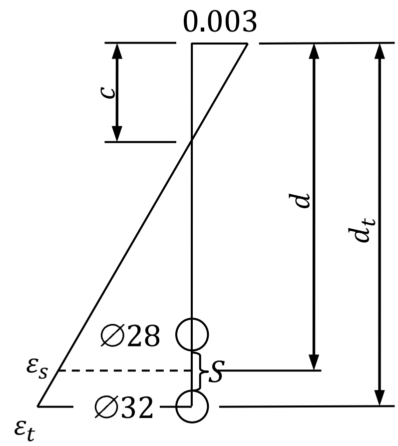
$$A_{s,min} = 0.25 \frac{\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$A_{s,min} = 0.25 \frac{\sqrt{20}}{400} 300 \cdot 510 = 428\ mm^2$$

$$A_{s,min} = \frac{1.4}{400} 300 \cdot 510 = 534\ mm^2 \quad -control$$

$A_s = 4243.2\ \ mm^2 > A_{s,min} = 534\ mm^2 \ \ -OK$

Use $3\varnothing 32 + 3\varnothing 28$ in two layers with




$$A_s = 24.127 + 18.473 = 42.6\ cm^2 > A_{s,req} = 42.43\ cm^2 \ - OK$$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{4260\ \cdot 400}{0.85 \cdot 20 \cdot 1300} = 77.1\ mm$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85$$

$$c = \frac{77.1}{0.85} = 90.71\ mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 510 + \frac{25}{2} + \frac{32}{2} = 538.5\ mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{538.5 - 90.71}{90.71}\right) = 0.0148 > 0.005 \qquad OK$$

Check for bar placement in one layer:

$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 3 \times 32}{2} = 52\ mm > d_b = 32\ mm, \quad > 25\ mm \qquad OK$$

**Example:**

Repeat the previous example using $M_u = 930\ KN \cdot m$.

**Solution:**

$$\bar{M}_{nf} = 1016.6\ KN \cdot m < \frac{M_u}{\phi} = \frac{930}{0.9} = 1033.3\ KN \cdot m \quad \Longrightarrow \quad a > h_f$$

The section will be designed as T-section section.

$$A_{sf} f_y = 0.85 f_c'(b - b_w) h_f, \quad from\ where \qquad A_{sf} = \frac{0.85 f_c'(b - b_w) h_f}{f_y},$$

$$A_{sf} = \frac{0.85 f_c'(b - b_w) h_f}{f_y} = \frac{0.85 \cdot 20(1300 - 300) \cdot 100}{400} = 4250\ mm^2$$

$$M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 4250 \cdot 400 \left(510 - \frac{100}{2}\right) \times 10^{-6} = 782\ KN \cdot m$$

$$M_{nw} = M_n - M_{nf} = \frac{M_u}{\phi} - M_{nf} = \frac{930}{0.9} - 782 = 251.3\ KN \cdot m$$

Here the web will be designed as rectangular section with $b = b_w = 300\ mm$ to resist $M_{nw} = 251.3\ KN \cdot m$

$$R_{nw} = \frac{M_{nw}}{b_w d^2} = \frac{251.3 \cdot 10^6}{300 \cdot 510^2} = 3.22\ MPa,$$

$$m = \frac{f_y}{0.85 f_c{}'} = \frac{400}{0.85 \cdot 20} = 23.53$$

$$\rho_w = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2R_{nw}m}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \cdot 3.22 \cdot 23.53}{400}}\right) = 0.009,$$

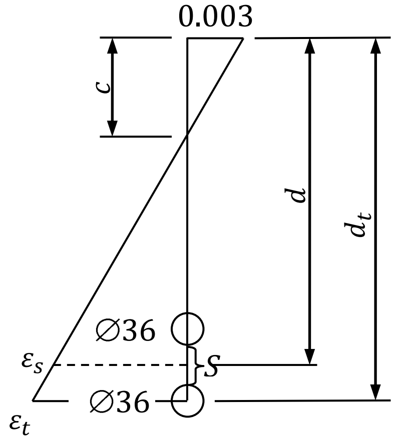
$A_{sw} = \rho_w b_w d = 0.009 \cdot 300 \cdot 510 = 1377\ mm^2,$

$A_s = A_{sf} + A_{sw} = 4250 + 1377 = 5627\ mm^2$

$A_s = 5627\ \ mm^2 > A_{s,min} = 534\ mm^2\ \ - OK$

Use 6 $\varnothing$ 36 in two layers with $A_s = 61.07\ cm^2 >\ A_{s,req} = 56.27\ cm^2$ – $OK$

$A_{sw,provided} = A_{s,provided} - A_{sf} = 6107 - 4250 = 1857\ mm^2$




Check for strain:

$$a = \frac{A_{sw} f_y}{0.85\ f_c' b_w} = \frac{1857\ \cdot 400}{0.85 \cdot 20 \cdot 300} = 145.65\ mm$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85$$

$$c = \frac{145.65}{0.85} = 171.35\ mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 510 + \frac{25}{2} + \frac{36}{2} = 540.5\ mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{540.5 - 171.35}{171.35}\right) = 0.00646 > 0.005 \qquad OK$$

Check for bar placement in one layer:

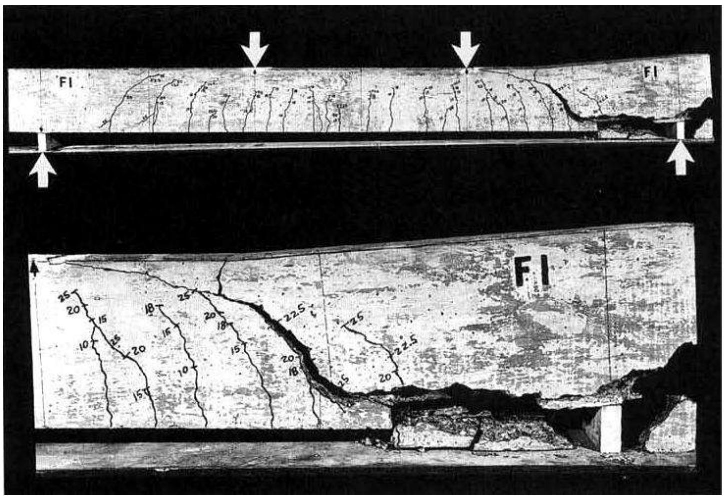
$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 3 \times 36}{2} = 46\ mm > d_b = 36\ mm, \quad > 25\ mm \qquad OK$$

# CHAPTER 5 SHEAR IN BEAMS

## 1.1. INTRODUCTION

When a simple beam is loaded, bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters.

The beam is then designed for shear. If shear reinforcement is not provided, shear failure may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.



Shear failure of reinforced concrete beam

By the traditional theory of homogeneous, elastic, uncracked beams, we can calculate shear stresses, $v$, using equation

$$v = \frac{VQ}{Ib}$$

where $V -$ total shear at the section considered,

$Q -$ statical moment about the neutral axis of that portion of cross-section lying between a line through the point in question parallel to the neutral axis and nearest face, upper or lower, of the beam,

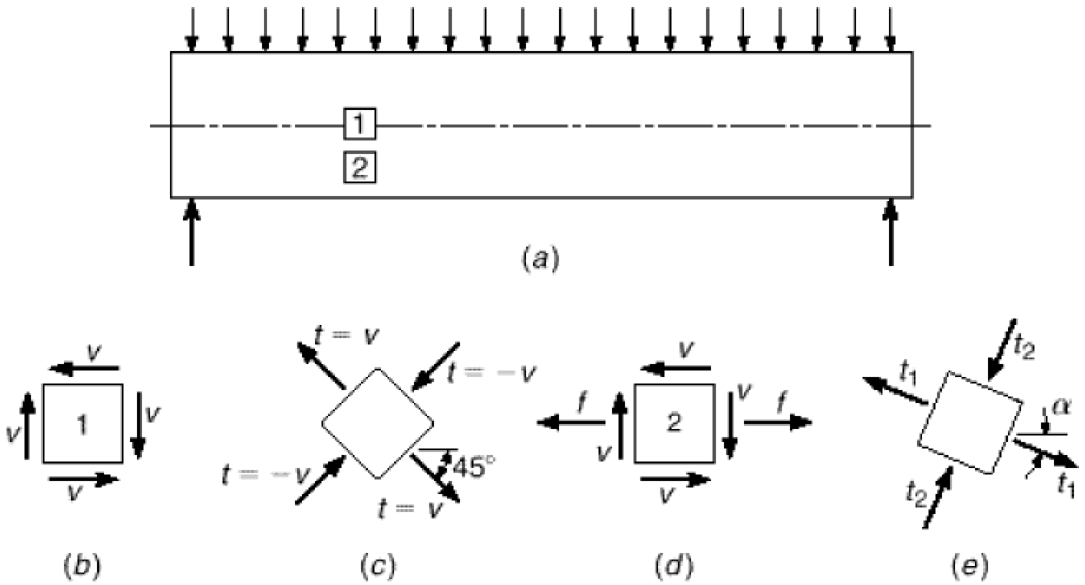
$I -$ moment of inertia of cross-section about the neutral axis,

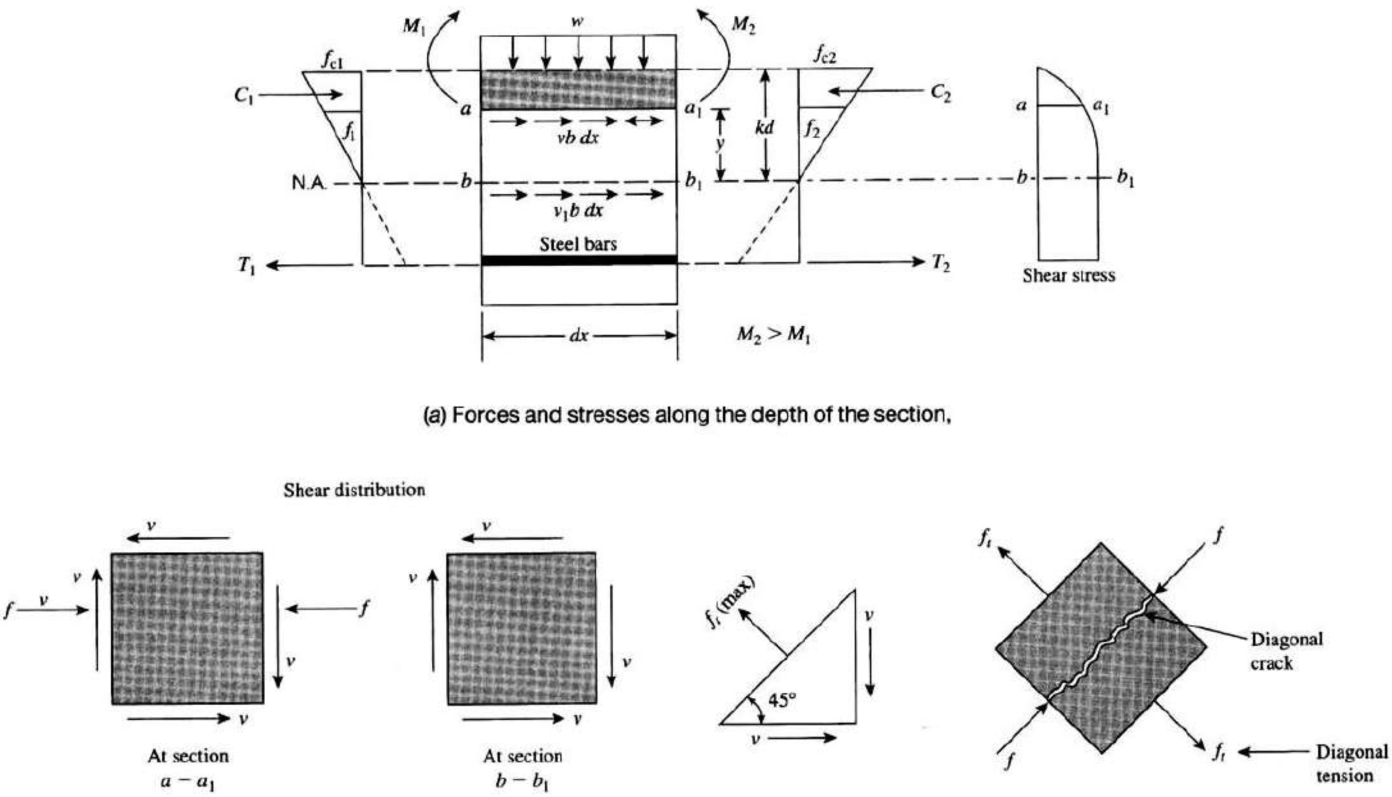
$b -$ width of beam at the given point.

The tensile stresses are equivalent to the principal stresses. Such principal stresses are traditionally called **diagonal tension stresses**. When the diagonal tension stresses reach the tensile strength of concrete, a diagonal crack develops. This brief analysis explains the concept of diagonal tension and diagonal cracking. The actual behavior is more complex, and it is affected by other factors. For the combined action of shear and normal stresses at any point in a beam, the maximum and minimum diagonal tension (principal stresses) $f_p$ are given by the equation

$$f_p = \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + v^2}$$

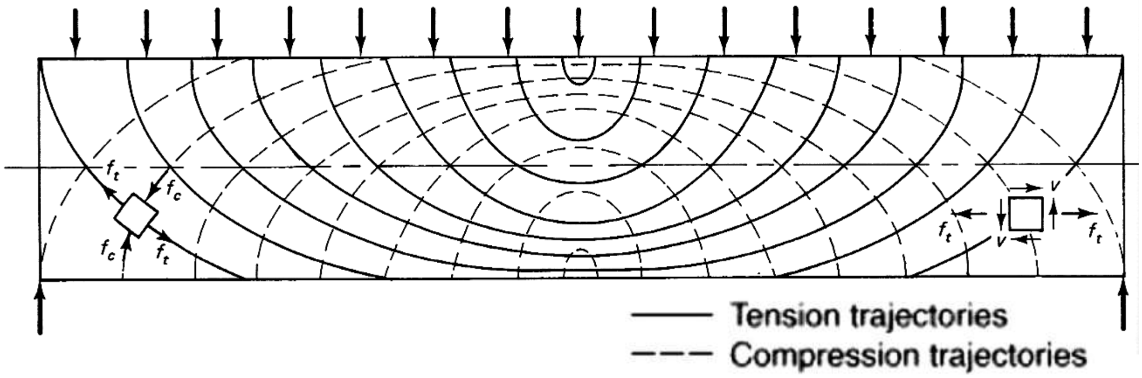
$f$ – intensity of normal stress due to bending,
$v$ – shear stress.



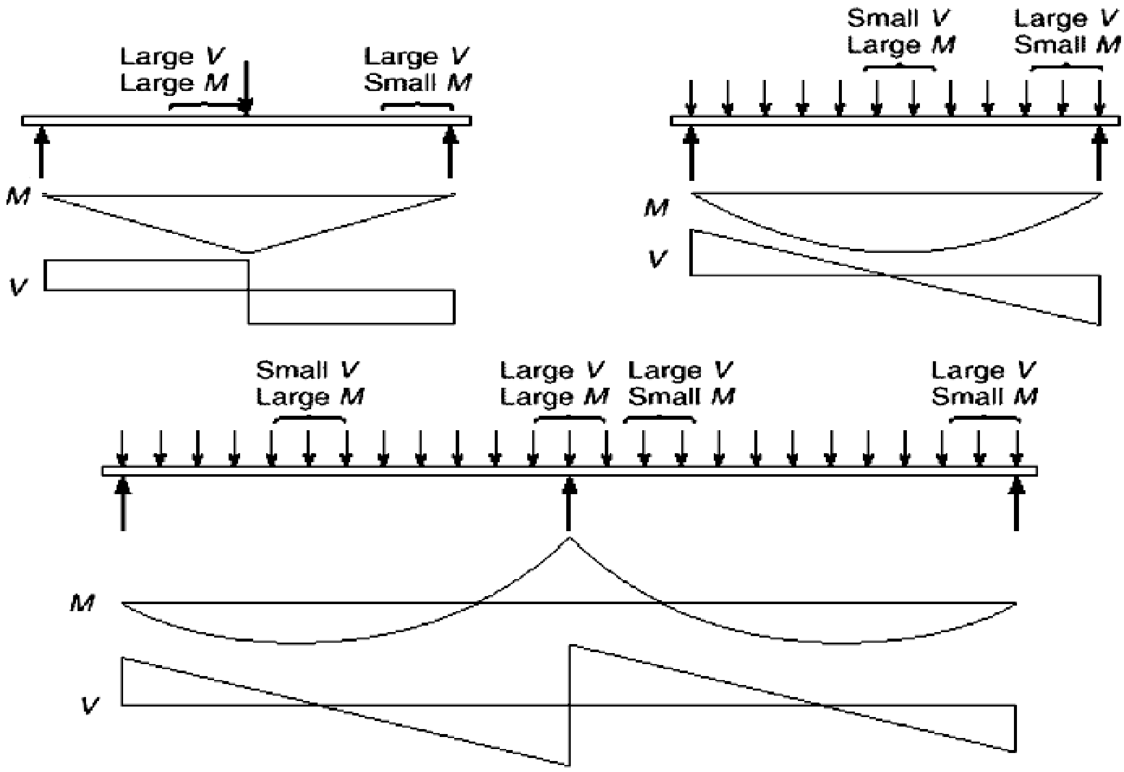

(a) Forces and stresses along the depth of the section,

(b) normal and shear stresses, (c) pure shear, (d) diagonal tension.




Trajectories of principal stresses in a homogeneous isotropic beam.

### 1.2. CRITICAL SECTIONS FOR SHEAR DESIGN




Typical Locations of critical combinations of shears and moment

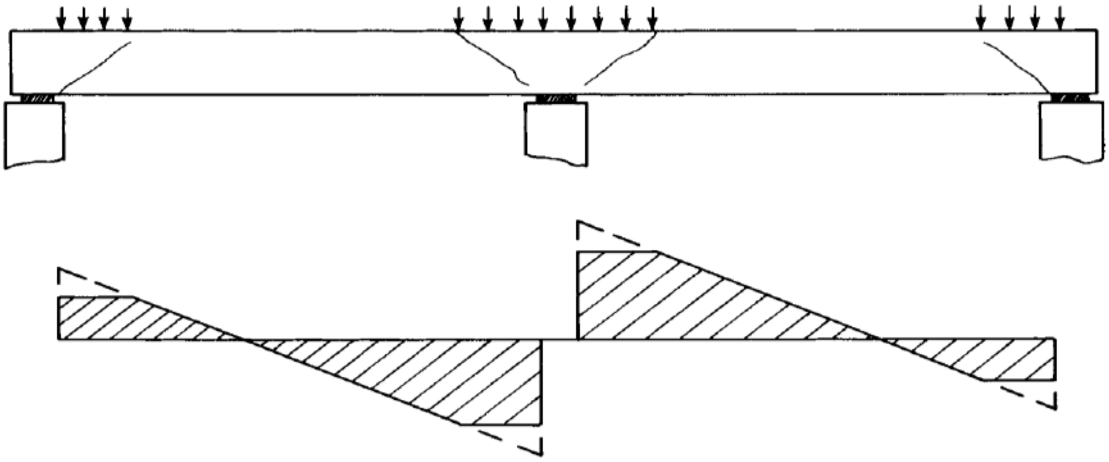
In a beam loaded on the top flange and supported on the bottom as shown in the figure below, the closest inclined cracks that can occur adjacent to the supports will extend outward from the supports at roughly 45°. Loads applied to the beam within a distance $d$ from the support in such a beam will be transmitted directly to the support by the compression fan above the 45° cracks and will not affect the stresses in the stirrups crossing the cracks shown. As a result, ACI Code Section 11.1.3.1 states:

For nonprestressed members, sections located less than a distance $d$ from the face of the support may be designed for the same shear, $V_u$, as that computed at a distance $d$.

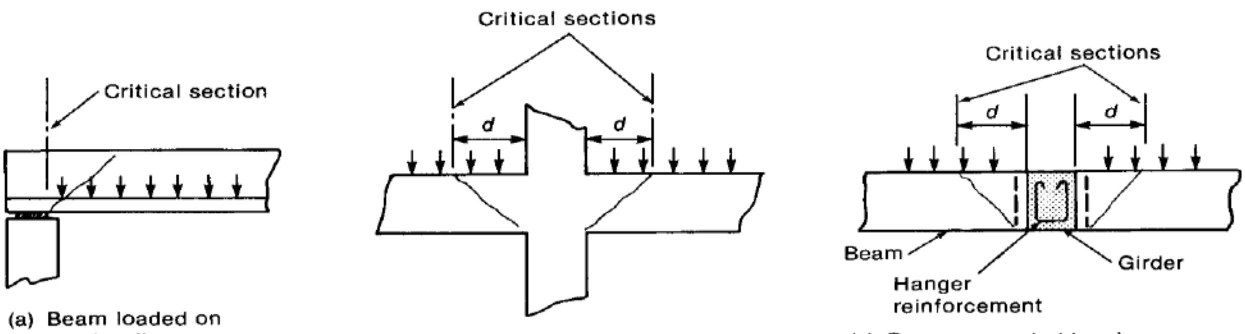
This is permitted only when

1. the support reaction, in the direction of the applied shear, introduces compression into the end regions of a member,
2. the loads are applied at or near the top of the beam, and
3. no concentrated load occurs within d from the face of the support.

Thus, for the beam shown below, the values of $V_u$ used in design are shown shaded in the shear force diagram.



This allowance must be applied carefully because it is not applicable in all cases. There are shows five other typical cases that arise in design. If the beam was loaded on the lower flange, as indicated in Fig. a, the critical section for design would be at the face of the support, because loads applied within $d$ of the support must be transferred across the inclined crack before they reach the support.

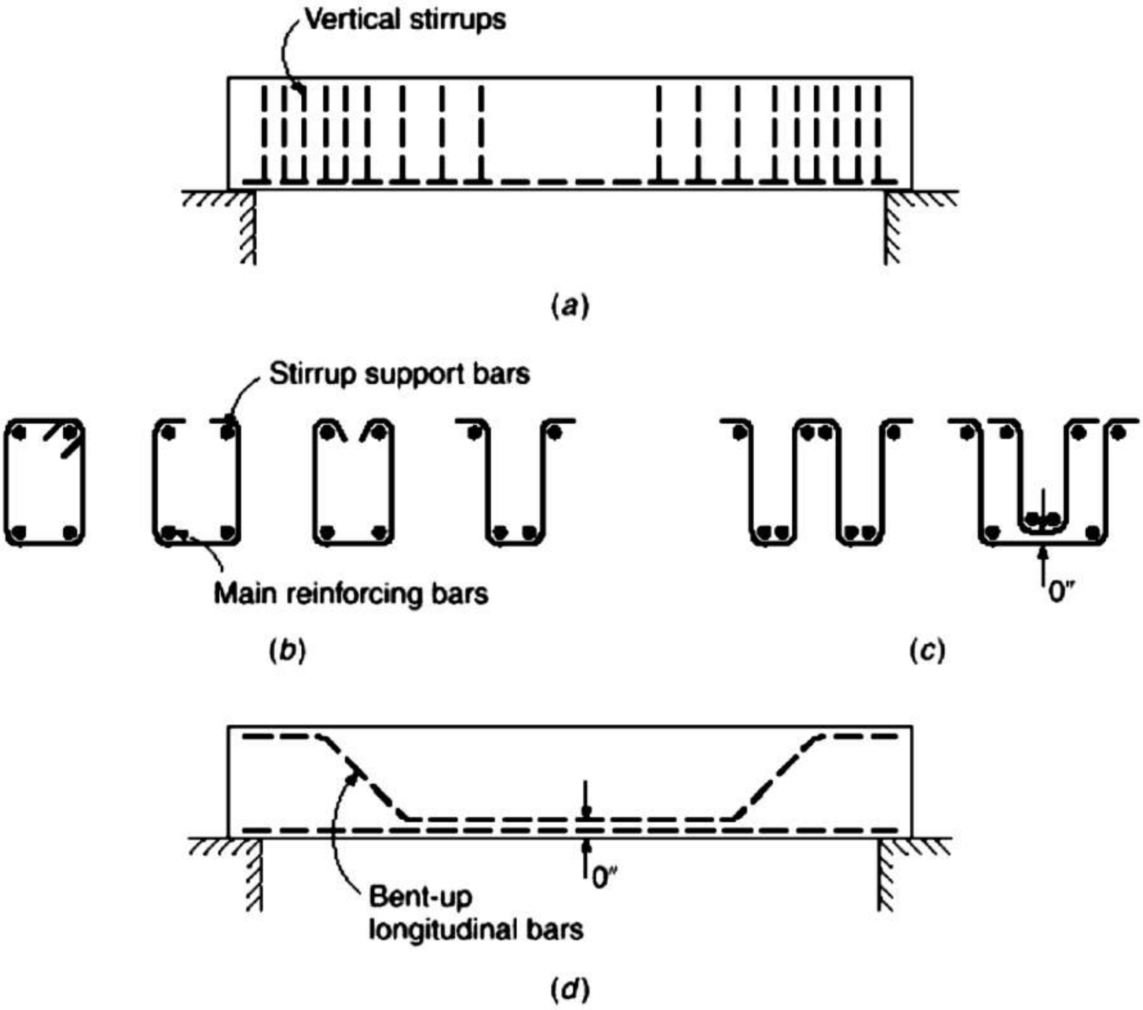



(a) Beam loaded on tension flange. (b) Beam column joint. (c) Beam supported by shear.

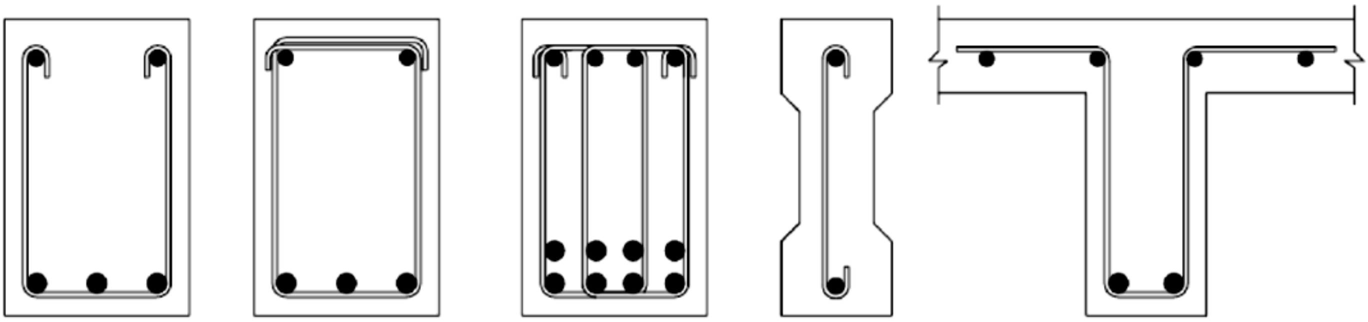

(d) Beam supported by tension force. (e) Beam with concentrated load close to support.

A typical beam-to-column joint is shown in Fig. b. Here the critical section for design is $d$ away from the section as shown.
If the beam is supported by a girder of essentially the same depth, as shown in Fig. c, the compression fans that form in the supported beams will tend to push the bottom off the supporting beam. The critical shear design sections in the supported beams normally are taken at the face of the supporting beam. The critical section may be taken at $d$ from the end of the beam if hanger reinforcement is provided to support the reactions from the compression fans.
Generally, if the beam is supported by a tensile force rather than a compressive force, the critical section will be at the face of the support, and the joint must be carefully detailed, because shear cracks will extend into the joint, as shown in Fig. d.
Occasionally, a significant part of the shear at the end of the beam will be caused by a concentrated load acting less than d from the face of the column, as shown in Fig. e. In such a case, the critical section must be taken at the support face.

## 1.3. TYPES OF WEB REINFORCEMENT




a) Vertical ***Stirrups,***
b) U-shaped bars single stirrups.
c) Multiple-leg stirrups
d) Bent-up longitudinal (inclined) bars



**The ACI Code defines the types of shear reinforcement as:**

**11.4.1.1** — Shear reinforcement consisting of the following shall be permitted:

(a) Stirrups perpendicular to axis of member;

(b) Welded wire reinforcement with wires located perpendicular to axis of member;

(c) Spirals, circular ties, or hoops.

**11.4.1.2** — For nonprestressed members, shear reinforcement shall be permitted to also consist of:

(a) Stirrups making an angle of 45 degrees or more with longitudinal tension reinforcement;

(b) Longitudinal reinforcement with bent portion making an angle of 30 degrees or more with the longitudinal tension reinforcement;

(c) Combinations of stirrups and bent longitudinal reinforcement.

## 1.4. DESIGN PROCEDURE FOR SHEAR

Design of cross section subjected to shear shall be based on:

$$\phi V_n \geq V_u$$

where $V_u -$ the factored shear force at the section,

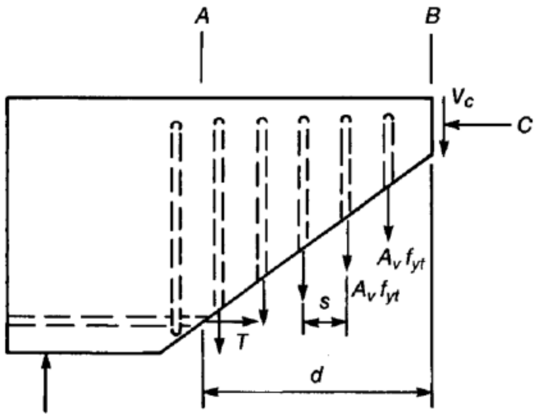
$V_n -$ the nominal shear strenght,

$$V_n = V_c + V_s,$$

$V_c -$ the nominal shear strenght provided by concrete,

$V_s -$ the nominal shear strenght provided by shear reinforcement (stirrups),

The figure shows a free body between the end of a beam and an inclined crack. The horizontal projection of the crack is taken as $d$, suggesting that the crack is slightly flatter than 45°. If $s$ is the stirrup spacing, the number of stirrups cut by the crack is $d/s$. Assuming that all the stirrups yield at failure, the shear resisted by the stirrups is




$$V_s = \frac{A_v f_{yt} d}{s}$$

ACI Code 11.2.1 states, for members subject to shear and flexure only

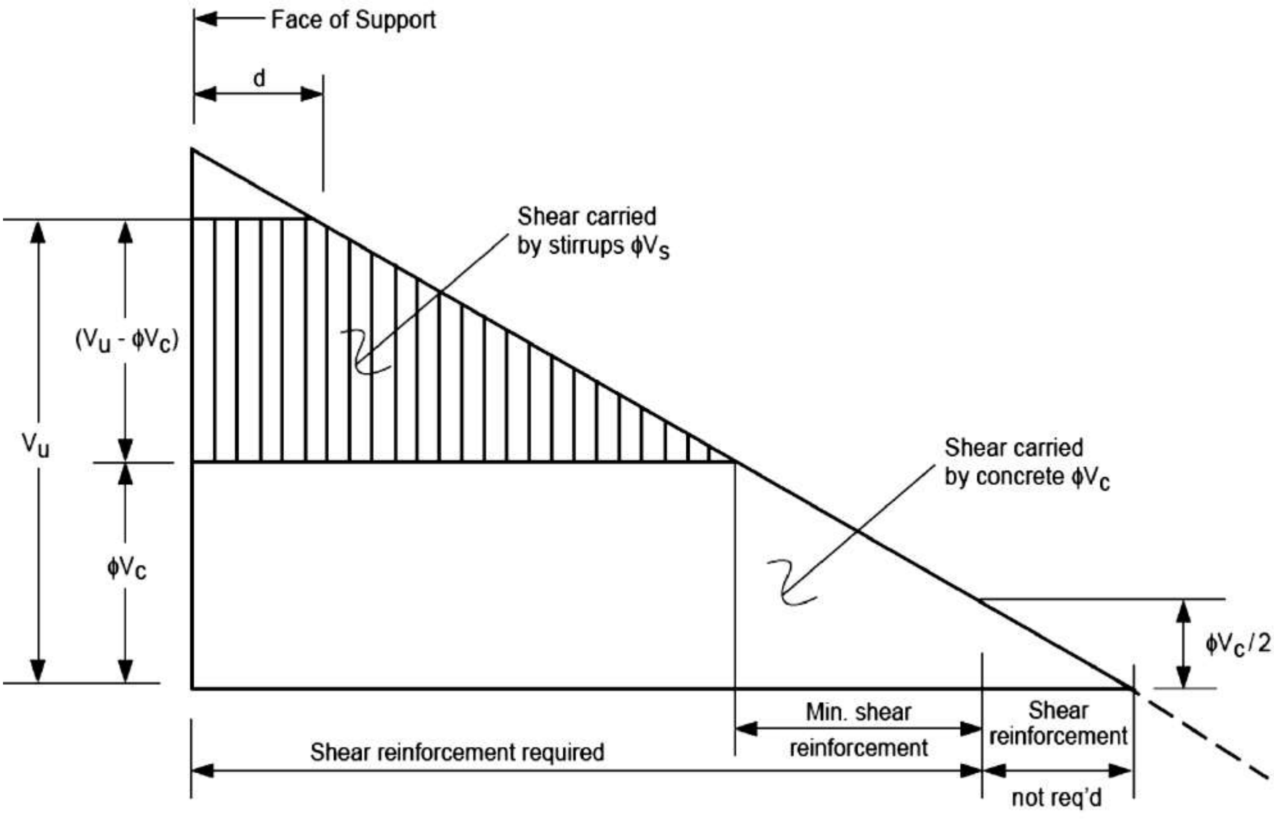
$$V_c = \frac{1}{6}\lambda\sqrt{f_c'}b_w d = 0.17\lambda\sqrt{f_c'}b_w d, \qquad \lambda = 1.0 \ for\ Normal - weight\ concrete$$

$V_c$ shall be permitted to be computed by the more detailed calculation

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{V_u d}{M_u}\right) b_w d \le 0.29\sqrt{f_c'}b_w d, \qquad where \qquad \frac{V_u d}{M_u} \le 1$$

To simplify the calculations the formula $V_c = 0.17\lambda\sqrt{f_c'}b_w d$ will be used.

**Shear conditions and cases (Items):**




**Check for dimensions:**

The ACI Code, 11.4.7.9, states that $V_s$ shall not be taken greater than $0.66\sqrt{f_c'}b_w d$.

$So, \quad if \quad V_s > V_{s,max} - The\ section\ must\ be\ enlarged\ (Dimenstions\ are\ not\ enough)$

$$where \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c, \qquad V_{s,max} = \frac{2}{3}\sqrt{f_c'}b_w d$$

**Case I:**

$$V_u \leq \frac{1}{2}\phi V_c \quad - No\ shear\ reinforcement\ is\ required$$

**Case II:**

$$\frac{1}{2}\phi V_c < V_u \leq \phi V_c \quad - Minimum\ shear\ reinforcement\ is\ required\ \left(A_{v,min}\right)\ except:$$

- footings and solid slabs,
- Hollow-core units with total untopped depth not greater than 315 mm and hollow-core units where $V_u$ is not greater than $0.5\phi V_{cw}$;
- Concrete joist construction;
- Beams with $h$ not greater than $250\ mm$;
- Beam integral with slabs with $h$ not greater than $600\ mm$ and not greater than the larger of 2.5 times thickness of flange, and 0.5 times width of web;
- Beams constructed of steel fiber-reinforced, normalweight concrete with $f_c'$ not exceeding 40 MPa, $h$ not greater than 600 mm, and $V_u$ not greater than $0.17\sqrt{f_c'}b_w d$.

For these cases no shear reinforcement is required unless $V_u > \phi V_c$.

Minimum shear reinforcement, $A_{v,min}$

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} = 0.062\sqrt{f_c'}\frac{b_w s}{f_{yt}} \quad \geq \quad \frac{1}{3}\frac{b_w s}{f_{yt}} = 0.35\frac{b_w s}{f_{yt}},$$

$$or\ in\ the\ form \qquad \left(\frac{A_{v,min}}{s}\right) \geq \frac{1}{3}\frac{b_w}{f_{yt}} \geq \frac{1}{16}\sqrt{f_c'}\frac{b_w}{f_{yt}},$$

$$Here \qquad s_{max} \leq \frac{d}{2} \qquad or \qquad s_{max} \leq 600\ mm$$

where $s -$ step of stirrups (spacing between stirrups),

$f_{yt} -$ yield stress of stirrups

**Case III:**

$$\phi V_c < V_u \leq \phi\left(V_c + V_{s,min}\right)$$

$$\frac{A_{v,min}}{s} = \frac{V_{s,min}}{f_{yt}d} \implies \qquad V_{s,min} = \left(\frac{A_{v,min}}{s}\right)f_{yt}d$$

then, $V_{s,min}$ is the maximum of

$$V_{s,min} = \frac{1}{16}\sqrt{f_c'}b_w d \qquad and \qquad V_{s,min} = \frac{1}{3}b_w d$$

Minimum shear reinforcement is provided $(A_{v,min})$ with

$$s_{max} \le \frac{d}{2} \qquad or \qquad s_{max} \le 600\ mm$$

**Case IV:**

$$\phi\left(V_c + V_{s,min}\right) < V_u \le \phi(V_c + V_s') \quad - stirrups\ are\ required$$

$$where \qquad V_{s,min} < V_s \le V_s', \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c \qquad V_s' = \frac{1}{3}\sqrt{f_c'}b_w d$$

$$and \qquad \frac{A_v}{s} = \frac{V_s}{f_{yt}d}.$$

$$here \qquad s_{max} \le \frac{d}{2} \qquad or \qquad s_{max} \le 600\ mm$$

**Case V:**

$$\phi(V_c + V_s') < V_u \le \phi\left(V_c + V_{s,max}\right) \quad - stirrups\ are\ required$$

$$where \qquad V_s' < V_s \le V_{s,max}, \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c\ , \qquad V_s' = \frac{1}{3}\sqrt{f_c'}b_w d,$$

$$V_{s,max} = \frac{2}{3}\sqrt{f_c'}b_w d \qquad and \qquad \frac{A_v}{s} = \frac{V_s}{f_{yt}d}.$$

$$here \qquad s_{max} \le \frac{d}{4} \qquad or \qquad s_{max} \le 300\ mm$$

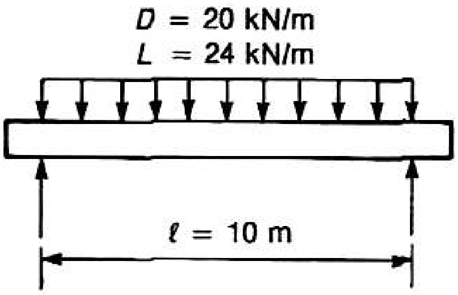
**Example:**

The Figure shows the elevation and cross section of a simply supported T-beam. This beam supports a uniformly distributed service (unfactored) dead load of 20 $KN/m$, including its own weight, and a uniformly distributed service live load of 24 $KN/m$. Design vertical stirrups for this beam. The concrete strength is 25 $MPa$, the yield strength of the flexural reinforcement is 420 $MPa$, and the yield strength of the stirrups is 300 $MPa$.

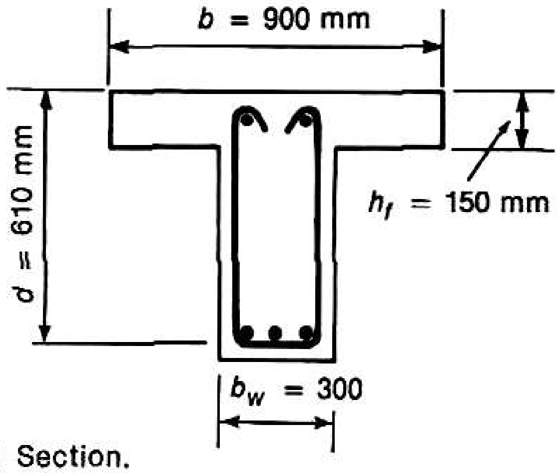
**The support reactions act usually at the center of supports with full span center to center of supports, in this example, we have no information about the support width, so we assumed that the shear calculations will be done for the given clear span with end reactions at the face of supports for the following all examples.**

**Important Note:**

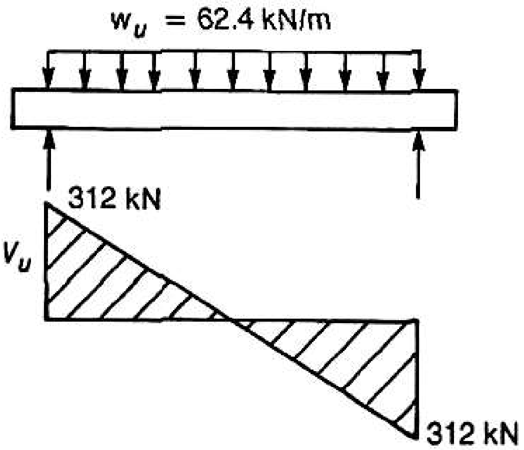
*In a normal building, the dead and live loads are assumed to be uniform loads. Although the dead load is always present over the full span, the live load may act over the full span, or over part of the span. Full uniform load over the full span gives the maximum shear at the ends of the beam. Full uniform load over half the span plus dead load on the remaining half gives the maximum shear at midspan. The maximum shear forces at other points in the span are closely approximated by a linear shear-force envelope resulting from these cases.*



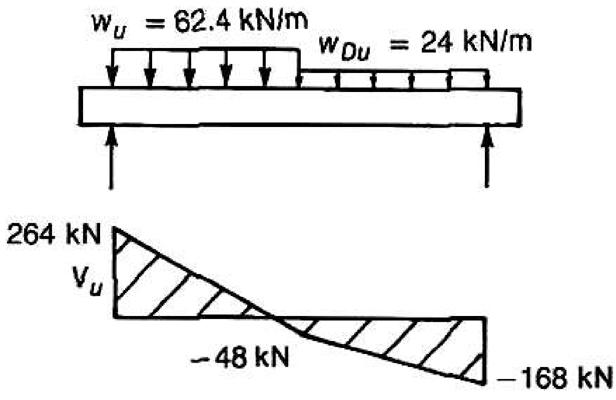

(a) Elevation.



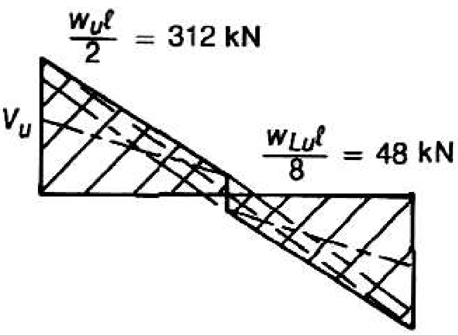

(b) Section.



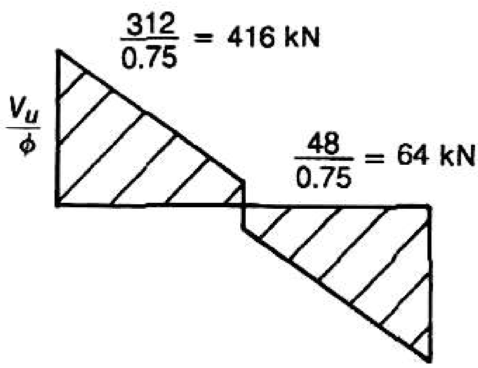

(c) Load case 1.




(d) Load case 2.

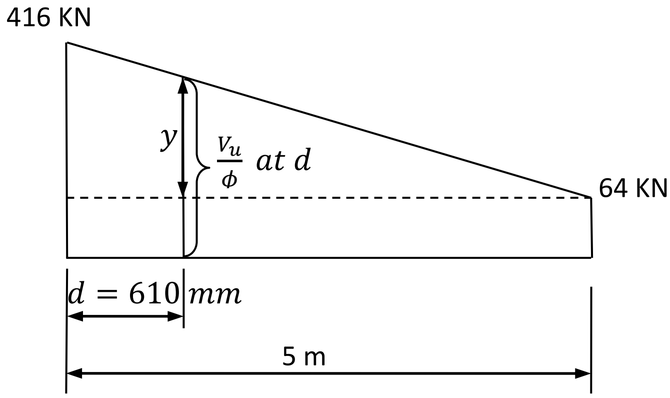



(e) Shear force envelope.




(f) $V_u/\phi$ diagram.

**Solution:**




Critical section at $d = 610\ mm$ from the face of support.

$V_u$ at $d = 610\ mm$.

$$\frac{416-64}{5} = \frac{y}{5-0.61} \quad \longrightarrow y = 309\ KN$$

$$V_n = \frac{V_u}{\phi} = y + 64 = 309 + 64 = 373\ KN$$

$$V_c = \frac{1}{6}\lambda\sqrt{f_c'}b_w d = \frac{1}{6}\cdot 1\cdot\sqrt{25}\cdot 300\cdot 610\cdot 10^{-3} = 152.5\ KN.$$

Check for section dimensions:

$$V_s = V_n - V_c = 373 - 152.5 = 220.5\ KN.$$

$$V_{s,max} = \frac{2}{3}\sqrt{f_c'}b_w d = \frac{2}{3}\sqrt{25}\cdot 300\cdot 610\cdot 10^{-3} = 610\ KN$$

$$V_s = 220.5\ KN < V_{s,max} = 610\ KN \quad - the\ section\ is\ large\ enough.$$

OR

$$V_{n,max} = V_c + V_{s,max} = \frac{1}{6}\sqrt{f_c'}b_w d + \frac{2}{3}\sqrt{f_c'}b_w d = \left(\frac{1}{6}+\frac{2}{3}\right)\sqrt{f_c'}b_w d = \frac{5}{6}\sqrt{f_c'}b_w d = 5V_c$$

$$V_{n,max} = 5\cdot 152.5 = 762.5\ KN$$

$$\frac{V_u}{\phi} = 373\ KN < V_{n,max} = 762.5\ KN \quad - the\ section\ is\ large\ enough$$

Find the maximum stirrups spacing:

$$if \quad V_s < V_s' = \frac{1}{3}\sqrt{f_c'}b_w d \quad then \quad s_{max} \le \frac{d}{2} \quad or \quad s_{max} \le 600\ mm$$

$$V_s' = \frac{1}{3}\sqrt{f_c'}b_w d = \frac{1}{3}\sqrt{25}\cdot 300\cdot 610\cdot 10^{-3} = 305\ KN$$

$$V_s = 220.5\ KN < V_s' = 305\ KN \quad then$$

$$s_{max} \le 600\ mm, \qquad s_{max} \le \frac{d}{2} = \frac{610}{2} = 305\ mm - control$$

$$V_n = 373\ KN > V_c = 152.5\ KN \quad or$$

$$V_u = \phi V_n = 0.75\cdot 373 = 279.75\ KN > \phi V_c = 0.75\cdot 152.5 = 114.375\ KN$$

Try minimum shear reinforcement:

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} \quad but\ not\ less\ than$$

$$A_{v,min} = \frac{1}{3}\frac{b_w s}{f_{yt}}, \quad - control \quad \left(\frac{1}{16}\sqrt{f_c'} = \frac{5}{16} < \frac{1}{3}\right)$$

Use stirrups U-shape (double-leg stirrups) $\varnothing 10$ with $A_v = 2\cdot 78.5 = 157.1\ mm^2$

$$s = \frac{3A_v f_{yt}}{b_w} = \frac{3 \cdot 157.1 \cdot 300}{300} = 471.3\ mm > \ s_{max} = 305\ mm, \quad take\ s = s_{max} = 305\ mm$$

$$V_{s(2\varnothing\,10)} = \frac{A_v f_{yt} d}{s} = \frac{157.1 \cdot 300 \cdot 610}{305} \cdot 10^{-3} = 94.26\ KN$$

$$V_s = 220.5\ KN > V_{s(2\varnothing\,10)} = 94.26\ \ KN, \quad find\ "s" - Case\ IV$$

Alternative step is to calculate $V_{s,min}$

$$V_{s,min} = \frac{1}{16}\sqrt{f_c'}b_w d = \frac{1}{16}\sqrt{25} \cdot 300 \cdot 610 \cdot 10^{-3} = 57.2\ KN$$

$$V_{s,min} = \frac{1}{3}b_w d = \frac{1}{3}300 \cdot 610 \cdot 10^{-3} = 61\ KN \qquad - control$$

$$\phi\left(V_c + V_{s,min}\right) < \ V_u \leq \ \phi(V_c + V_s')$$

$$0.75(152.5 + 61) = 160.13\ KN < \ V_u = 279.75\ KN < \ 0.75(152.5 + 305) = 343.13\ KN$$

Or $V_s = 220.5\ KN > V_{s,min} = 61\ KN$ – Case IV

Compute the stirrups spacing required to resist the shear forces.

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} \quad \Longrightarrow \quad s = \frac{A_v f_{yt} d}{V_s} = \frac{157.1 \cdot 300 \cdot 610}{220.5 \cdot 10^3} = 130.4\ mm.$$

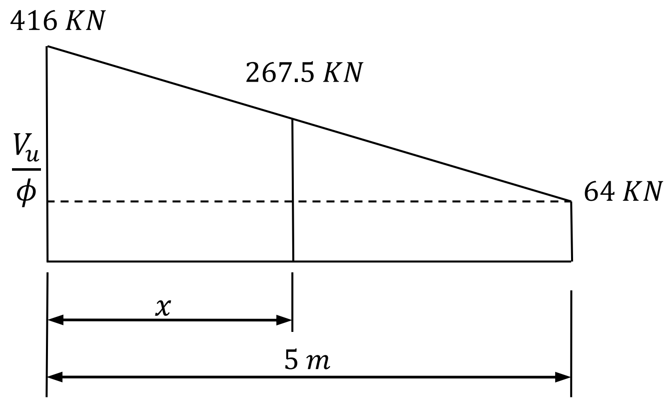
Take U-shape (double-leg stirrups) $\varnothing\,10@125\ mm\ < s_{max} = 305\ mm.$

Changing "$s$" to $s_2 = 2s_1 = 2 \cdot 125 = 250\ mm$ for another region.

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} = \frac{\left(\frac{V_u}{\phi} - V_c\right)}{f_{yt} d} \Longrightarrow \frac{V_u}{\phi} = \frac{A_v f_{yt} d}{s} + V_c$$

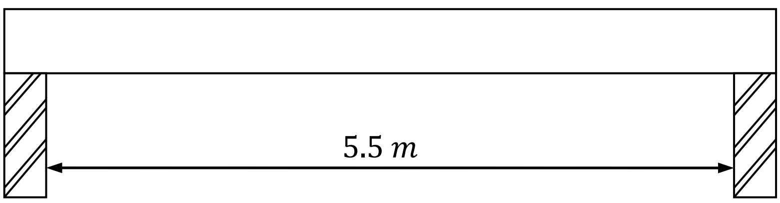
$$\frac{V_u}{\phi} = \frac{157.1 \cdot 300 \cdot 610}{250 \cdot 10^3} + 152.5 = 267.5\ KN$$

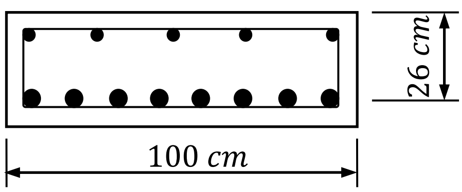
$$\frac{416 - 64}{5} = \frac{267.5 - 64}{5 - x} \quad \rightarrow x = 2.1\ m$$




**Example:**

The simply supported beam shown below is loaded by a service dead load of 40 $KN/m$, and a uniformly distributed service live load of 25 $KN/m$. Design vertical stirrups for this beam. The concrete strength is 25 $MPa$, and the yield strength of the stirrups is 412 $MPa$.



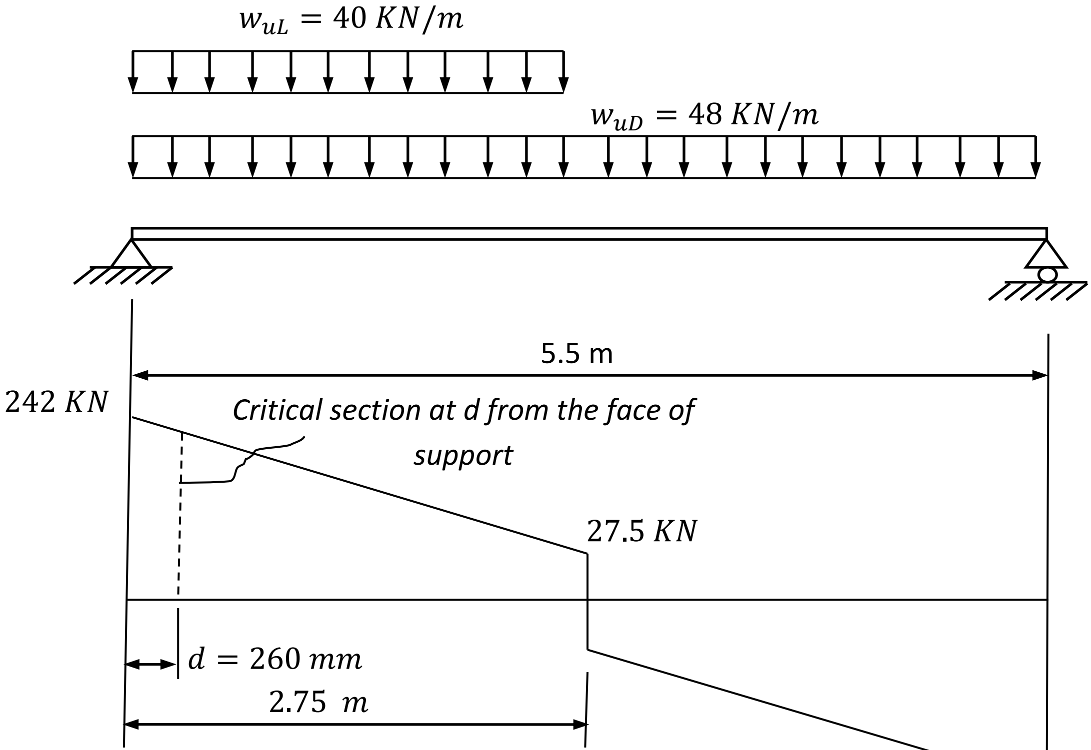



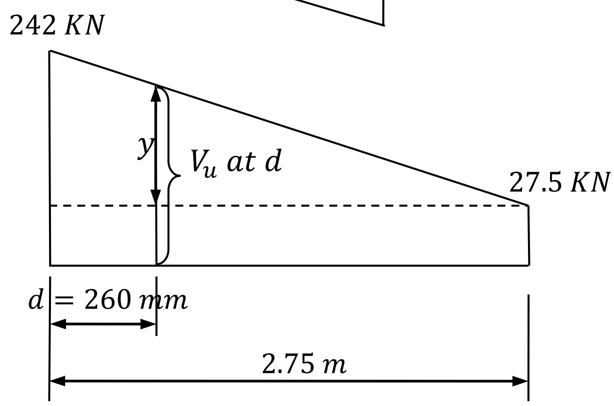
**Solution:**

$w_{uD} = 1.2 \cdot 40 = 48 \quad KN/m, \qquad\qquad w_{uL} = 1.6 \cdot 25 = 40 \ KN/m$

$$V_u \ at \ face \ of \ support = \frac{wl}{2} = \frac{(48+40)\cdot 5.5}{2} = 242 \ KN,$$

$$V_u \ at \ midspan = \frac{w_{uL} l}{8} = \frac{40 \cdot 5.5}{8} = 27.5 \ KN,$$




Critical section at $d = 260 \ mm$ from the face of support.

$V_u$ at $d = 260 \ mm$.

$$\frac{242 - 27.5}{2.75} = \frac{y}{2.75 - 0.26} \quad \rightarrow y = 194.22 \ KN$$

$$V_u = y + 27.5 = 194.22 + 27.5 = 221.72 \ KN$$

$$V_c = \frac{1}{6}\lambda\sqrt{f_c'} b_w d = \frac{1}{6} \cdot 1 \cdot \sqrt{25} \cdot 1000 \cdot 260 \cdot 10^{-3} = 216.67 \ KN.$$

Check for section dimensions:

$$V_s = \frac{V_u}{\phi} - V_c = \frac{221.72}{0.75} - 216.67 = 79 \ KN.$$

$$V_{s,max} = \frac{2}{3}\sqrt{f_c'} b_w d = \frac{2}{3}\sqrt{25} \cdot 1000 \cdot 260 \cdot 10^{-3} = 866.67 \ KN$$

$V_s = 79\ KN < V_{s,max} = 866.67\ KN \quad - the\ section\ is\ large\ enough.$

Check for $V_{s,min}$:

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} \qquad but\ not\ less\ than$$

$$A_{v,min} = \ \frac{1}{3}\frac{b_w s}{f_{yt}}, \qquad - control \qquad (\frac{1}{16}\sqrt{f_c'} = \frac{5}{16} < \frac{1}{3})$$

$$V_{s,min} = \frac{1}{16}\sqrt{f_c'}b_w d = \frac{1}{16}\sqrt{25}\cdot 1000\cdot 260\cdot 10^{-3} = 81.25\ KN$$

$$V_{s,min} = \frac{1}{3}b_w d = \frac{1}{3}1000\cdot 260\cdot 10^{-3} = 86.67\ KN \qquad - control$$

$$\phi V_c < \ V_u \le \ \phi\left(V_c + V_{s,min}\right)$$

$$0.75(216.67) = 162.5\ KN < \ V_u = 221.72\ KN < \ 0.75(216.67 + 86.67) = 227.51\ KN$$

Or $V_s = 79\ KN < V_{s,min} = 86.67\ KN$ – Case III

$$\frac{A_{v,min}}{s} = \frac{1}{16}\sqrt{f_c'}\frac{b_w}{f_{yt}} \qquad but\ not\ less\ than \qquad \frac{A_{v,min}}{s} = \ \frac{1}{3}\frac{b_w}{f_{yt}},$$

$$\frac{A_{v,min}}{s} = \frac{1}{16}\sqrt{25}\frac{1000}{412} = 0.7585$$

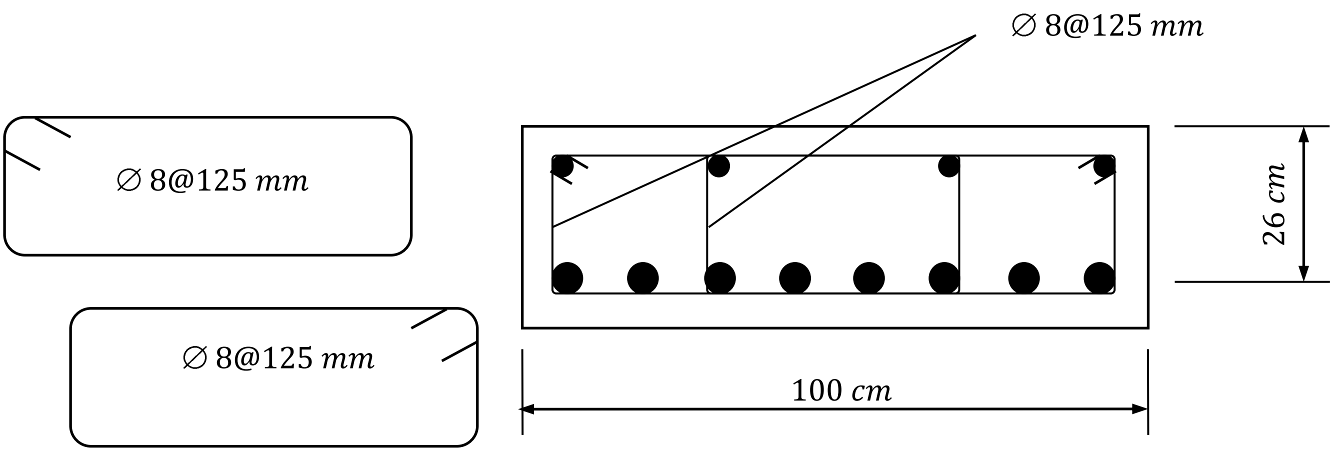
$$\frac{A_{v,min}}{s} = \ \frac{1}{3}\times\frac{1000}{412} = 0.80906 \quad - control$$

Use stirrups 2U-shape (4-leg stirrups) $\varnothing\ 8\ mm$ with $A_v = 4\cdot 50.27 = 201.1\ mm^2$

$$\frac{201.1}{s} = \ = 0.80906 \qquad \Longrightarrow \qquad s = 248.6\ mm$$

$$s_{max} \le 600\ mm, \qquad s_{max} \le \frac{d}{2} = \frac{260}{2} = 130\ mm - control$$

Take 2U-shape (4-leg stirrups) $\varnothing$ 8@125 $mm$ $< s_{max} = 130\ mm$



# CHAPTER 6 COLUMNS: COMBINED AXIAL LOAD AND BENDING

## 6.1 TYPES OF COLUMNS.

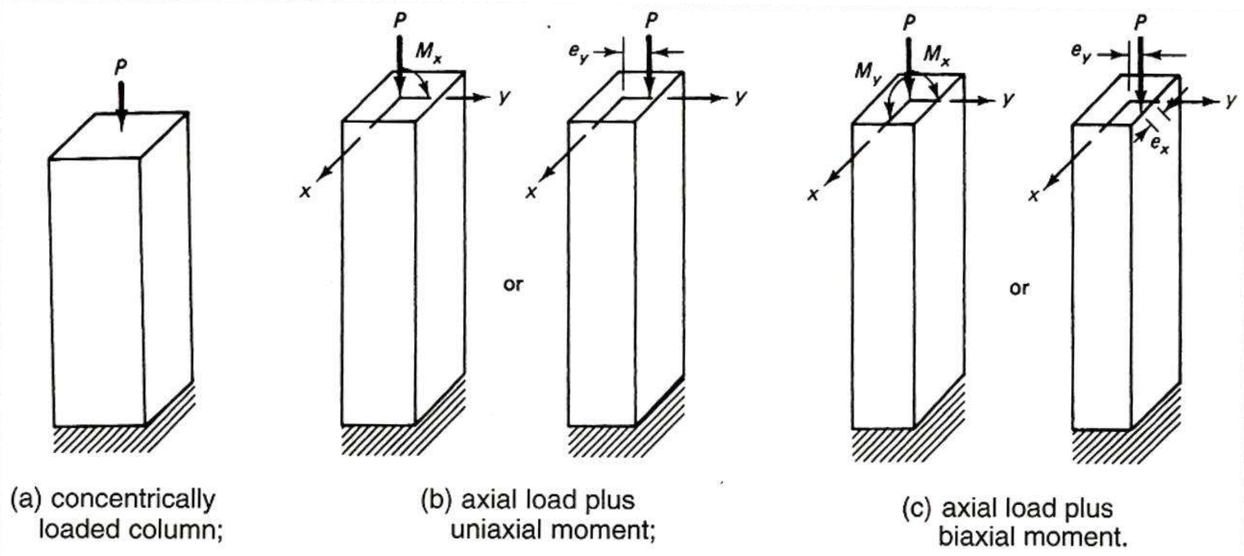
Columns are vertical compression members of a structural frame intended to support the load-carrying beams. They transmit loads from the upper floors to the lower levels and, then to the soil through the foundations. Since columns are compression elements, failure of one column in a critical location can cause the progressive collapse of the adjoining floors and the ultimate total collapse of the entire structure.

Structural column failure is of major significance in terms of economic as well as human loss. Thus extreme care needs to be taken in column design, with a higher reserve strength than in the case of beams and other horizontal structural elements, particularly since compression failure provides little visual warning.

In reinforced concrete buildings, concrete beams, floors, and columns are cast monolithically, causing some moments in the columns due to end restraint. Moreover, perfect vertical alignment of columns in a multistory building is not possible, causing loads to be eccentric relative to the center of columns. The eccentric loads will cause moments in columns. Therefore, a column subjected to pure axial loads does not exist in concrete buildings. However, it can be assumed that axially loaded columns are those with relatively small eccentricity, $e$, of about $0.1h$ or less, where $h$ is the total depth of the column and $e$ is the eccentric distance from the center of the column. Because concrete has a high compressive strength and is an inexpensive material, it can be used in the design of compression members economically.

Columns may be classified based on the following different categories:

1. Based on loading, columns may be classified as follows:
    a. Axially loaded columns, where loads are assumed acting at the center of the column section.



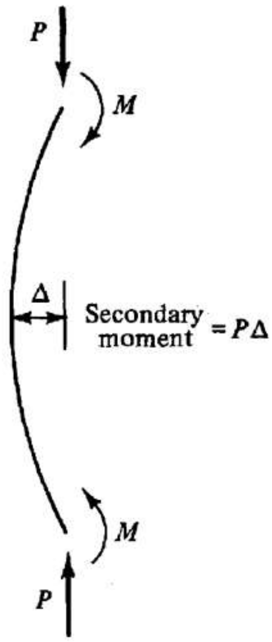

Types of columns based on the position of the load on the cross section.

b. Eccentrically loaded columns, where loads are acting at a distance e from the center of the column section. The distance $e$ could be along the $x-$ or $y-$ axis, causing moments either about the $x-$ or $y-$axis.

c. Biaxially loaded columns, where the load is applied at any point on the column section, causing moments about both the $x-$ and $y-$ axes simultaneously.

2. Based on length, columns may be classified as follows:
    a. Short columns, where the column's failure is due to the crushing of concrete or the yielding of the steel bars under the full load capacity of the column.
    b. Long columns, where buckling effect and slenderness ratio must be taken into consideration in the design, thus reducing the load capacity of the column relative to that of a short column.

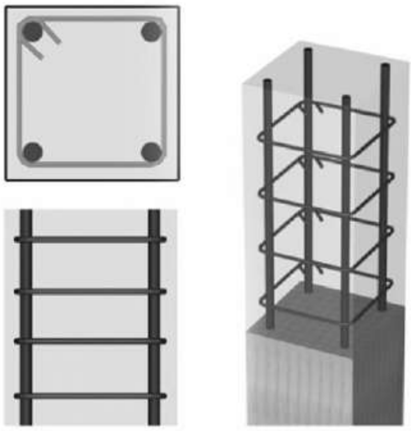
A column that has large secondary moments is said to be a slender column, and it is necessary to size its cross section for the sum of both the primary and secondary moments. The ACI's intent is to permit columns to be designed as short columns if the secondary or $P\Delta$ effect does not reduce their strength by more than $5\%$.



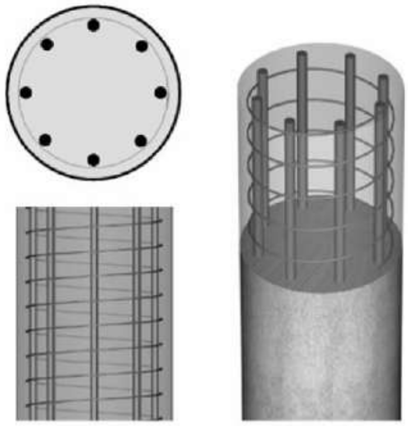

Therefore, the transition from the short column (material failure) to the long column (failure due to buckling) is defined by using the ratio of the effective length $kl_u$ to the radius of gyration $r$. The height, $l_u$, is the unsupported length of the column, and $k$ is a factor that depends on end conditions of the column and whether it is braced or unbraced. For example, in the case of unbraced columns, if $\frac{kl_u}{r}$ is less than or equal to 22, such a column is classified as a short column, in accordance with the ACI load criteria. Otherwise, it is defined as a long or a slender column. The ratio $\frac{kl_u}{r}$ is called the **slenderness ratio**.

3. Based on the shape of the cross-section, column sections may be square, rectangular, round, L-shaped, octagonal, or any desired shape with an adequate side width or dimensions.

4. Based on column ties, columns may be classified as follows:
    a. Tied columns containing steel ties to confine the main longitudinal bars in the columns. Ties are normally spaced uniformly along the height of the column.
    b. Spiral columns containing spirals (spring-type reinforcement) to hold the main longitudinal reinforcement and to help increase the column ductility before failure. In general, ties and spirals prevent the slender, highly stressed longitudinal bars from buckling and bursting the concrete cover.

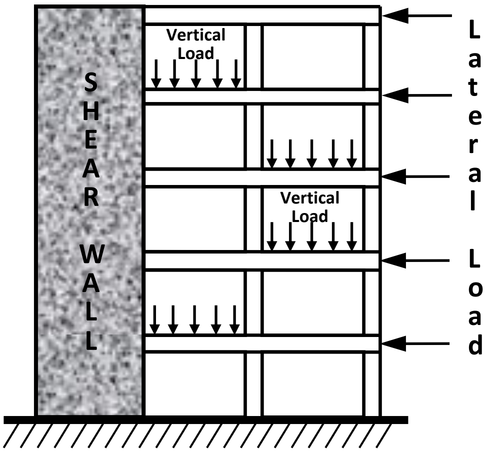


(a) Rectangular tied Column

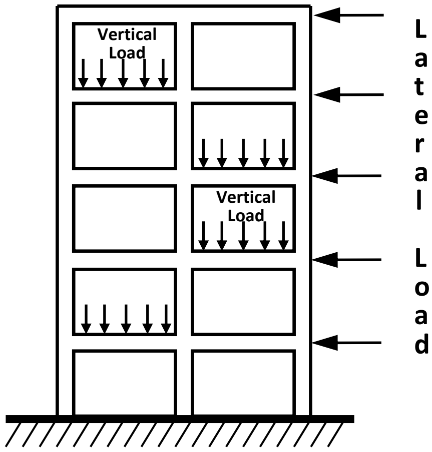


(b) Round spiral Column

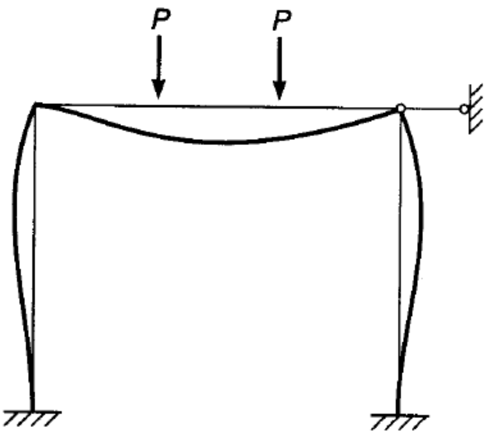
5. Based on frame bracing, columns may be part of a frame that is braced against sidesway or unbraced against sidesway. Bracing may be achieved by using shear walls or bracings in the building frame. In braced frames, columns resist mainly gravity loads, and shear walls resist lateral loads and wind loads. In unbraced frames, columns resist both gravity and lateral loads, which reduce the load capacity of the columns.



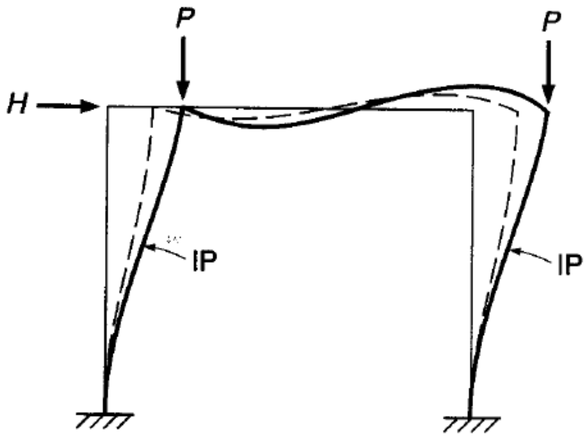

Braced (Nonsway) Frame




Unbraced (sway) Frame

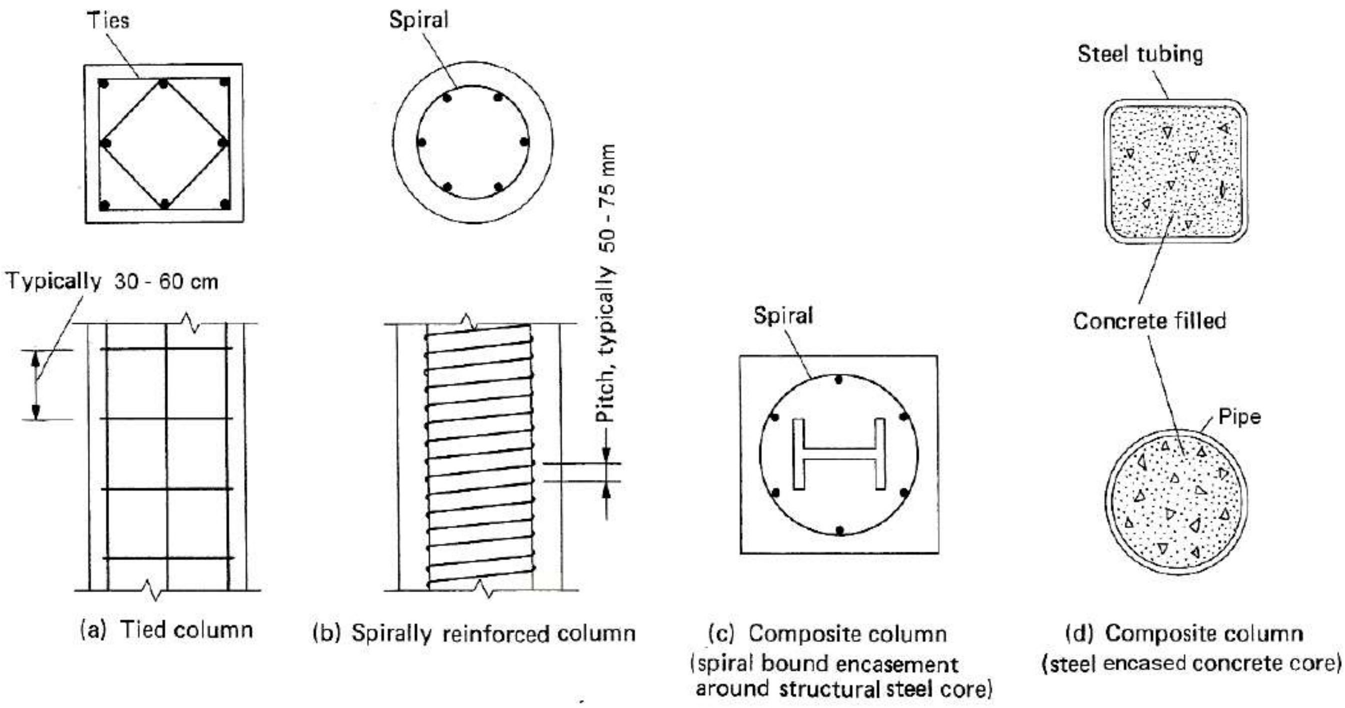



Laterally braced frame




Laterally unbraced frame

6. Based on materials, columns may be reinforced, prestressed, composite (containing rolled steel sections such as I-sections), or a combination of rolled steel sections and reinforcing bars. Concrete columns reinforced with longitudinal reinforcing bars are the most common type used in concrete buildings.




### 6.2 BEHAVIOR OF AXIALLY LOADED COLUMNS.

When an axial load is applied to a reinforced concrete short column, the concrete can be considered to behave elastically up to a low stress of about $\frac{1}{3}(f_c')$. If the load on the column is increased to reach its ultimate strength, the concrete will reach the maximum strength and the steel will reach its yield strength, $f_y$. The nominal load capacity of the column can be written as follows:

$$P_o = 0.85 f_c' A_n + A_{st} f_y$$

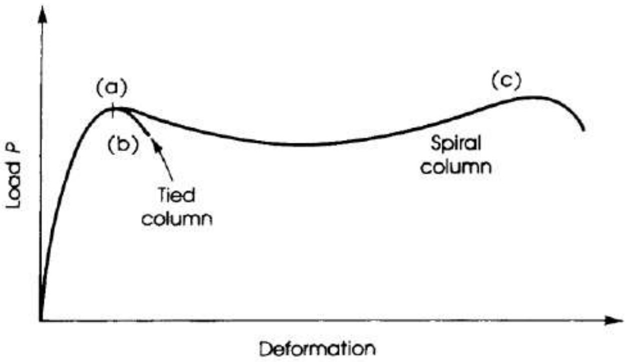
where $A_n$ and $A_{st}$ – the net concrete and total steel compressive areas, respectively.

$$A_n = A_g - A_{st}$$

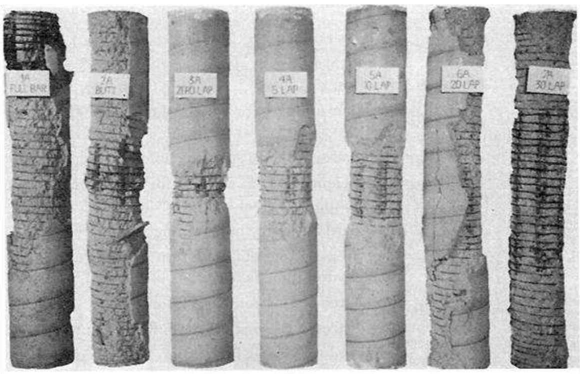
$A_g$ – gross concrete area.

Two different types of failure occur in columns, depending on whether ties or spirals are used. For a tied column, the concrete fails by crushing and shearing outward, the longitudinal steel bars fail by buckling outward between ties, and the column failure occurs suddenly, much like the failure of a concrete cylinder.

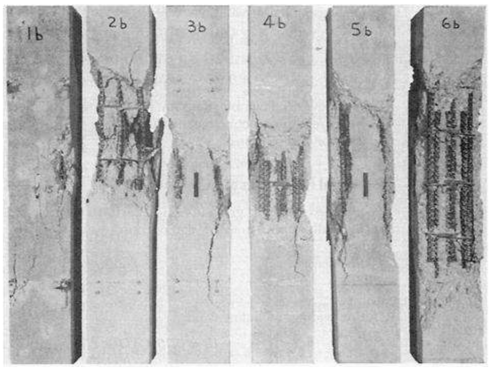
A spiral column undergoes a marked yielding, followed by considerable deformation before complete failure. The concrete in the outer shell fails and spalls off. The concrete inside the spiral is confined and provides little strength before the initiation of column failure. A hoop tension develops in the spiral, and for a closely spaced spiral, the steel may yield. A sudden failure is not expected. The Figure shows typical load deformation curves for tied and spiral columns. Up to point a, both columns behave similarly. At point a, the longitudinal steel bars of the column yield, and the spiral column shell spalls off. After the factored load is reached, a tied column fails suddenly (curve b), whereas a spiral column deforms appreciably before failure (curve c).



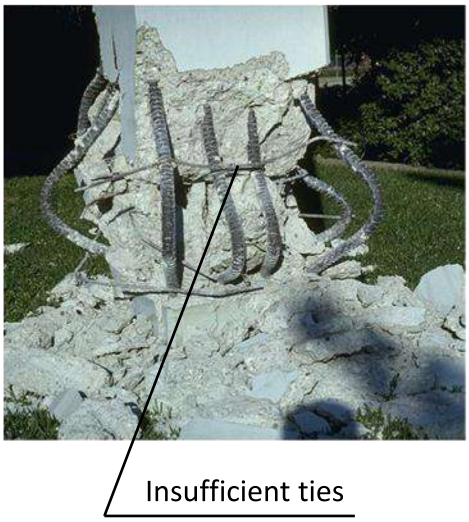

Behavior of tied and spiral columns.

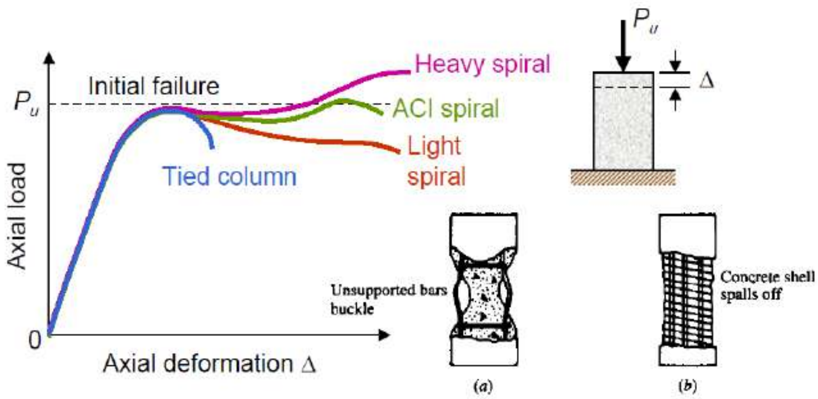


Spirally reinforced column behavior.



Tied column behavior.




Column Failure by Axial Load

### 6.3 ACI CODE LIMITATIONS

The ACI Code presents the following limitations for the design of compression members:

1. For axially as well as eccentrically loaded columns, the ACI Code sets the strength-reduction factors at $\phi = 0.65$ for tied columns and $\phi = 0.75$ for spirally reinforced columns. The difference of 0.1 between the two values shows the additional ductility of spirally reinforced columns. The strength-reduction factor for columns is much lower than those for

flexure ($\phi = 0.9$) and shear ($\phi = 0.75$). This is because in axially loaded columns, the strength depends mainly on the concrete compression strength, whereas the strength of members in bending is less affected by the variation of concrete strength, especially in the case of an under-reinforced section. Furthermore, the concrete in columns is subjected to more segregation than in the case of beams. Columns are cast vertically in long, narrow forms, but the concrete in beams is cast in shallow, horizontal forms. Also, the failure of a column in a structure is more critical than that of a floor beam.

2. The minimum longitudinal steel percentage is 1%, and the maximum percentage is 8% of the gross area of the section (ACI Code, Section 10.9.1). Minimum reinforcement is necessary to provide resistance to bending, which may exist, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses. Practically, it is very difficult to fit more than 8% of steel reinforcement into a column and maintain sufficient space for concrete to flow between bars.

$$0.01 \leq \rho_g = \frac{A_{st}}{A_g} \leq 0.08$$

3. At least four bars are required for tied circular and rectangular members and six bars are needed for circular members enclosed by spirals (ACI Code, Section 10.9.2). For other shapes, one bar should be provided at each corner, and proper lateral reinforcement must be provided. For tied triangular columns, at least three bars are required.

4. The ties shall be arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of s tie having an included angle of not more than $135^{\text{o}}$ and no bar shall be farther 150 mm clear on either side from such a laterally supported bar. The Figures below show the arrangement of longitudinal bars in tied columns and the distribution of ties. The minimum concrete cover in columns is 40 mm.

5. The minimum of volumetric spiral reinforcement ratio which defined as the ratio of the volume of spiral steel to the volume of core concrete, $\rho_s$, according to the ACI Code, Eq. 10.5, and as explained in Section 10.9.3, is limited to

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

where $A_g$ − gross area of section.

$A_{ch}$ − area of core of spirally reinforced column measured to the outside diameter of spiral.

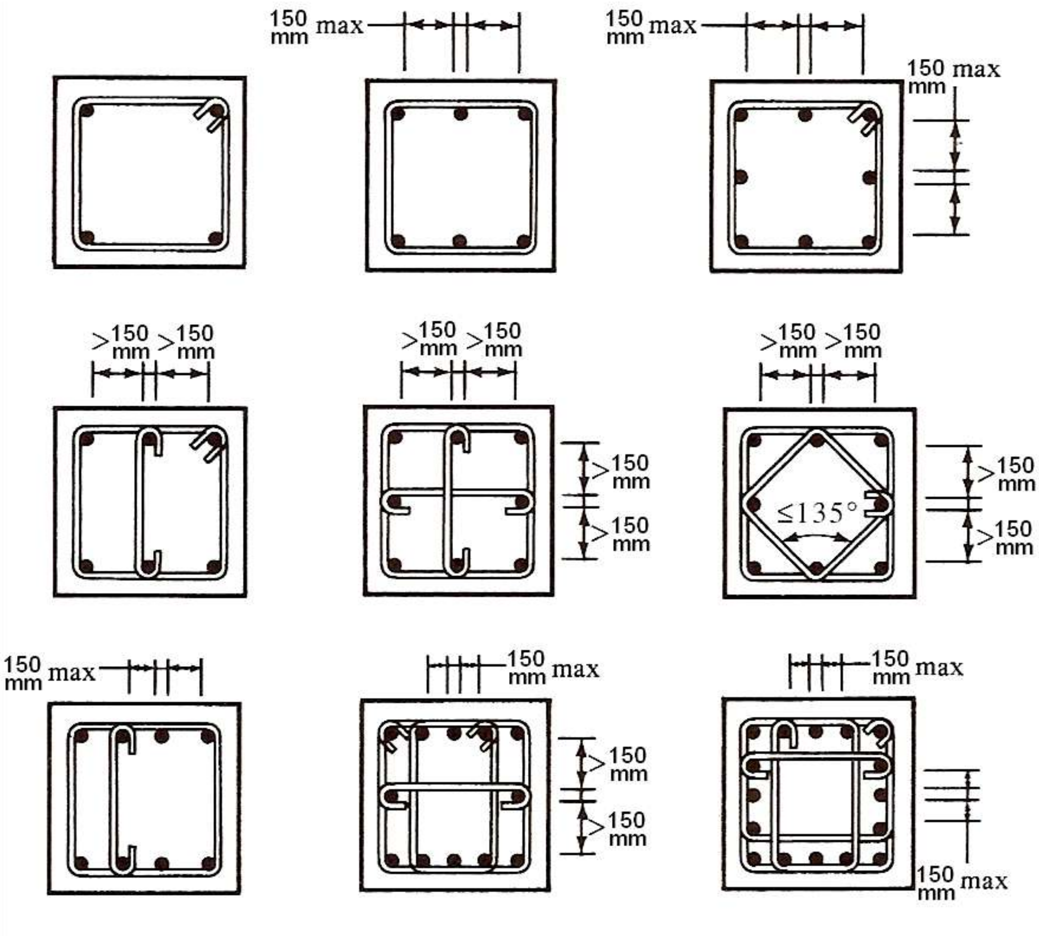
$f_{yt}$ − yield strength of spiral reinforcement.

6. The minimum diameter of spirals is 10 mm, and their clear spacing should not be more than 75 mm nor less than 25 mm, according to the ACI Code, Section 7.10.4. Anchorage of spiral reinforcement shall be provided by $1\frac{1}{2}$ extra turns of spiral bar or wire at each end of a spiral unit.

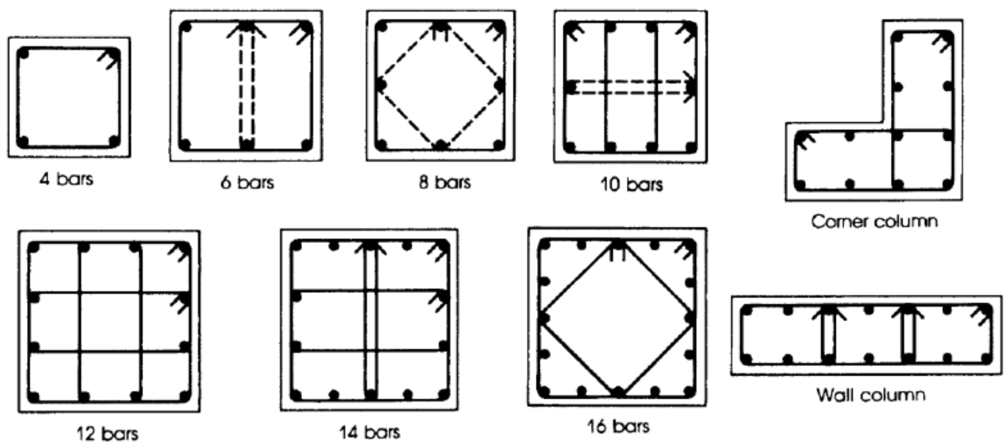
7. Ties for columns must have a minimum diameter of $\varnothing\, 10\, mm$ to enclose longitudinal bars of $\varnothing\, 32\, mm$ or smaller and a minimum diameter of $\varnothing\, 13\, mm$ for larger bar diameters (ACI Code, Section 7.10.5).

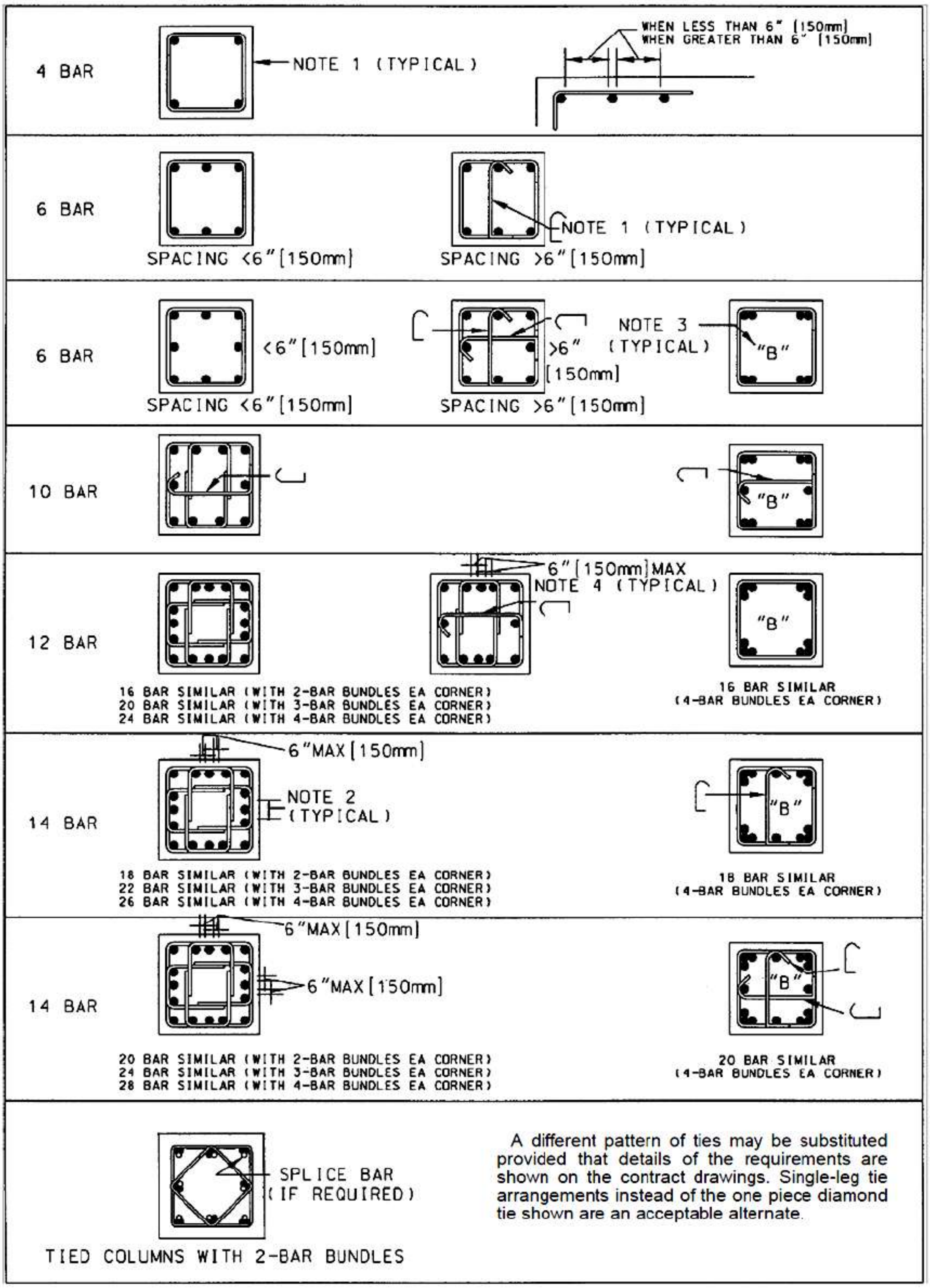
8. Spacing of ties shall not exceed the smallest of:
   - 48 times the tie diameter,
   - 16 times the longitudinal bar diameter, or
   - the least dimension of the column.

The Code does not give restrictions on the size of columns to allow wider utilization of reinforced concrete columns in smaller sizes.




Typical tie arrangements.

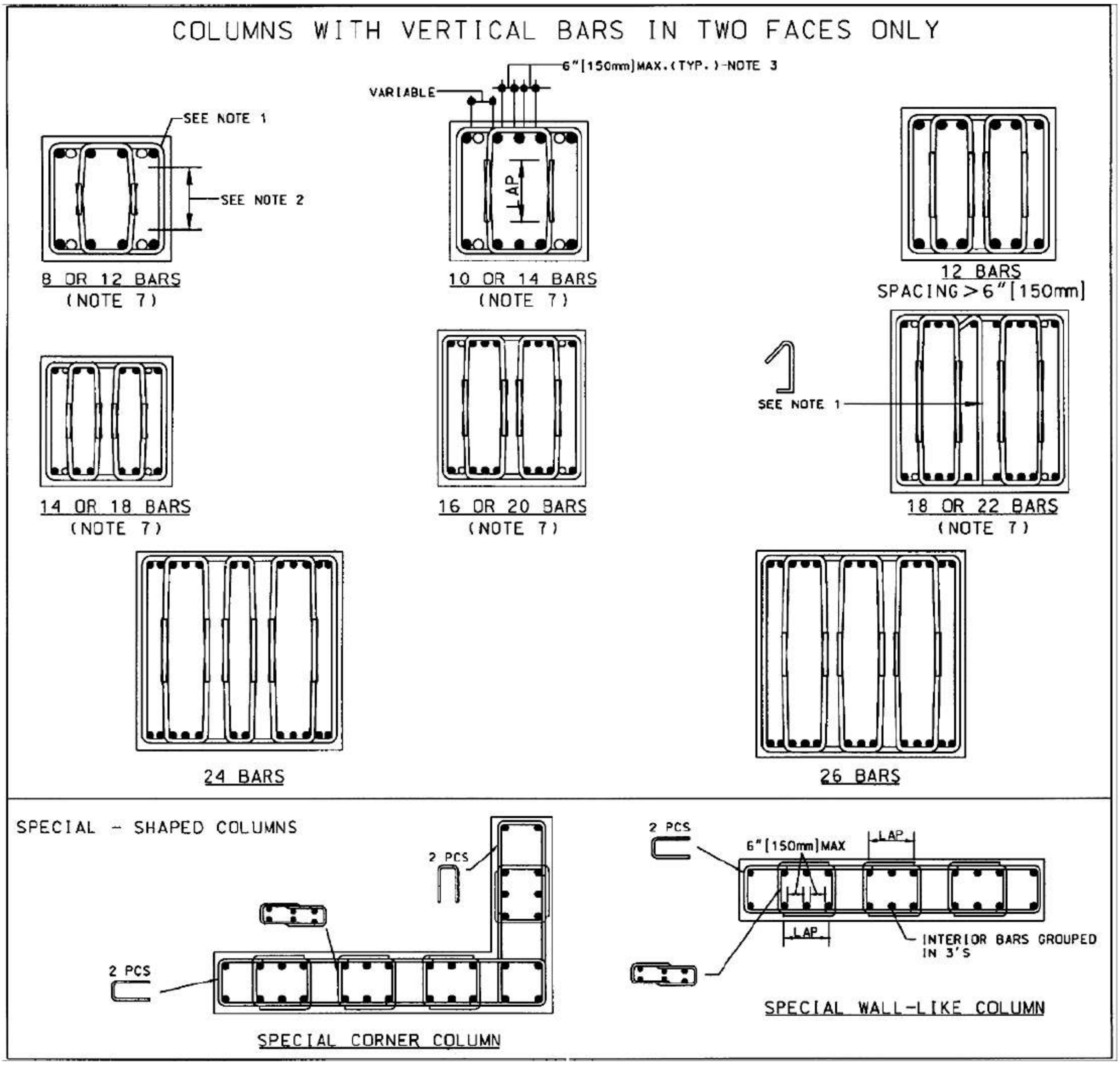





Notes:

1. Alternate position of hooks in placing successive sets of ties.
2. Minimum lap shall be 12 in. (300 mm).
3. *B* indicates bundled bars. Bundles shall not exceed four bars.
4. Elimination of tie for center bar in groups of three limits clear spacing to be 6 in. (150 mm) maximum. Unless otherwise specified, bars should be so grouped.
5. Note to Architect/Engineer: Accepted practice requires that design drawings show all requirements for splicing column verticals, that is, type of splice, lap length if lapped, location in elevation, and layout in cross section.
6. Note to Detailer: Dowel erection details are required for any design employing special large vertical bars, bundled vertical bars, staggered splices, or specially grouped vertical bars as shown.
7. Bars must be securely supported to prevent displacement during concreting.
8. Tie patterns shown may accommodate additional single bars between tied groups provided clear spaces between bars do not exceed 6 in. (150 mm).
9. Minimum cover to ties, 11/2 in. (40 mm) for nonprestressed cast-in-place concrete.
10. Spaces between corner bars and interior groups of three and between interior groups may vary to accommodate average spacing > 6 in. (150 mm).
11. For average spacing < 6 in. (150 mm), one untied bar may be located between each tied group of three and between a tied group and a corner bar.

*Fig. 13—Standard column ties applicable for either preassembled cages or field erection.*




Notes:
1. Alternate position of hooks in placing successive sets of ties.
2. Minimum lap shall be 12 in. (300 mm).
3. Elimination of tie for center bar in groups of three limits clear spacing to be 6 in. (150 mm) maximum. Unless otherwise specified, bars should be so grouped.
4. Note to Architect/Engineer: Accepted practice requires that design drawings show all requirements for splicing column verticals, that is, type of splice, lap length if lapped, location in elevation, and layout in cross section.
5. Note to Detailer: Dowel erection details are required for any design employing special large vertical bars, bundled vertical bars, staggered splices, or specially grouped vertical bars as shown.
6. Bars must be securely supported to prevent displacement during concreting.
7. Bars shown as open circles may be accommodated provided clear spaces between bars do not exceed 6 in. (150 mm).
8. Tie patterns shown may accommodate additional single bars between tied groups provided clear spaces between bars do not exceed 6 in. (150 mm).
9. Minimum cover to ties, 1 1/2 in. (40 mm) for nonprestressed cast-in-place concrete.
10. Spaces between corner bars and interior groups of three and between interior groups may vary to accommodate average spacing > 6 in. (150 mm).
11. For average spacing < 6 in. (150 mm), one untied bar may be located between each tied group of three and between a tied group and a corner bar.

*Fig. 14—Standard column ties applicable for either preassembled cages or field erection, special-shaped columns, and columns with bars in two faces only.*

## 6.4 DESIGN OF TIED AND SPIRAL SHORT CONCENTRICALLY LOADED COLUMNS.

It is highly improbable to attain zero eccentricity in actual structures. Eccentricities could easily develop because of factors such as slight inaccuracies in the layout of columns and unsymmetric loading due to the difference in thickness of the slabs in adjacent spans or imperfections in the alignment. For many years the Code specified that such columns had to be designed for certain minimum moments even though no calculated moments were present. This was accomplished by requiring designers to assume certain minimum eccentricities, $e$, for their column loads. These minimum values were $25\,mm$, or $0.05h$, whichever was larger, for spiral columns and $25\ mm$, or $0.10h$ for tied columns. (The term $h$ represents the outside diameter of round columns or the total depth of square or rectangular columns.) A moment equal to the axial load times the minimum eccentricity was used for design $M_u = P_u \cdot e$.

To reduce the calculations necessary for analysis and design for minimum eccentricity, the ACI Code specifics a reduction of 20% in the axial load for tied columns and a 15% reduction for spiral columns. Using these factors, the maximum nominal axial load capacity or columns cannot be taken greater than:

$$P_{n,max} = 0.8\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right] -$$ for tied reinforced columns, and

$$P_{n,max} = 0.85\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right] -$$ for spirally reinforced columns.

**Spiral Reinforcement.** Spiral reinforcement in compression members prevents a sudden crushing of concrete and buckling of longitudinal steel bars. It has the advantage of producing a tough column that undergoes gradual and ductile failure. The minimum spiral ratio required by the ACI Code is meant to provide an additional compressive capacity to compensate for the spalling of the column shell.

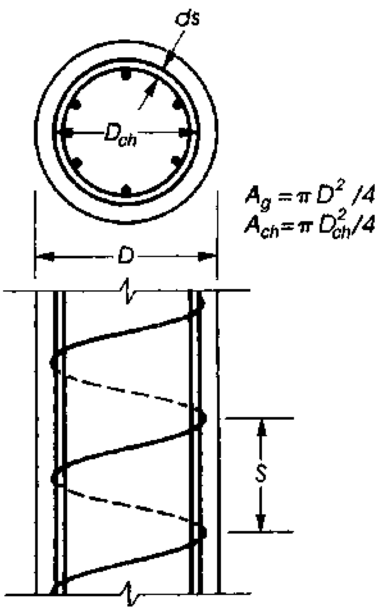
Volumetric spiral reinforcement ratio, $\rho_s$, shall be not less than the value given by:

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}}$$

where $A_g -$ gross area of section.

$A_{ch} -$ area of core of spirally reinforced column measured to the outside diameter of spiral.

$f_{yt} -$ yield strength of spiral reinforcement.

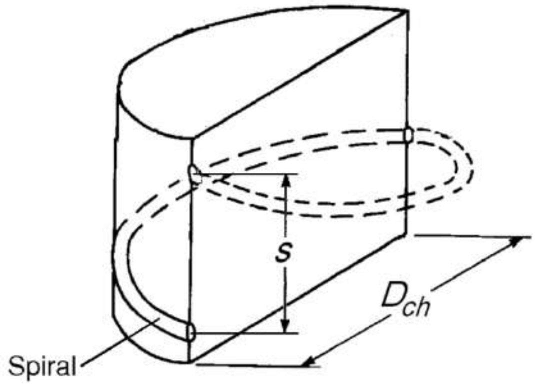



Once the required percentage of spiral steel is determined, the spiral may be selected with the expression to follow, in which $\rho_s$ is written in terms of the volume of the steel in one loop:

$$\rho_s = \frac{volume\ of\ spiral\ in\ one\ loop}{volume\ of\ concrete\ core\ for\ a\ pitch\ "s"} = \frac{V_{spiral}}{V_{core}}$$

$$\rho_s = \frac{V_{spiral}}{V_{core}} = \frac{a_s \pi (D_{ch} - d_s)}{\left(\frac{\pi D_{ch}^2}{4}\right) s} = \frac{4 a_s (D_{ch} - d_s)}{s D_{ch}^2}$$

In this expression, $D_{ch}$ is the diameter of the core out to out of the spiral, $a_s$ is the cross-sectional area of the spiral bar, and $d_s$ is the diameter of the spiral bar. Here reference is made to next Figure. The designer can assume a diameter for the spiral bar and solve for the pitch required. If the results do not seem reasonable, he or she can try another diameter. The pitch used must be within the limitations listed before (ACI requirements).




**Example:**
Design an axially loaded short square tied column for $P_u = 2600\ KN$.
Given: $f_c' = 28\ MPa$, $f_y = 350\ MPa$.

**Solution:**

- Assume $0.01 \le \rho_g = \frac{A_{st}}{A_g} \le 0.08$, $\rho_g = 0.02$ $\Longrightarrow$ $A_{st} = \rho_g A_g = 0.02 A_g$
- Selecting column dimensions:

$$\phi P_{n,max} = P_u = \phi\ 0.8\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right], \quad \phi = 0.65 - for\ tied\ column$$

$$2600 \times 10^3 = 0.65 \cdot 0.8\left[0.85 \cdot 28\left(A_g - 0.02A_g\right) + 0.02A_g \cdot 350\right]$$

$$A_g = \frac{2600 \times 10^3}{15.768} = 164885.9\ mm^2$$

$$A_g = a^2 \quad \Longrightarrow \quad a = \sqrt{A_g} = \sqrt{164885.9} = 406\ mm.$$

Try $a = 400\ mm$, $A_g = a^2 = 400^2 = 160000\ mm^2$.

- Selecting longitudinal bars:

$$2600 \times 10^3 = 0.65 \cdot 0.8[0.85 \cdot 28(160000 - A_{st}) + A_{st} \cdot 350]$$

$$A_{st} = \left[\frac{2600 \times 10^3}{0.65 \cdot 0.8} - 3808000\right]\frac{1}{326.2} = 3654.2\ mm^2.$$

Use $6 \varnothing 28$ with $A_{st} = 36.945\ cm^2 > A_{st,req} = 36.542\ cm^2$.

$$\rho_g = \frac{A_{st}}{A_g} = \frac{36.945}{160000} = 0.023$$

- Design of Ties:

Use ties $\varnothing 10$ with spacing of ties shall not exceed the smallest of:

1. 48 times the tie diameter, $48 d_s = 48 \cdot 10 = 480\ mm$,
2. 16 times the longitudinal bar diameter, $16 d_b = 16 \cdot 28 = 448\ mm$,
3. the least dimension of the column $= 400\ mm$. $-control$

Use ties $\varnothing\, 10\ @\ 400\ mm$.

- Check for code requirements:

1. Clear spacing between longitudinal bars:

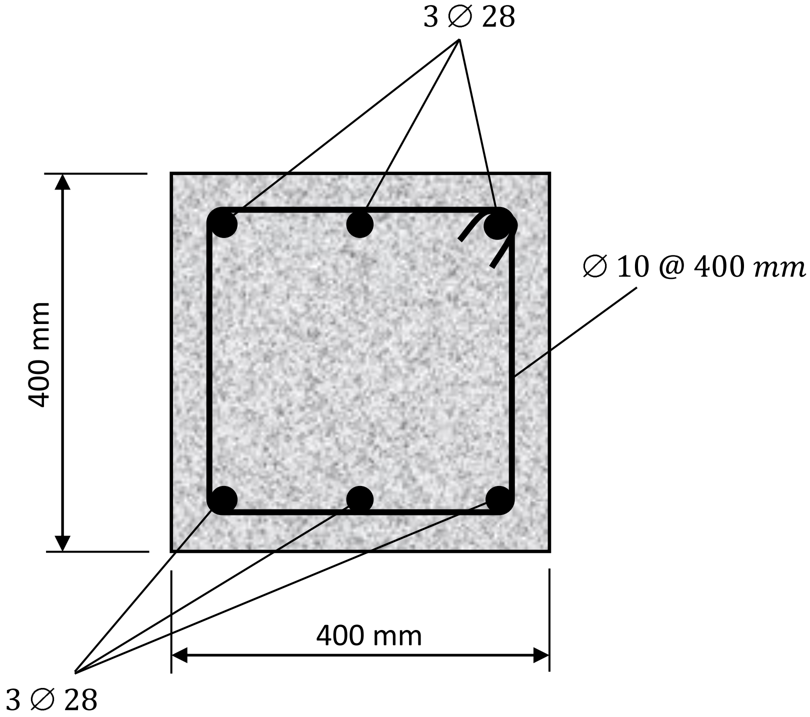
$$Clear\ space = \frac{400 - 40 \cdot 2 - 10 \cdot 2 - 28 \cdot 3}{2} = 108\ mm > 40\ mm, \qquad and$$

$$> 1.5d_b = 1.5 \cdot 28 = 42\ mm - OK.$$

2. Gross reinforcement ratio:

$$0.01 < \rho_g = 0.023 < 0.08 \qquad - OK$$

3. Number of bars: $6 > 4 - for\ square\ section$ $\qquad - OK$
4. Minimum tie diameter: $\varnothing\, 10\ for\ \varnothing\, 28\ bars$ $\qquad - OK$
5. Spacing of ties: $s = 400\ mm$ $\qquad - OK$
6. Arrangement of ties: $108 < 150\ mm.$ $\qquad - OK$




**Example:**

Design an axially loaded short round spiral column to support an axial dead load $DL = 800\ KN$ and an axial live load $LL = 1610\ KN$.
Given: $f_c' = 30\ MPa$, $f_y = 400\ MPa$, and $f_{yt} = 400\ MPa$.
Assume $\rho_g = 0.02$

**Solution:**

- $P_u = 1.2DL + 1.6LL = 1.2 \cdot 800 + 1.6 \cdot 1610 = 3536\ KN$

- $A_{st} = 0.02A_g$
- Selecting column dimensions:

$$\phi P_{n,max} = P_u = \phi\ 0.85\left[0.85f_c'\left(A_g - A_{st}\right) + A_{st}f_y\right], \qquad \phi = 0.75 - for\ spiral\ column$$

$$3536 \times 10^3 = 0.75 \cdot 0.85\left[0.85 \cdot 30\left(A_g - 0.02A_g\right) + 0.02A_g \cdot 400\right]$$

$$A_g = \frac{5546666.7}{33} = 168081\ mm^2$$

$$A_g = \frac{\pi D^2}{4} \quad \Longrightarrow \quad D = \sqrt{\frac{4A_g}{\pi}} = \sqrt{\frac{4 \cdot 168081}{\pi}} = 462.6\ mm.$$

Try $D = 450\ mm,$ $\qquad A_g = \frac{\pi D^2}{4} = \frac{\pi \cdot 450^2}{4} = 159043.13\ mm^2$

- Selecting longitudinal bars:

$$3536 \times 10^3 = 0.75 \cdot 0.85[0.85 \cdot 30(159043.13 - A_{st}) + A_{st} \cdot 400]$$

$$A_{st} = \left[\frac{3536 \times 10^3}{0.75 \cdot 0.85} - 4055599.8\right]\frac{1}{374.5} = 3981.5\ mm^2.$$

Use 11 $\varnothing$ 22 with $A_{st} = 41.815\ cm^2 > A_{st,req} = 39.815\ cm^2$.

$$\rho_g = \frac{A_{st}}{A_g} = \frac{4181.5}{159043.13} = 0.0263$$

- Design of spiral reinforcement:

Use spiral $\varnothing$ 10 with$a_s = 78.54\ mm^2$:

$D_{ch} = D - 2cover = 450 - 2 \cdot 40 = 370\ mm,$

$A_g = \frac{\pi D^2}{4} = \frac{\pi \cdot 450^2}{4} = 159043.13\ mm^2,$

$A_{ch} = \frac{\pi D_{ch}^2}{4} = \frac{\pi \cdot 370^2}{4} = 107521\ mm^2,$

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}} = 0.45\left(\frac{159043.13}{107521} - 1\right)\frac{30}{400} = 0.01617$$

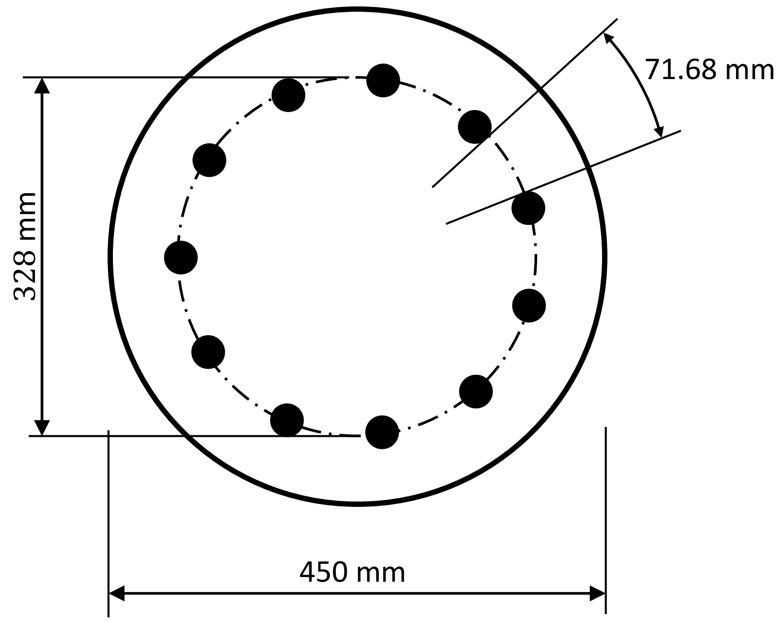
$$\rho_s = \frac{4a_s(D_{ch} - d_s)}{sD_{ch}^2} = \frac{4 \cdot 78.54(370 - 10)}{s \cdot 370^2} = 0.01617 \qquad \Longrightarrow \quad s = 51.09\ mm$$

Take $s = 50\ mm$

- Check for code requirements:

1. Clear spacing between longitudinal bars:

$diameter\ of\ the\ centroidal\ circle\ of\ bars = 450 - 40 \cdot 2 - 10 \cdot 2 - 22 = 328\ mm$




$$Clear\ space = \frac{\pi \cdot 328 - 11 \cdot 22}{11} = 71.68\ mm > 40\ mm, \qquad and$$

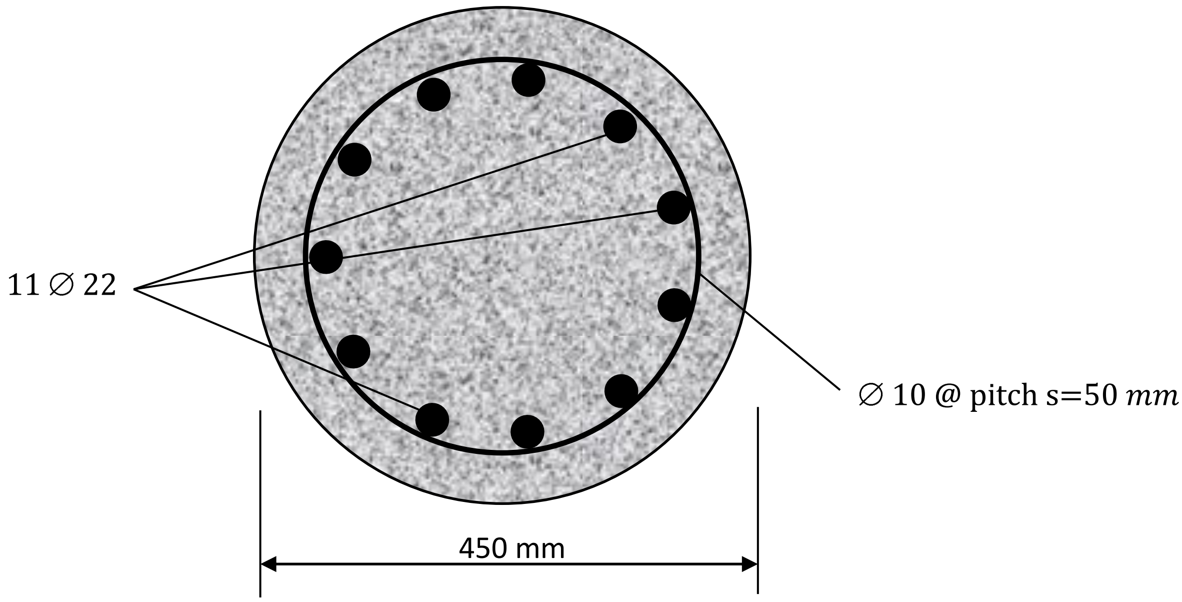
$$> 1.5\ d_b = 1.5 \cdot 22 = 33\ mm - OK.$$

2. Gross reinforcement ratio:

$$0.01 < \rho_g = 0.0263 < 0.08 \qquad - OK$$

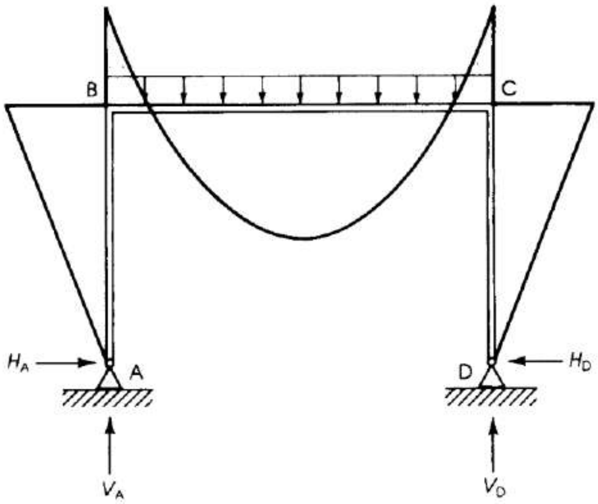
3. Number of bars: $11 > 6$ – for circular members enclosed by spirals $- OK$
4. Minimum spiral diameter: $\varnothing\ 10 \quad - OK$
5. Clear spacing for one loop: $clear\ spacing = s - d_s = 50 - 10 = 40\ mm$

$$25\ mm < 40\ mm < 75\ mm \qquad - OK$$



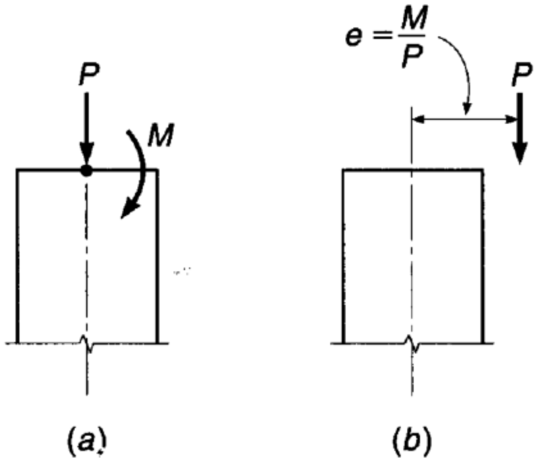
### 6.5 ECCENTRICALLY LOADED COLUMNS: AXIAL LOAD AND BENDING.

Members that are axially, i.e., concentrically, compressed occur rarely, if ever, in buildings and other structures. Components such as columns and arches chiefly carry loads in compression, but simultaneous bending is almost always present. Bending moments are caused by continuity, i.e., by the fact that building columns are parts of monolithic frames in which the support moments of the girders are partly resisted by the abutting columns, by transverse loads such as wind forces, by loads carried eccentrically on column brackets, or in arches when the arch axis does not coincide with the pressure line. Even when design calculations show a member to be loaded purely axially, inevitable imperfections of construction will introduce eccentricities and consequent bending in the member as built. For this reason members that must be designed for simultaneous compression and bending are very frequent in almost all types of concrete structures.



Two-hinged portal frame with bending moment diagram drawn on the tension side.

When a member is subjected to combined axial compression $P$ and moment $M$, such as in the figure (a), it is usually convenient to replace the axial load and moment with an equal load $P$ applied at eccentricity $e = M/P$, as in figure (b). The two loadings are statically equivalent. All columns may then be classified in terms of the equivalent eccentricity. Those having relatively small $e$ are generally characterized by compression over the entire concrete section, and if overloaded, will fail by crushing of the concrete accompanied by yielding of the steel in compression on the more heavily loaded side. Columns with large eccentricity are subject to tension over at least a part of the section, and if overloaded, may fail due to tensile yielding of the steel on the side farthest from the load.




For columns, load stages below the ultimate are generally not important. Cracking of concrete, even for columns with large eccentricity, is usually not a serious problem, and lateral deflections at service load levels are seldom, if ever, a factor. Design of columns is therefore based on the factored load, which must not exceed the design strength, as usual, i.e.,

$$\phi M_n \geq M_u$$
$$\phi P_n \geq P_u$$

The design limitations for columns, according to the ACI Code, Section 10.2, are as follows:

1. Strains in concrete and steel are proportional to the distance from the neutral axis.
2. Equilibrium of forces and strain compatibility must be satisfied.
3. The maximum usable compressive strain in concrete is 0.003.
4. Strength of concrete in tension can be neglected.
5. The stress in the steel is $f_s = E_s \varepsilon \le f_y$.
6. The concrete stress block may be taken as a rectangular shape with concrete stress of $0.85 f_c'$ that extends from the extreme compressive fibers a distance $a = \beta_1 c$, where $c$ is the distance to the neutral axis and where $\beta_1$ as defined in ACI 10.2.7.3 equal:
   $\beta_1 = 0.85 - 0.007(f_c' - 28)$  $0.65 \le \beta_1 \le 0.85$  (see page 23)

The eccentricity, $e$, represents the distance from the plastic centroid of the section to the point of application of the load. The plastic centroid is obtained by determining the location of the resultant force produced by the steel and the concrete, assuming that both are stressed in compression to $f_y$ and $0.85 f_c'$, respectively. For symmetrical sections, the plastic centroid coincides with the centroid of the section. For nonsymmetrical sections, the plastic centroid is determined by taking moments about an arbitrary axis, as explained in example below.

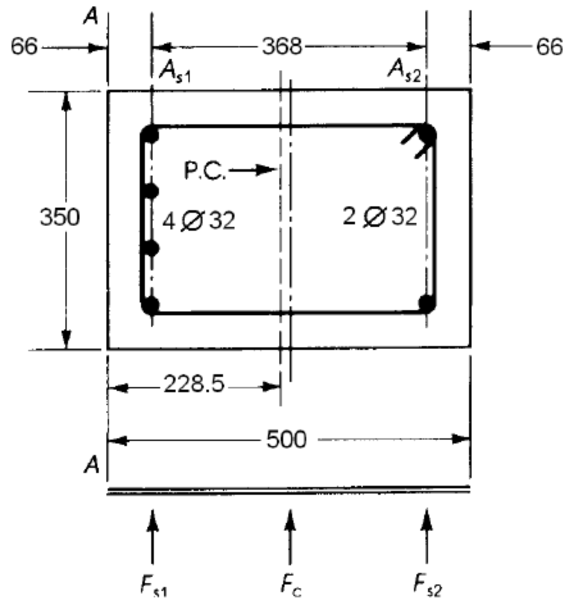
**Example**

Determine the plastic centroid of the section in figure. Take $f_c' = 24\ MPa,\ \ f_y = 420\ MPa.$

**Solution:**

$A_s(4\varnothing 32)$=3217 $mm^2$

$A_s(2\varnothing 32)$=1608.5 $mm^2$




1. It is assumed that the concrete is stressed in compression to $0.85 f_c'$:

$$F_c = 0.85 f_c' A_g = 0.85 \cdot 24 \cdot 500 \cdot 350 \cdot 10^{-3} = $$
$$= 3570\ KN$$

$F_c$ is located at the centroid of the concrete section (at 250 mm from axis A-A).

2. Forces in steel bars:

$$F_{s1} = A_{s1}(f_y - 0.85 f_c') =$$
$$= 3217(420 - 0.85 \cdot 24) \cdot 10^{-3} =$$
$$= 1285.5\ KN$$

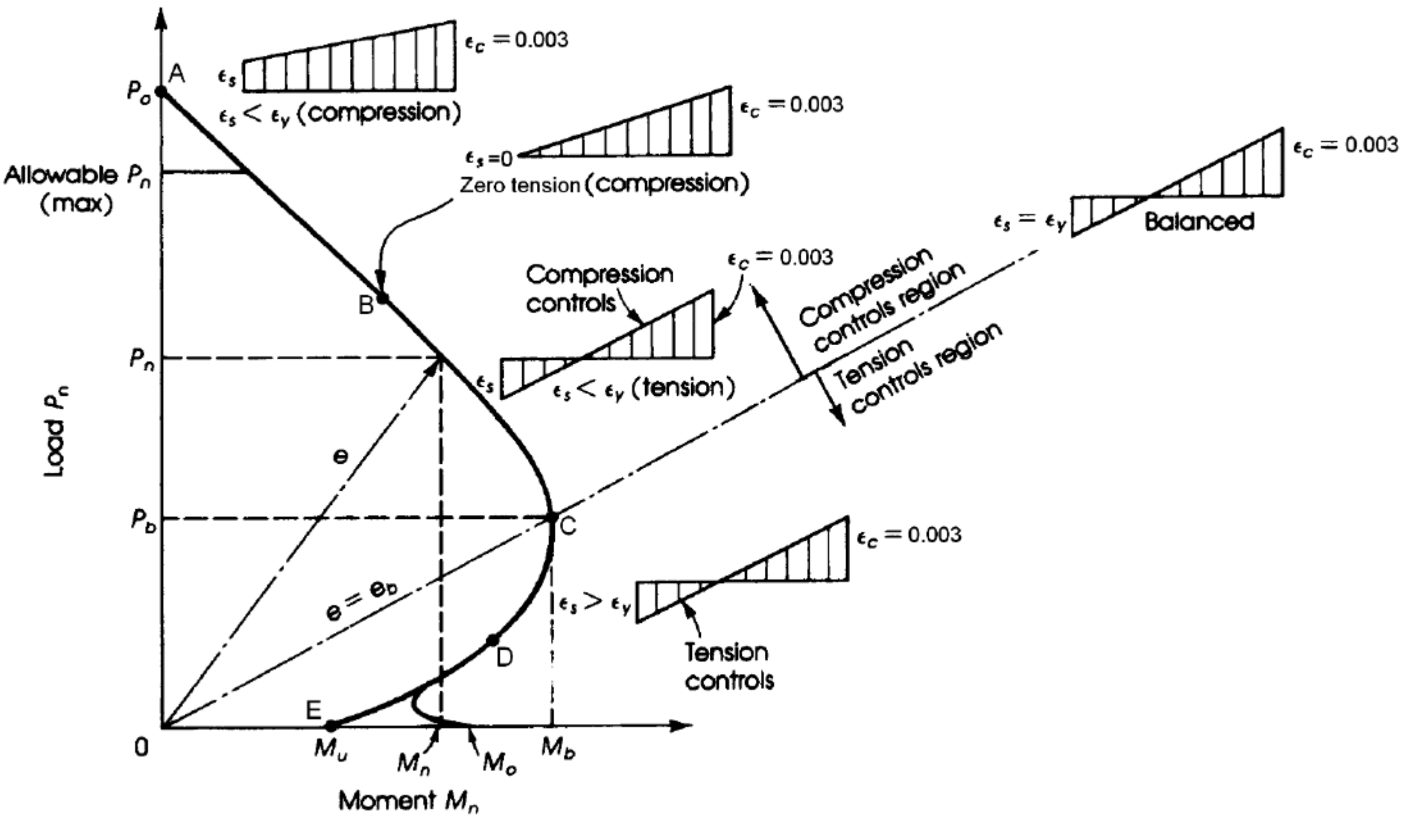
$$F_{s2} = A_{s2}(f_y - 0.85 f_c') =$$
$$= 1608.5(420 - 0.85 \cdot 24) \cdot 10^{-3} = 642.8\ KN$$

3. Take moments about A-A:

$$x = \frac{3570 \cdot 250 + 1285.5 \cdot 66 + 642.8 \cdot 434}{3570 + 1285.5 + 642.8} = 228.5\ mm$$

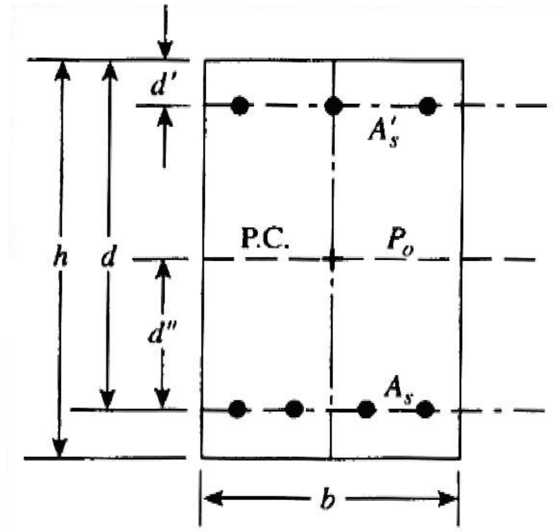
### 6.6 LOAD-MOMENT (STRENGTH) INTERACTION DIAGRAM

When a normal force is applied on a short reinforced concrete column, the following cases may arise, according to the location of the normal force with respect to the plastic centroid.

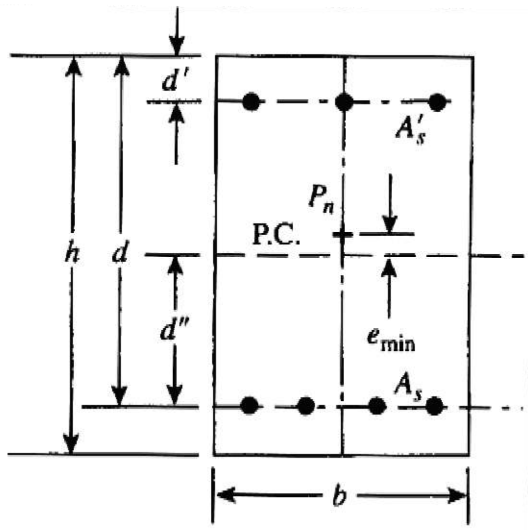



Load-moment strength interaction diagram showing ranges of cases discussed in text.

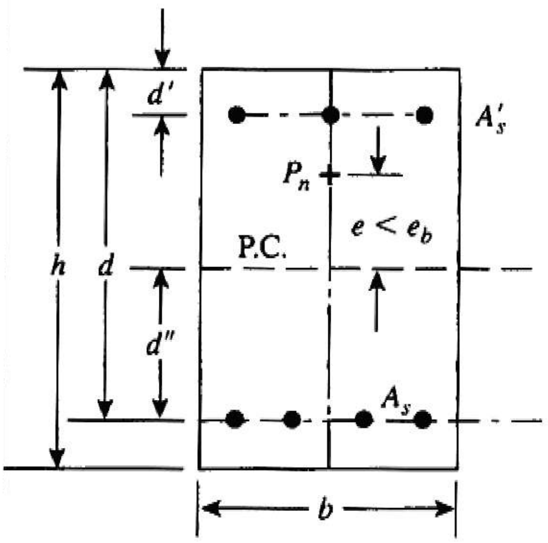
1. **Axial compression $(P_0)$ – Point A**. This is a theoretical case assuming that a large axial load is acting at the plastic centroid; $e = 0$ and $M_n = 0$. Failure of the column occurs by crushing of the concrete and yielding of steel bars. This is represented by $P_0$ on the curve of Fig. a.
2. **Maximum nominal axial load $P_{n,max}$:** This is the case of a normal force acting on the section with minimum eccentricity. According to the ACI Code, $P_{n,max} = 0.80P_0$ for tied columns and $0.85P_0$ for spirally reinforced columns, as explained in before in this chapter. In this case, failure occurs by crushing of the concrete and the yielding of steel bars.
3. **Compression failure:** This is the case of a large axial load acting at a small eccentricity. The range of this case varies from a maximum value of $P_n = P_{n,max}$ to a minimum value of $P_n = P_b$ (balanced load). Failure occurs by crushing of the concrete on the compression side with a strain of 0.003, whereas the stress in the steel bars (on the tension side) is less than the yield strength, $f_y$ $(f_s < f_y)$. In this case $P_n > P_b$ and $e < e_b$.



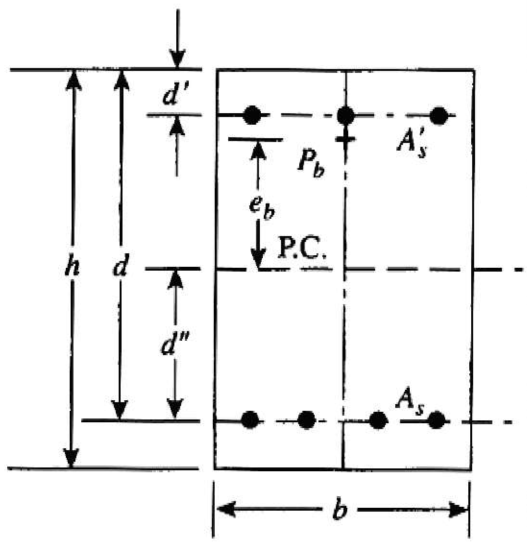

Case 1: Axial load, $P_0$



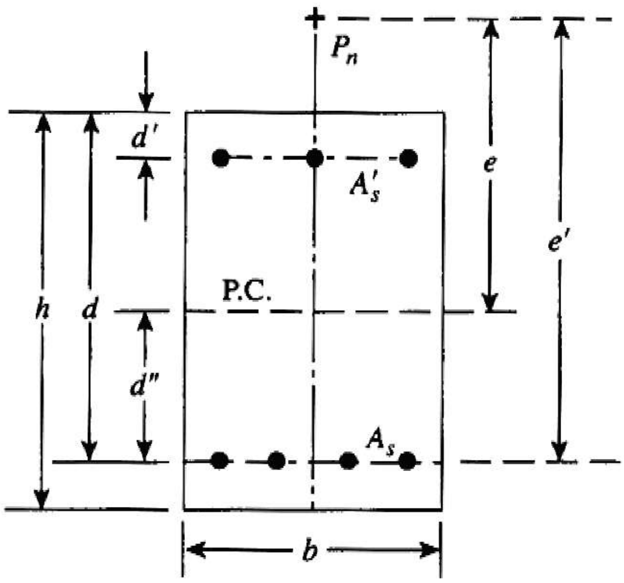

Case 2: $P_n = 0.8\ P_0$



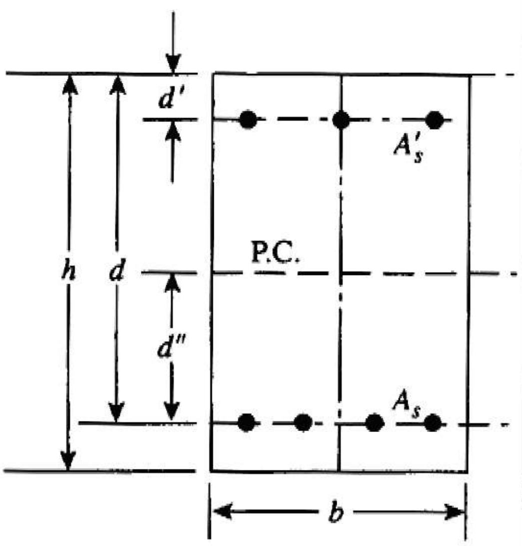

Case 3: Compression controls, $P_n > P_b$




Case 4: Balanced load, $P_b$




Case 5: Tension controls, $P_n < P_b$




Case 6: Pure moment, $P_n = 0$

The strain distribution at **Point B** corresponds to the axial load and moment at the onset of crushing of the concrete just as the strains in the concrete on the opposite face of the column reach zero. Case B represents the onset of cracking of the least compressed side of the column. Because tensile stresses in the concrete are ignored in the strength calculations, failure loads below point B in the interaction diagram represent cases where the section is partially cracked.

**Region A–C** - Compression-Controlled Failures. Columns with axial loads $P_n$ and moments $M_n$ that fall on the upper branch of the interaction diagram between points A and C initially fail due to crushing of the compression face before the extreme tensile layer of reinforcement yields. Hence, they are called compression-controlled columns.

4. **Balanced condition $(P_b)$ – Point C:** A balanced condition is reached when the compression strain in the concrete reaches $0.003$ and the strain in the tensile reinforcement reaches $\varepsilon_y = \frac{f_y}{E_s}$ simultaneously; failure of concrete occurs at the same time as the steel yields. The moment that accompanies this load is called the balanced moment, $M_b$, and the relevant balanced eccentricity is $e_b = \frac{M_b}{P_b}$.
5. **Tension failure:** This is the case of a small axial load with large eccentricity, that is, a large moment. Before failure, tension occurs in a large portion of the section, causing the tension steel bars to yield before actual crushing of the concrete. At failure, the strain in the tension steel is greater than the yield strain, $\varepsilon_y$, whereas the strain in the concrete reaches $0.003$. The range of this case extends from the balanced to the case of pure flexure. When tension controls, $P_n < P_b$ and $e > e_b$.

**Point D -** Tensile-Controlled Limit. Point D corresponds to a strain distribution with $0.003$ compressive strain on the top face and a tensile strain of $0.005$ in the extreme layer of tension steel (the layer closest to the tensile face of the section.) The failure of such a column will be ductile, with steel strains at failure that are about two and a half times the yield strain (for Grade-420 steel). ACI Code Section 10.3.4 calls this the tension-controlled strain limit.

**Region C–D** - Transition Region. Flexural members and columns with loads and moments which would plot between points C and D are called transition failures because the mode of failure is transitioning from a brittle failure at point C to a ductile failure at point D, corresponding respectively to steel strains of $0.002$ and $0.005$ in the extreme layer of tension steel. This is reflected in the transition of the $\phi -$ factor, which equals 0.65 (tied column) or $0.75$ (spiral column) at point C and equals $0.9$ at point D.

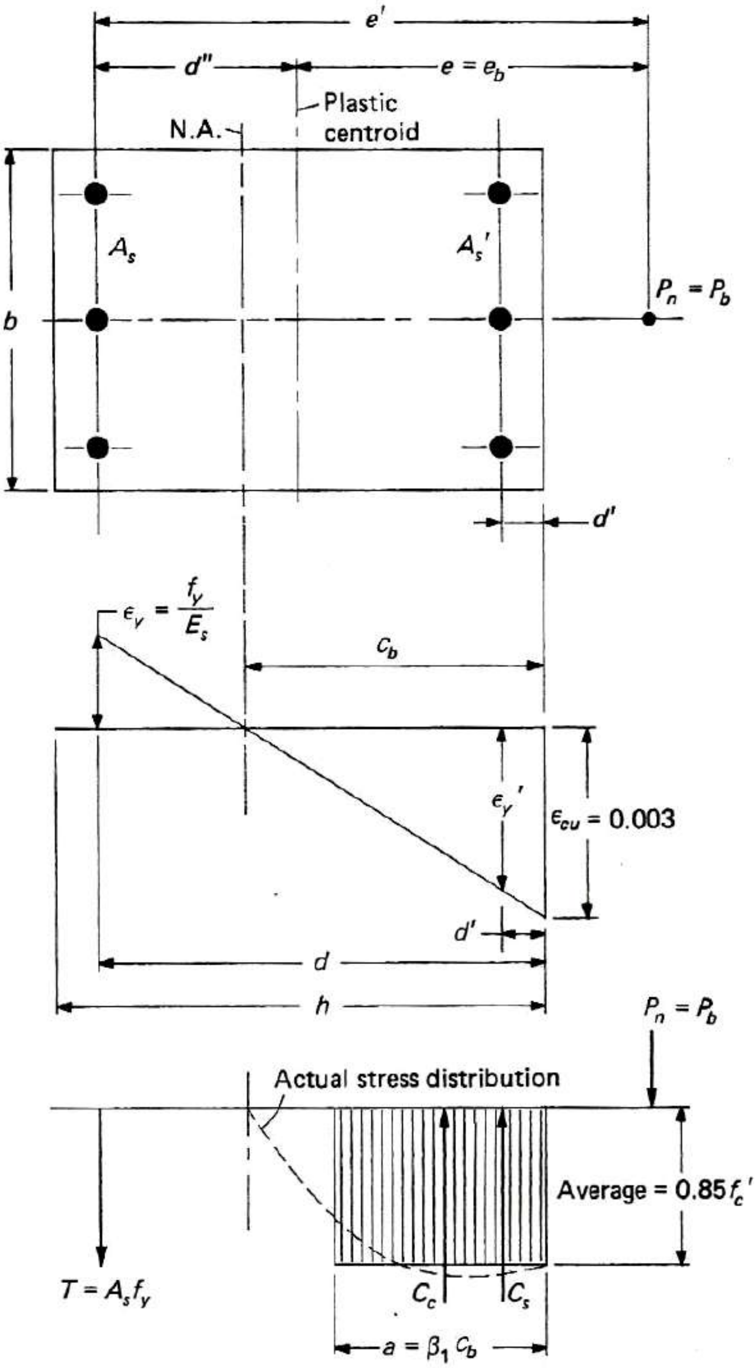
6. **Pure flexure:** The section in this case is subjected to a bending moment, $M_n$, whereas the axial load is $P_n = 0$. Failure occurs as in a beam subjected to bending moment only. The eccentricity is assumed to be at infinity. Note that radial lines from the origin represent constant ratios of $\frac{M_n}{P_n} = e =$ eccentricity of the load $P_n$ from the plastic centroid.

Cases 1 and 2 were discussed in section 6.4 of this chapter, and Case 6 was discussed in detail in Chapter 4. The other cases will be discussed later in this chapter.

### 6.7 BALANCED STRAIN CONDITION – RECTANGULAR SECTIONS.

The balanced strain condition represents the dividing point between the "section compression-controlled" and the "transition zone" of the strength interaction diagram (Point C on the interaction diagram). Defined in the same manner as in Chapter 4, section 4.5, it is the simultaneous occurrence of a strain of 0.003 in the extreme fiber of concrete and the strain $\varepsilon_y = \frac{f_y}{E_s}$ on the tension steel.

It may be noted that in the case of bending moment without axial load, the balanced strain condition is not permitted by ACI – 10.3.5. However, in the case of combined bending and axial load, the balanced strain condition is only one point on an acceptable interaction diagram.




$$\frac{c_b}{d} = \frac{0.003}{0.003 + \frac{f_y}{E_s}}$$

Substituting $E_s = 200\ 000\ MPa$

$$c_b = \frac{0.003}{0.003 + \frac{f_y}{200\ 000}} d$$

or

$$c_b = \left(\frac{600}{600 + f_y}\right) d$$

Force equilibrium requires

$$P_b = C_c + C_s - T \qquad (*)$$

where

$$C_c = 0.85\, f_c' ab = 0.85\, f_c' \beta_1 c_b b,$$

$$T = A_s f_y,$$

and if compression steel yields at balanced strain condition

$$f_s' = 600\left(\frac{c - d'}{c}\right) = f_y$$

$$C_s = A_s'\left(f_y - 0.85\, f_c'\right)$$

Thus the equation (*) becomes

$$P_b = 0.85\, f_c' \beta_1 c_b b + A_s'\left(f_y - 0.85\, f_c'\right) - A_s f_y \qquad (**)$$

The eccentricity $e_b$ is measured from the plastic centroid, which has been defined before in this chapter. For symmetrical sections the plastic centroid is at the middepth of the section. Rotational equilibrium of the forces is satisfied by taking moments about any point such as the plastic centroid,

$$P_b e_b = C_c\left(d - \frac{a}{2} - d''\right) + C_s(d - d' - d'') + Td'' \qquad (***)$$

$$or \quad P_b e_b = M_b = 0.85\, f_c' ab\left(d - \frac{a}{2} - d''\right) + A_s'(f_y - 0.85\, f_c')(d - d' - d'') + A_s f_y d''$$

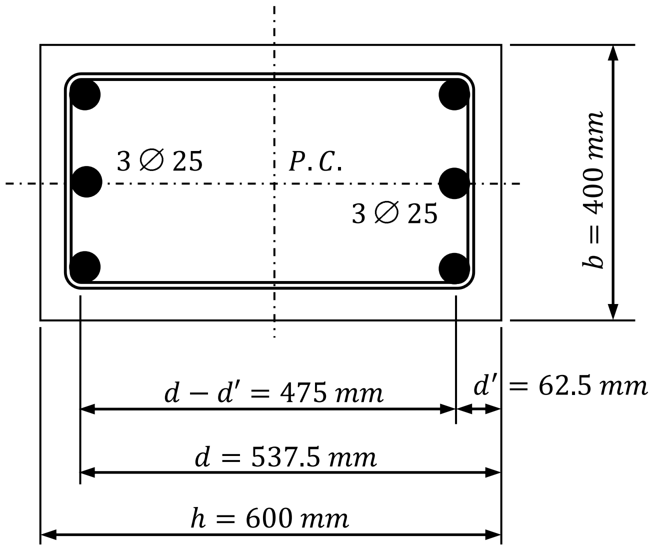
Equations (**) and (***) may be solved simultaneously to obtain $P_b$ and $e_b$.

The balanced eccentricity is

$$e_b = \frac{M_b}{P_b}$$

**Example**

Determine the eccentric balanced compressive strength $P_b$ and the eccentricity $e_b$ for a balanced strain condition on the section below. Take $f_c' = 20\ MPa,\ \ f_y = 380\ MPa.$




<u>**Solution:**</u>

$$c_b = \left(\frac{600}{600 + f_y}\right)d$$

$$c_b = \left(\frac{600}{600 + 380}\right)537.5 = 329.1\ mm$$

$$a_b = \beta_1 c_b$$

$$a_b = 0.85 \cdot 329.1 = 279.74\ mm$$

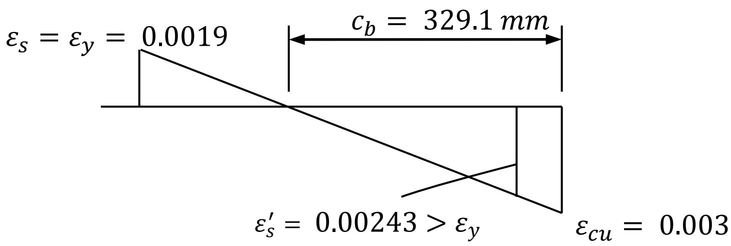
$$C_c = 0.85\, f_c' ab$$

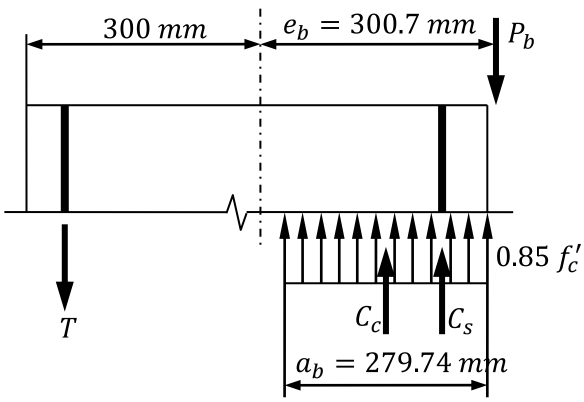
$$C_c = 0.85 \cdot 20 \cdot 279.74 \cdot 400 \cdot 10^{-3} = $$

$$= 1902.23\ KN,$$

$$T = A_s f_y = 1472.62 \cdot 380 \cdot 10^{-3} =$$

$$= 559.6\ KN$$




Check if compression steel yields

$$\varepsilon_s' = 0.003\left(\frac{c - d'}{c}\right) \geq \varepsilon_y = \frac{f_y}{E_s}$$

$$or \qquad f_s' = 600\left(\frac{c - d'}{c}\right) \geq f_y$$

$$\varepsilon_s' = 0.003\left(\frac{329.1 - 62.5}{329.1}\right) = 0.00243 > \varepsilon_y = \frac{f_y}{E_s} = \frac{380}{200000} = 0.0019$$

$$or \qquad f_s' = 600\left(\frac{329.1 - 62.5}{329.1}\right) = 486.1\ MPa > f_y = 380\ MPa$$

Compression steel yields. $f_s' = f_y = 380\ MPa$

$$C_s = A_s'(f_y - 0.85\ f_c') = 1472.62(380 - 0.85 \cdot 20)10^{-3} = 534.56\ KN$$

$$P_b = C_c + C_s - T = 1902.23 + 534.56 - 559.6 = 1877.19\ KN$$

For rotational equilibrium about the plastic centroid, for symmetrical sections, the plastic centroid coincides with the centroid of the section,

$$P_b e_b = C_c\left(d - \frac{a}{2} - d''\right) + C_s(d - d' - d'') + Td''$$

For symmetrical section, it can be written in the form

$$P_b e_b = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T\left(\frac{h}{2} - d'\right)$$

$$1877.19 \cdot e_b = 1902.23\left(\frac{600}{2} - \frac{279.74}{2}\right) + 534.56\left(\frac{600}{2} - 62.5\right) + 559.6\left(\frac{600}{2} - 62.5\right)$$

$$e_b = \frac{564467.1}{1877.19} = 300.7\ mm$$

On the given section:

If $P_n > P_b = 1877.19\ KN$ or $e < e_b = 300.7\ mm$ – the section is Compression-controlled

If $P_n < P_b = 1877.19\ KN$ or $e > e_b = 300.7\ mm$ – the section may be in the “transition zone”.

If $e$ is large enough, the section would be tension controlled.

**Example**

Determine the eccentric balanced compressive strength $P_b$ and the eccentricity $e_b$ for a balanced strain condition on the section below. Take $f_c' = 25\ MPa,\ \ f_y = 345\ MPa$.

**Solution:**

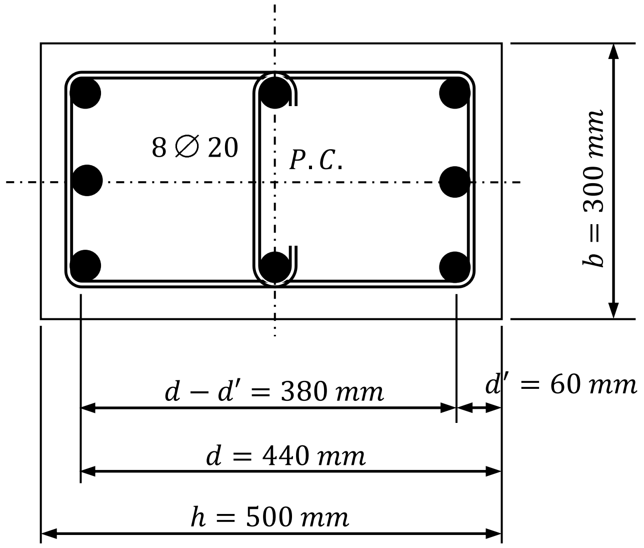
$$c_b = \left(\frac{600}{600 + f_y}\right)d = \left(\frac{600}{600 + 345}\right)440 = 279.4\ mm$$

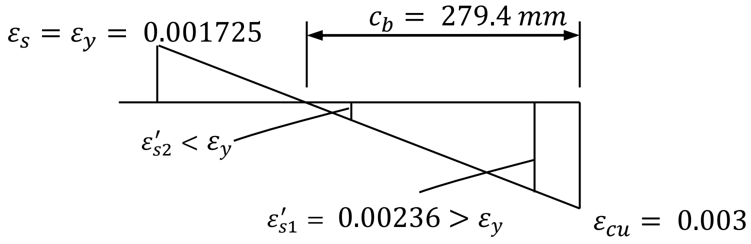
$$a_b = \beta_1 c_b = 0.85 \cdot 279.4 = 237.5\ mm$$

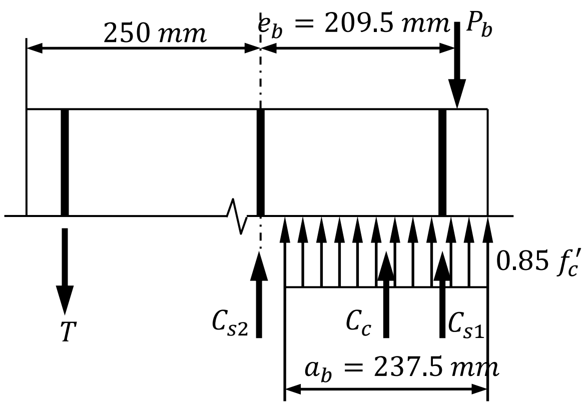
$$C_c = 0.85\ f_c' ab = 0.85 \cdot 25 \cdot 237.5 \cdot 300 \cdot 10^{-3} = 1514.1\ KN,$$

$$T = A_s f_y = 942.47 \cdot 345 \cdot 10^{-3} = 325.15\ KN$$

Check if compression steel $A_{s1}'$ yields,




$$\varepsilon'_{s1} = 0.003\left(\frac{c-d'}{c}\right) \geq \varepsilon_y = \frac{f_y}{E_s} \quad or \qquad f'_{s1} = 600\left(\frac{c-d'}{c}\right) \geq f_y$$

$$\varepsilon'_{s1} = 0.003\left(\frac{279.4-60}{279.4}\right) = 0.00236 > \varepsilon_y = \frac{f_y}{E_s} = \frac{345}{200000} = 0.001725$$

$$or \qquad f'_{s1} = 600\left(\frac{279.4-60}{279.4}\right) = 471.15\ MPa > f_y = 345\ MPa$$

Compression steel yields. $f'_{s1} = f_y = 345\ MPa$

$C_{s1} = A'_{s1}\left(f_y - 0.85\ f'_c\right) = 942.47(345 - 0.85 \cdot 25)10^{-3} = 305.12\ KN$

Compression steel $A'_{s2}$

$$f'_{s2} = 600\left(\frac{c-h/2}{c}\right) = 600\left(\frac{279.4-250}{279.4}\right) = 63.14\ MPa < f_y = 345\ MPa$$

$C_{s2} = A'_{s2}(f'_{s2} - 0.85\, f'_c) = 628.32(63.14 - 0.85 \cdot 25)10^{-3} = 26.32\, KN$

$P_b = C_c + C_{s1} + C_{s2} - T = 1514.1 + 305.12 + 26.32 - 325.15 = 1520.39\, KN$

For rotational equilibrium about the plastic centroid, for symmetrical sections, the plastic centroid coincides with the centroid of the section,

$$P_b e_b = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_{s1}\left(\frac{h}{2} - d'\right) + T\left(\frac{h}{2} - d'\right)$$

$$1520.39 \cdot e_b = 1514.1\left(\frac{500}{2} - \frac{237.5}{2}\right) + 305.12\left(\frac{500}{2} - 60\right) + 325.15\left(\frac{500}{2} - 60\right)$$

$$e_b = \frac{318476.9}{1520.39} = 209.5\, mm$$

$$M_b = P_b e_b = 1520.39 \cdot \frac{209.5}{1000} = 318.5\, KN \cdot m$$

## 6.8 NOMINAL STRENGTH OF A COMPRESSION-CONTROLLED RECTANGULAR SECTION ($e < e_b$).

When the nominal compression strength $P_n$ exceeds the balanced nominal strength $P_b$, or when the eccentricity $e$ is less than the balanced value $e_b$, or when $\varepsilon_t$ at the extreme layer of steel at the face opposite the maximum compression face is less than $\varepsilon_y$, the section is “compression controlled”. The tensile force $T$ will then be based on a tensile strain less than $\varepsilon_y$ and may actually be a compressive force if the eccentricity is small enough.

**Example**

Determine the nominal compressive strength $P_n$ for the section below for an eccentricity $e = 200\, mm$.

Take $f'_c = 20\, MPa,\;\; f_y = 380\, MPa$.

**<u>Solution:</u>**

Determine whether the given eccentricity $e = 200\, mm$ is larger or smaller than $e_b$.

The balanced strain condition was computed in the previous example (page 92) as

$$P_b = 1877.19\, KN$$

$$e_b = 300.7\, mm$$

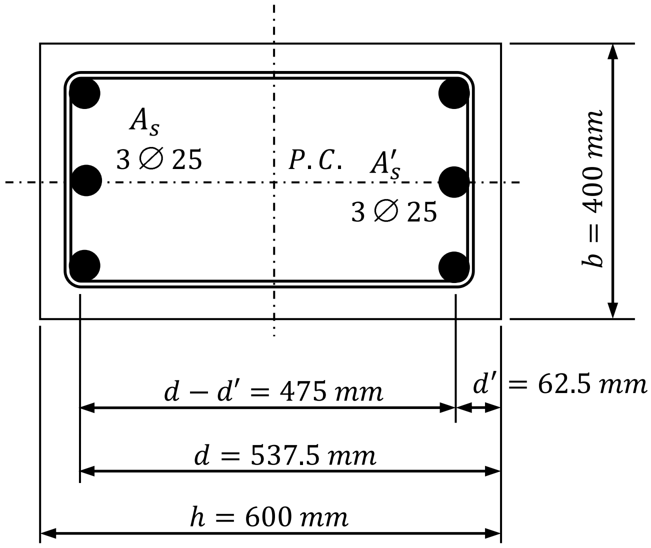
$$e = 200\, mm < e_b = 300.7\, mm$$

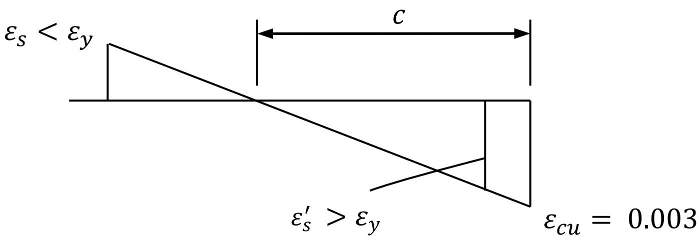
The section is compression-controlled.

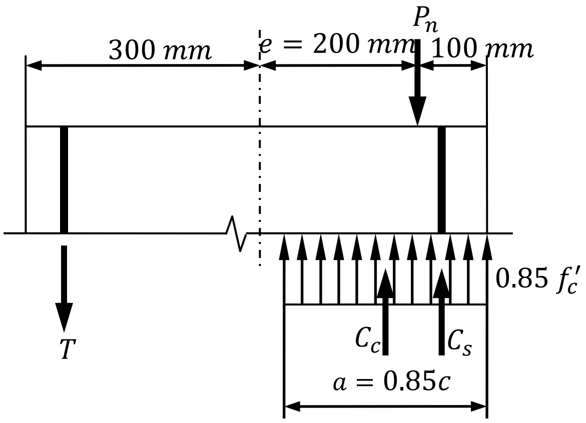
It is certain that $\varepsilon'_s > \varepsilon_y$ or $f'_s = f_y$.

$$C_s = A'_s\left(f_y - 0.85\, f'_c\right) = 1472.62(380 - 0.85 \cdot 20) \cdot 10^{-3} = 534.56\, KN$$

$$C_c = 0.85\, f'_c ab = 0.85\, f'_c \beta_1 cb = 0.85 \cdot 20 \cdot 0.85c \cdot 400 \cdot 10^{-3} = 5.78c$$




$$T = A_s f_s = A_s \cdot 600\left(\frac{d-c}{c}\right) = 1472.62 \cdot 600\left(\frac{537.5-c}{c}\right) \cdot 10^{-3} = \frac{474919.95 - 883.572c}{c}$$

Taking moments about $P_n$, $\curvearrowright + \sum M_{P_n} = 0$,

$$\left[C_c\left(\frac{0.85c}{2} - 100\right) - C_s(100 - 62.5) - T(600 - 62.5 - 100)\right] \cdot 10^{-3} = 0$$

$$\left[5.78c\left(\frac{0.85c}{2} - 100\right) - 534.56(100 - 62.5) - \left(\frac{474919.95 - 883.572c}{c}\right)(600 - 62.5 - 100)\right] 10^{-3} = 0$$

$$0.0024565c^2 - 0.578c - 20.046 - \frac{207777.48}{c} + 386.56 = 0$$

$$c^3 - 235.29c^2 + 149202.81c - 84582730.77 = 0$$

"$c$" can be determined by trial or directly by a scientific calculator. Also, the solution of the cubic equation can be obtained by using the well known Newton-Raphson method. This method is very powerful for finding a root of $f(x) = 0$. It involves a simple technique, and the solution converges rapidly by using the following steps:

1. Let $f(c) = Ac^3 + Bc^2 + C\,c + D$, and calculate $A, B, C, and\ D$.
2. Calculate the first derivative of $f(c)$:
$$f'(c) = 3Ac^2 + 2Bc + C$$
3. Assume any initial value of $c$, say, $c_0$, and compute the next value:
$$c_1 = c_0 - \frac{f(c_0)}{f'(c_0)}$$
4. Use the obtained value $c_1$ in the same way to get
$$c_2 = c_1 - \frac{f(c_1)}{f'(c_1)}$$

5. Repeat the same steps to get the answer up to the desired accuracy. In the case of the analysis of columns when compression controls, the value $c$ is greater than the balanced $c_b$. Therefore, start with $c_0 = c_b$, and repeat twice to get reasonable results.

**Trial and error method**

$$f(c) = c^3 - 235.29c^2 + 149202.81c - 84582730.77 = 0$$

| $c$ | $f(c) = 0$ |
|---|---|
| 330 | – 2.503·10$^7$ |
| 350 | – 1.831·10$^7$ |
| 400 | + 1.452·10$^6$ |
| 390 | – 2.862·10$^6$ |
| 395 | – 7.289·10$^5$ |
| 396 | – 2.965·10$^5$ |
| 397 | + 1.377·10$^5$ |
| 396.5 | – 7.963·10$^4$ |
| 396.6 | – 3.619·10$^4$ |
| 396. 7 | + 7.259·10$^3$ |

The reasonable result for $c$ is 396.6 $mm$.

**Newton-Raphson method**

$$f(c) = c^3 - 235.29c^2 + 149202.81c - 84582730.77$$
$$f'(c) = 3c^2 - 470.58c + 149202.81$$

| $c_i$ | $f(c)$ | $f'(c)$ | $c_{i+1} = c_i - \frac{f(c_i)}{f'(c_i)}$ |
|---|---|---|---|
| 330 | – 2.503·10$^7$ | 3.206·10$^5$ | 408.075 |
| 408.075 | 5.076·10$^6$ | 4.567·10$^5$ | 396.961 |
| 396.961 | 1.208·10$^5$ | 4.351·10$^5$ | 396.683 |
| 396.683 | 73.614 | 4.346·10$^5$ | 396.683 |

The reasonable result for $c$ is 396.6 $mm$ as the result obtained by trial and error method.

Check for $C_s$, it was assumed that compression steel yields $(f_s' = f_y)$,

$$f_s' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{396.6-62.5}{396.6}\right) = 505.45\ MPa > f_y = 380\ MPa - OK$$

$C_s = 534.56\ KN$

$C_c = 5.78c = 5.78 \cdot 396.6 = 2292.35\ KN$

$T = A_s f_s, \qquad f_s = 600\left(\frac{d-c}{c}\right) = 600\left(\frac{537.5-396.6}{396.6}\right) = 213.16\ MPa < f_y = 380\ MPa$

$T = 1472.62 \cdot 213.16 \cdot 10^{-3} = 313.91\ KN$

$P_n = C_c + C_s - T = 2292.35 + 534.56 - 313.91 = 2513\ KN$

$M_n = P_n e = 2513 \cdot 0.2 = 502.6\ KN \cdot m$

$$P_u = \phi P_n$$

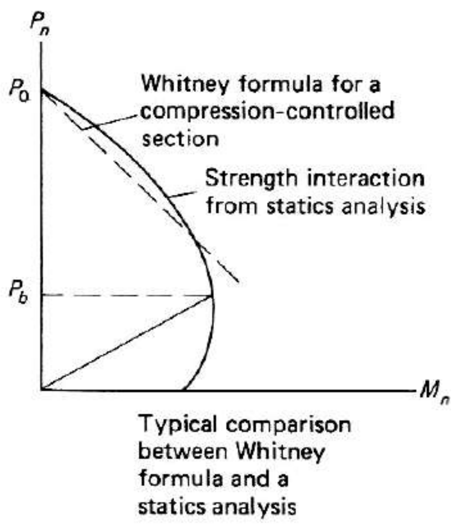
The tension steel does not yield - $f_s = 213.16\ MPa < f_y = 380\ MPa$ or by check for strain:

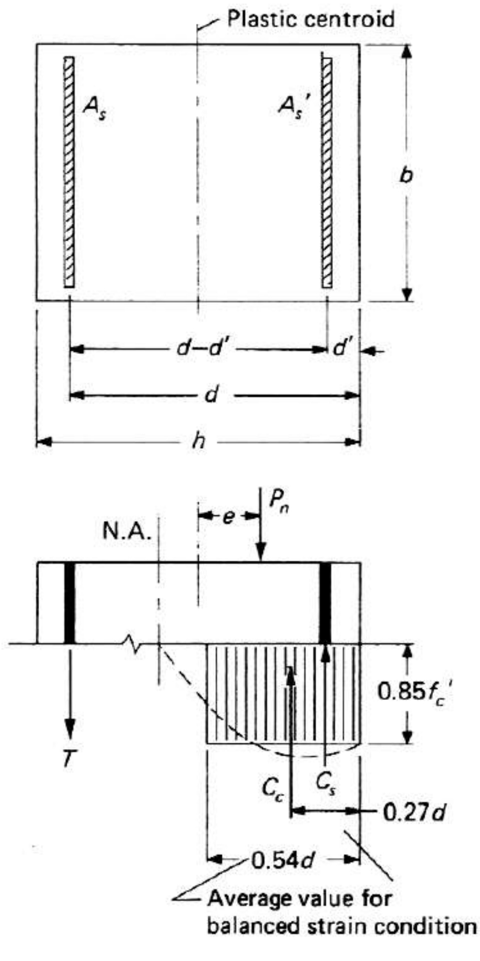
$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{537.5-396.6}{396.6}\right) = 0.00107 < \varepsilon_y = 0.0019$$

So $\phi = 0.65$ and,

$$P_u = 0.65 \cdot 2513 = 1633.5\ KN$$

**Whitney Formula – compression-controlled sections.**




One approximate procedure that may be applied to the case when ***the reinforcement is symmetrically placed in single layers parallel to the axis of bending*** is the one proposed by Whitney.

Taking moments of the forces about the tension steel gives

$$P_n\left(e+\frac{d-d'}{2}\right)=C_c\left(d-\frac{a}{2}\right)+C_s(d-d')$$

Whitney used for the depth of the rectangular stress distribution an average value based on the balanced strain condition, $a = 0.54d$

$$C_c = 0.85\, f_c' ab = 0.85 f_c'(0.54d)b = 0.459bdf_c'$$

and

$$C_c\left(d-\frac{a}{2}\right) = 0.459bdf_c'\left(d-\frac{0.54d}{2}\right)=\frac{1}{3}f_c' bd^2$$

When a section is compression-controlled, compression steel usually yields when $\varepsilon_{cu} = 0.003$ at the extreme fibers in compression. Neglecting displaced concrete,

$$C_s = A_s' f_y$$

Substituting in first equation for $P_n$ gives

$$P_n=\frac{\frac{1}{3}f_c' bd^2}{e+\frac{1}{2}(d-d')}+\frac{A_s' f_y(d-d')}{e+\frac{1}{2}(d-d')}$$

From which

$$P_n=\frac{f_c' bh}{\frac{3he}{d^2}+\frac{3(d-d')h}{2d^2}}+\frac{A_s' f_y}{\frac{e}{d-d'}+\frac{1}{2}}$$

One of the boundary conditions of this relationship is that it must satisfy the condition

$$P_n = P_o \qquad \text{at} \qquad e = 0$$

in which

$$P_o = 0.85f_c' bh + 2f_y A_s'$$

$$\frac{3(d-d')h}{2d^2}=\frac{1}{0.85}=1.18$$

$$P_n=\frac{bhf_c'}{\frac{3he}{d^2}+1.18}+\frac{A_s' f_y}{\frac{e}{d-d'}+0.5} \qquad -\ (Whitney\ formula)$$

which is Whitney formula for symmetrical steel with no correction for concrete displaced by compression steel.

A more useful expression for Whitney formula in terms of dimensionless ratios may be obtained by letting:

$$A_g = bh,\quad \xi h = d,\quad A_s = A_s'\ (for\ symmetrical\ reinforcement),\quad \rho_g=\frac{2A_s'}{A_g},\quad \gamma h = d-d'$$

$$P_n = A_g\left[\frac{f_c'}{\left(\frac{3}{\xi^2}\right)\left(\frac{e}{h}\right)+1.18}+\frac{\rho_g f_y}{\left(\frac{2}{\gamma}\right)\left(\frac{e}{h}\right)+1}\right]$$

**Example**

Resolve the previous example using Whitney Formula.

**Solution:**

$$\xi = \frac{d}{h} = \frac{537.5}{600} = 0.896, \qquad \frac{e}{h} = \frac{200}{600} = 0.333, \qquad \gamma = \frac{d-d'}{h} = \frac{537.5-62.5}{600} = 0.792$$

$$\rho_g = \frac{2A'_s}{A_g} = \frac{2 \cdot 1472.62}{600 \cdot 400} = 0.0123$$

$$P_n = A_g \left[ \frac{f'_c}{\left(\frac{3}{\xi^2}\right)\left(\frac{e}{h}\right) + 1.18} + \frac{\rho_g f_y}{\left(\frac{2}{\gamma}\right)\left(\frac{e}{h}\right) + 1} \right]$$

$$P_n = (600 \cdot 400) \left[ \frac{20}{\left(\frac{3}{0.896^2}\right)(0.333) + 1.18} + \frac{0.0123 \cdot 380}{\left(\frac{2}{0.792}\right)(0.333) + 1} \right] \cdot 10^{-3} = 2589.2\,KN$$

**Application of Interaction Diagram**

$$\rho_g = \frac{A_s}{A_g} = \frac{2945.2}{600 \cdot 400} = 0.0123 = 1.23\%$$

for $e = 200\,mm = 0.2\,m$, choose $P_u = 1000\,KN \quad and \quad M_u = 200\,KN \cdot m$

Use structural analysis and design programs (such as Prokon, CSI Column, ..etc.) for constructing interaction diagrams for the column section.

From the interaction diagram constructed in Prokon Program:

$$P_u = 1650\,KN, \qquad M_u = 335\,KN \cdot m$$

$$\phi = 0.65, \quad P_n = \frac{P_u}{\phi} = \frac{1650}{0.65} = 2538.5\,KN,$$

| PROKON Software Consultants (Pty) Ltd Internet: http://www.prokon.com E-Mail : mail@prokon.com | Job Number | | Sheet |
|---|---|---|---|
| | Job Title | | |
| | Client | | |
| | Calcs by | Checked by | Date |

# Rectangular column design

C11

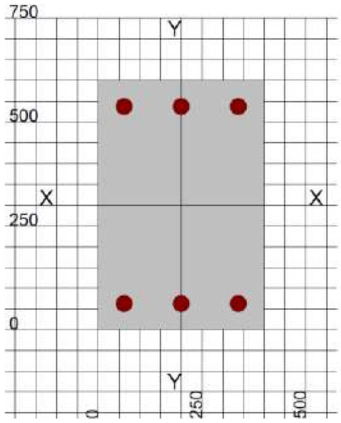
Rectangular column design by *PROKON.* (RecCol Ver W2.3.05 - 18 Jul 2007)

Design code : ACI 318 - 2005

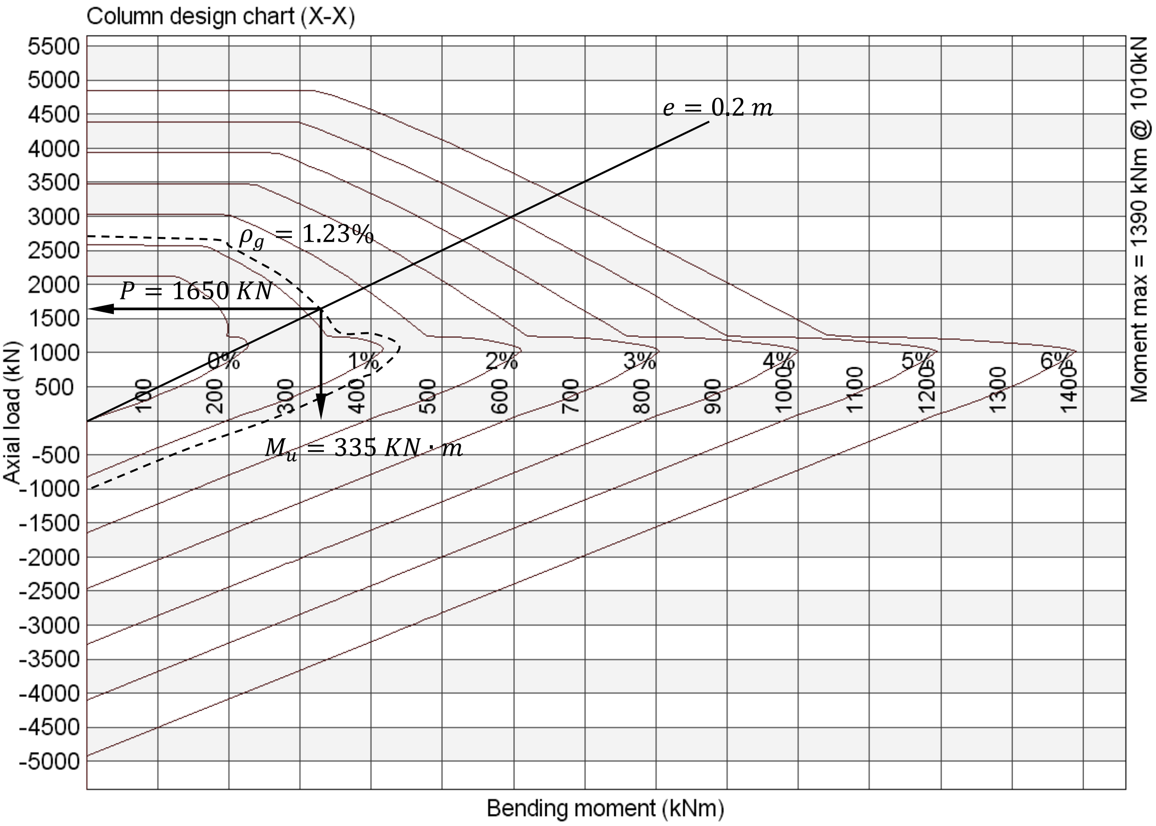
## General design parameters:

Given:

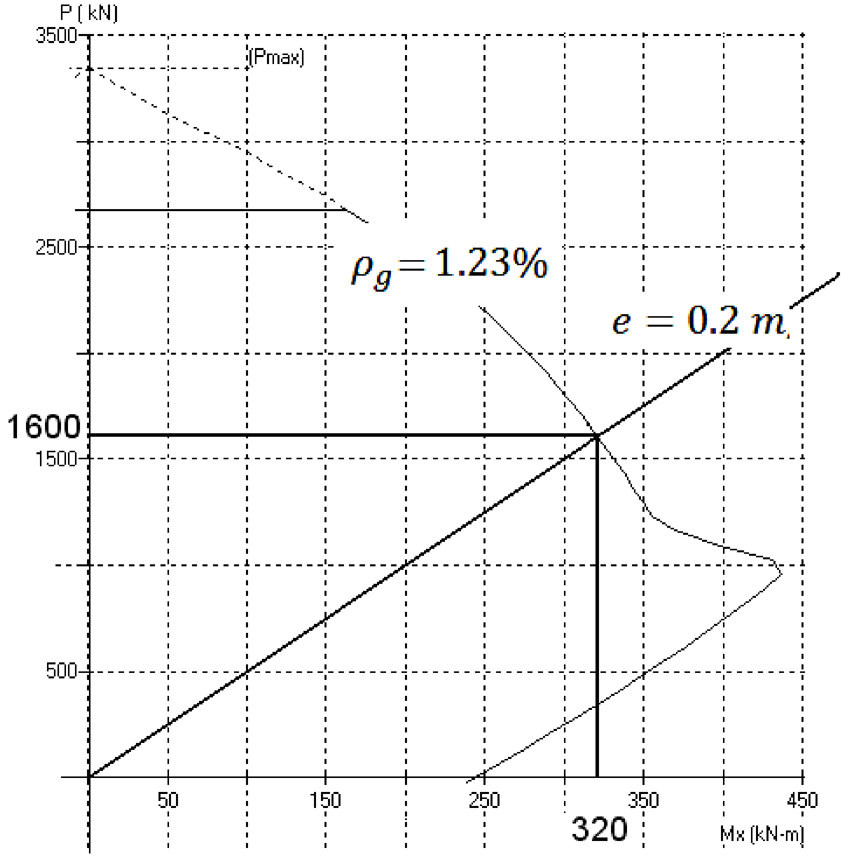
h = 600 mm
b = 400 mm
d'x = 62.5 mm
d'y = 62.5 mm
Lo = 2.000 m
fc' = 20 MPa
fy = 380 MPa

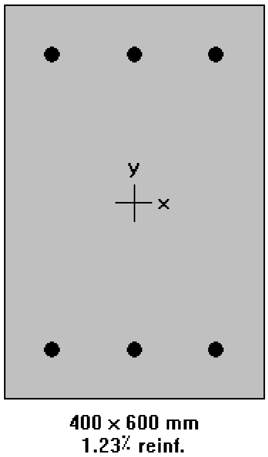



Design chart for bending about the X-X axis:



**Another solution,**




$\rho_g = 1.23\%$

for $e = 0.2\ m$, choose $P_u = 1500\ KN \quad and \quad M_u = 300\ KN \cdot m$

From the interaction diagram constructed in PCA-column Program:

$$P_u = 1600\ KN, \qquad M_u = 320\ KN \cdot m$$

$$\phi = 0.65, \quad P_n = \frac{P_u}{\phi} = \frac{1600}{0.65} = 2461.5\ KN,$$

## 6.9 NOMINAL STRENGTH OF A RECTANGULAR SECTION HAVIN $(e > e_b)$.

When the nominal compression strength $P_n$ is less than the balanced nominal strength $P_b$, or when the eccentricity $e$ is greater than the balanced value $e_b$, or when the net tensile strain $\varepsilon_t$ at the extreme layer of steel at the face opposite the maximum compression face is greater than $\varepsilon_y = \frac{f_y}{E_s}$, the section is more like a beam than a column (see interaction diagram in section 7.6).

**Example**

Determine the nominal compressive strength $P_n$ for the section below for an eccentricity $e = 500\ mm$.
Take $f_c' = 20\ MPa,\ \ f_y = 380\ MPa$.

**Solution:**

Determine whether the given eccentricity $e = 500\ mm$ is larger or smaller than $e_b$.
The balanced strain condition was computed in the previous example (page 92) as

$$P_b = 1877.19\ KN, \qquad e_b = 300.7\ mm$$

$$e = 500\ mm > e_b = 300.7\ mm,$$

therefore, the strain $\varepsilon_s$ on the tension steel exceeds $\varepsilon_y$. It is assumed (initially) that the strain $\varepsilon_s'$ on the compression steel is at least equal to the yield strain $\varepsilon_y$ although the validity of the assumption must be verified before the solution is accepted.

$$C_c = 0.85\ f_c' ab = 0.85\ f_c' \beta_1 cb = \ 0.85 \cdot 20 \cdot 0.85c \cdot 400 \cdot 10^{-3} = 5.78c$$

$$C_s = A_s'(f_y - 0.85\ f_c') = 1472.62(380 - 0.85 \cdot 20) \cdot 10^{-3} = 534.56\ KN$$

$$T = A_s f_y = 1472.62 \cdot 380 \cdot 10^{-3} = 559.6\ KN$$

Force equilibrium requires

$$P_n = C_c + C_s - T = 5.78c + 534.56 - 559.6 = 5.78c - 25.04$$

Taking moments about the tension steel $\sum M_{A_s} = 0$,

$$P_n\left(e + \frac{d - d'}{2}\right) = C_c\left(d - \frac{a}{2}\right) + C_s(d - d')$$

$$(5.78c - 25.04)\left(500 + \frac{537.5 - 62.5}{2}\right) = 5.78c\left(537.5 - \frac{0.85c}{2}\right) + 534.56(537.5 - 62.5)$$

$$\left[(5.78c - 25.04)\left(500 + \frac{537.5 - 62.5}{2}\right) = 5.78c\left(537.5 - \frac{0.85c}{2}\right) + 534.56(537.5 - 62.5)\right] \cdot 10^{-3}$$

$$4.26c - 18.47 = 3.11c - 0.002457c^2 + 253.92$$

$$0.002457c^2 + 1.15c - 272.39 = 0$$

$$c^2 + 468.1c - 110862.8 = 0$$

$$c_{1,2} = \frac{-468.1 \pm \sqrt{468.1^2 - 4 \cdot 1 \cdot (-110862.8)}}{2} = \frac{-468.1 \pm 814}{2}$$

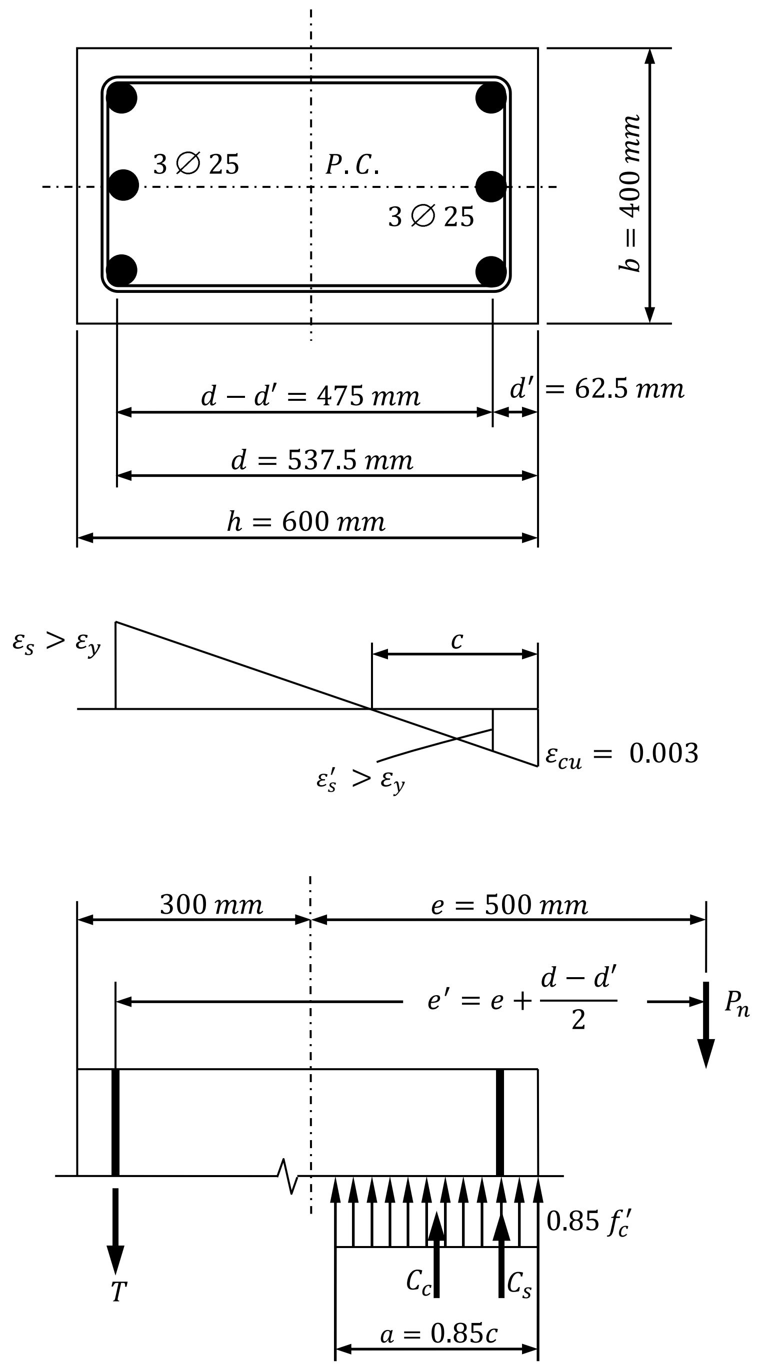
$$c = 172.95\ mm > 0$$

Check for the initial assumption for $\varepsilon_s' = \varepsilon_y$ or $f_s' = f_y$,

$$f_s' = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{172.95 - 62.5}{172.95}\right) = 383.17\ MPa > f_y = 380\ MPa$$

Take $f_s' = f_y = 380\ MPa$ as assumed before.

$$P_n = 5.78 \cdot 172.95 + 534.56 - 559.6 = 974.61\ KN$$



In addition we can calculate $M_n = P_n e = 974.61 \ \cdot 500 \cdot 10^{-3} = 487.31\ KN \cdot m$
Check for tension steel strain:

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{537.5-172.95}{172.95}\right) = 0.0063 > 0.005$$

$\varepsilon_s = 0.0063 > 0.005$ - tension controlled section and $\phi = 0.9$

$$P_u = \phi P_n = 0.9 \cdot 974.61 = 877.15\ KN$$

$$M_u = \phi M_u = 0.9 \cdot 487.31 = 438.58\ KN \cdot m$$

**Approximate Formulas – Rectangular Sections Having $e > e_b$.**

Using symbols in the same previous procedure to determine $P_n$ for rectangular section with $e > e_b$ we get the general solution in the form:

$$P_n = 0.85 f_c' bd \left\{\rho'(m-1) - \rho m + \left(1-\frac{e'}{d}\right) + \sqrt{\left(1-\frac{e'}{d}\right)^2 + 2\left[\left(\frac{e'}{d}\right)(\rho m - \rho' m + \rho') + \rho'(m-1)\left(1-\frac{d'}{d}\right)\right]}\right\}$$

When $\rho = \rho'$ then

$$P_n = 0.85 f_c' bd \left\{-\rho + 1 - \frac{e'}{d} + \sqrt{\left(1-\frac{e'}{d}\right)^2 + 2\rho\left[(m-1)\left(1-\frac{d'}{d}\right) + \frac{e'}{d}\right]}\right\}$$

where

$$\rho = \frac{A_s}{bd}, \qquad \rho' = \frac{A_s'}{bd}, \qquad m = \frac{f_y}{0.85 f_c'}, \qquad e' = e + \frac{d-d'}{2}$$

$e'$ – the distance between the applied load and tension steel.

For the previous example:

$$\rho = \rho' = \frac{A_s}{bd} = \frac{1472.62}{400 \cdot 537.5} = 0.00685$$

$$m = \frac{f_y}{0.85 f_c'} = \frac{380}{0.85 \cdot 20} = 22.35$$

$$e' = e + \frac{d-d'}{2} = 500 + \frac{537.5-62.5}{2} = 737.5\ mm$$

$$P_n = 0.85 \cdot 20 \cdot 400 \cdot 537.5 \left\{-0.00685 + 1 - \frac{737.5}{537.5} + \right.$$

$$\left. + \sqrt{\left(1-\frac{737.5}{537.5}\right)^2 + 2 \cdot 0.00685\left[(22.35-1)\left(1-\frac{62.5}{537.5}\right) + \frac{737.5}{537.5}\right]}\right\} 10^{-3} = 971.62\ KN$$

**Application of Interaction Diagram**

$$\rho_g = \frac{A_s}{A_g} = \frac{2945.2}{600 \cdot 400} = 0.0123 = 1.23\%$$

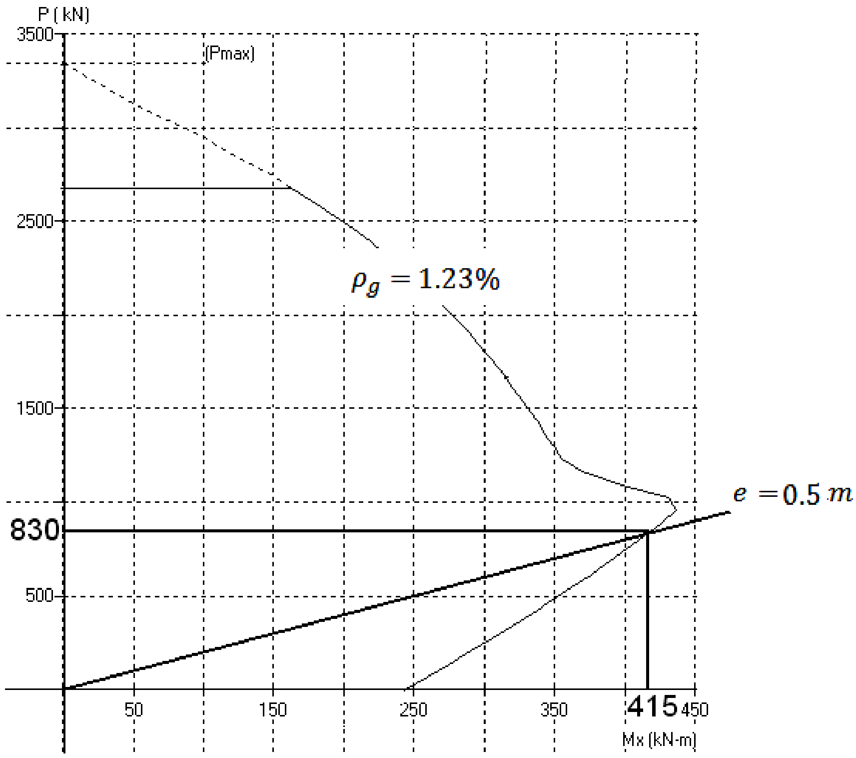
for $e = 500\ mm = 0.5\ m$, choose $P_u = 500\ KN \quad and \quad M_u = 250\ KN \cdot m$

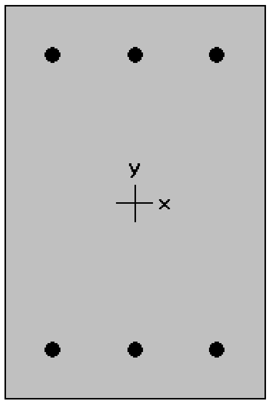
Use structural analysis and design programs (such as Prokon, CSI Column, ..etc.) for constructing interaction diagrams for the column section.

From the interaction diagram constructed in PCA-column Program:

$$P_u = 830\ KN, \qquad M_u = 415\ KN \cdot m$$

$$\phi = 0.9, \quad P_n = \frac{P_u}{\phi} = \frac{830}{0.9} = 922.2\ KN,$$




400 × 600 mm
1.23% reinf.

### 6.10 DESIGN FOR STRENGTH. PRACTICAL DESIGN APPROACH.

Generally, designers have access to published interaction diagrams or computer programs to compute interaction diagrams based on a sectional analysis for use in design. Occasionally, this is not true, as, for example, in the design of hollow bridge piers, elevator shafts, or unusually shaped members. Typical interaction charts are given here.

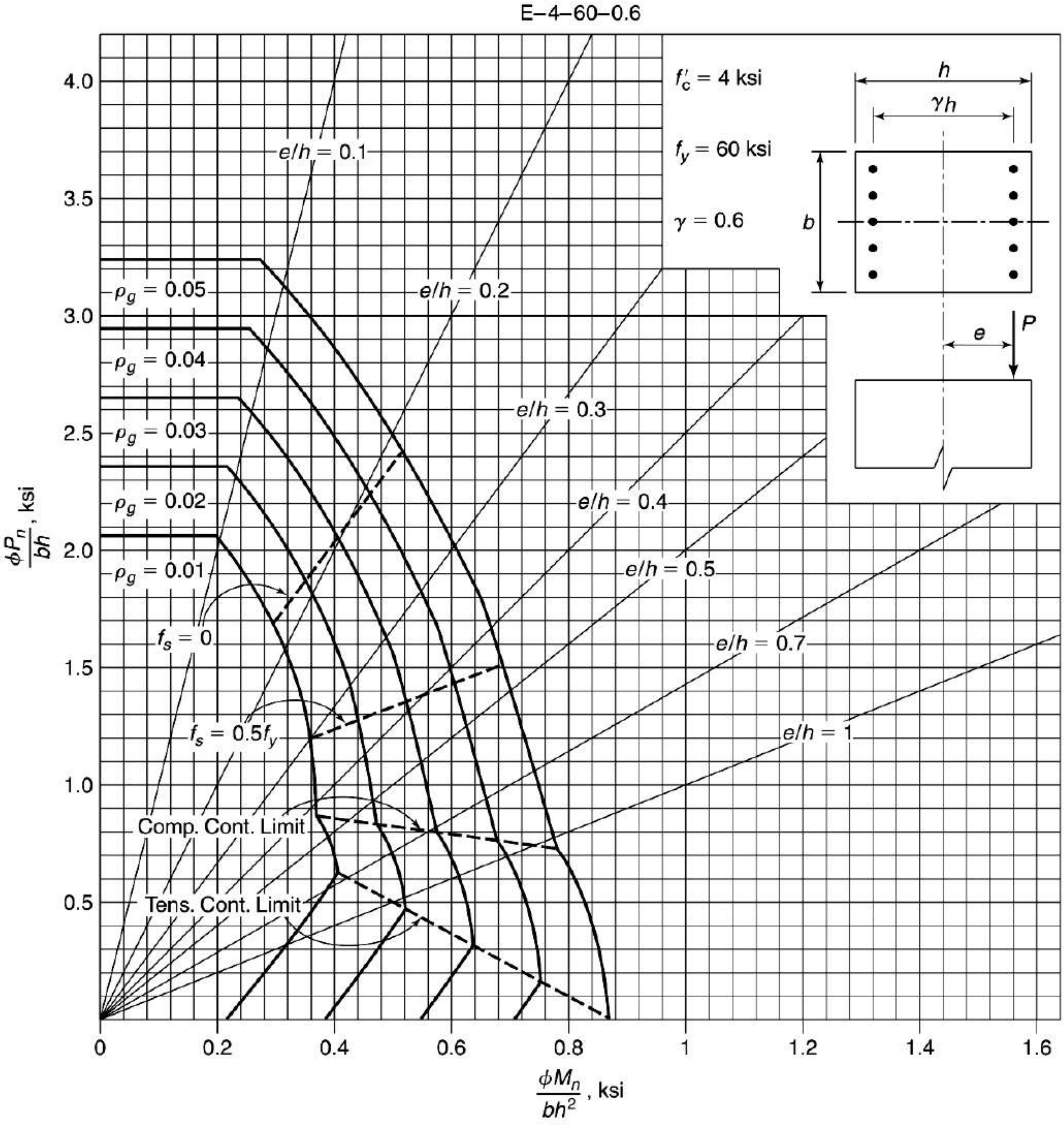



Fig. A-6a
Nondimensional interaction diagram for rectangular tied columns with bars in two faces: $f'_c = 4000$ psi and $\gamma = 0.60$.

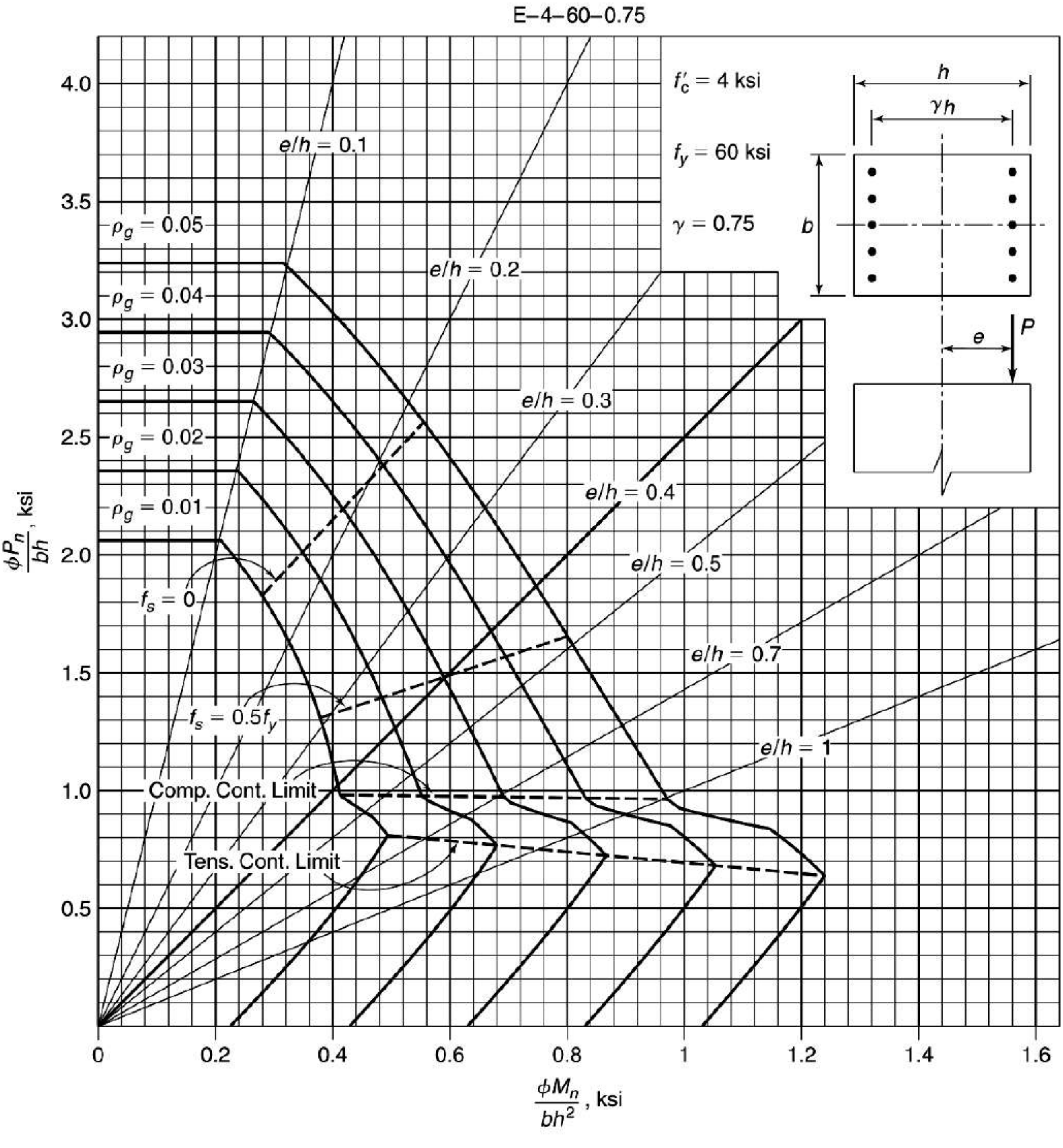



Fig. A-6b
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c$ = 4000 psi and $\gamma$ = 0.75.

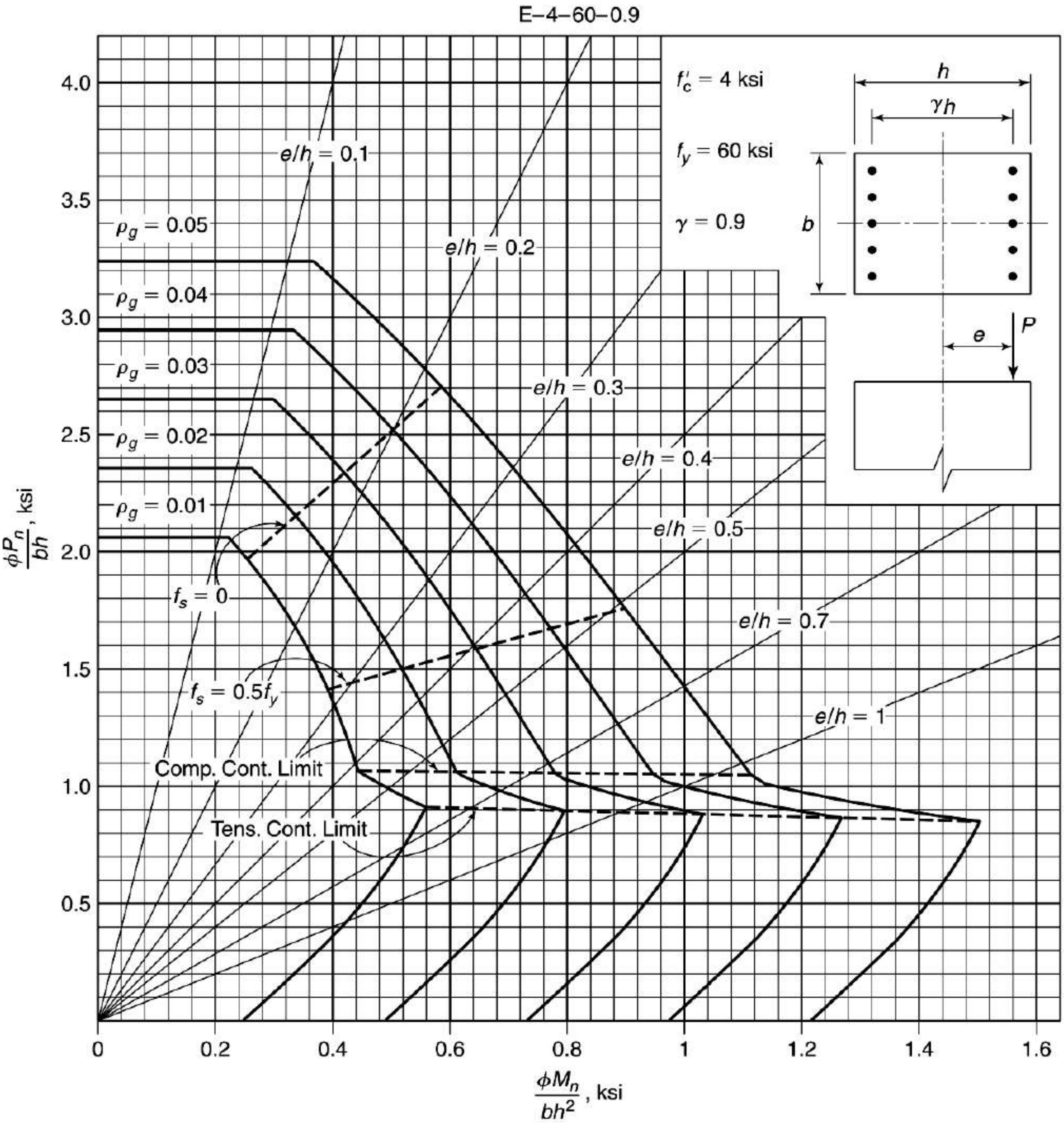



Fig. A-6c
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c$ = 4000 psi and $\gamma$ = 0.90.

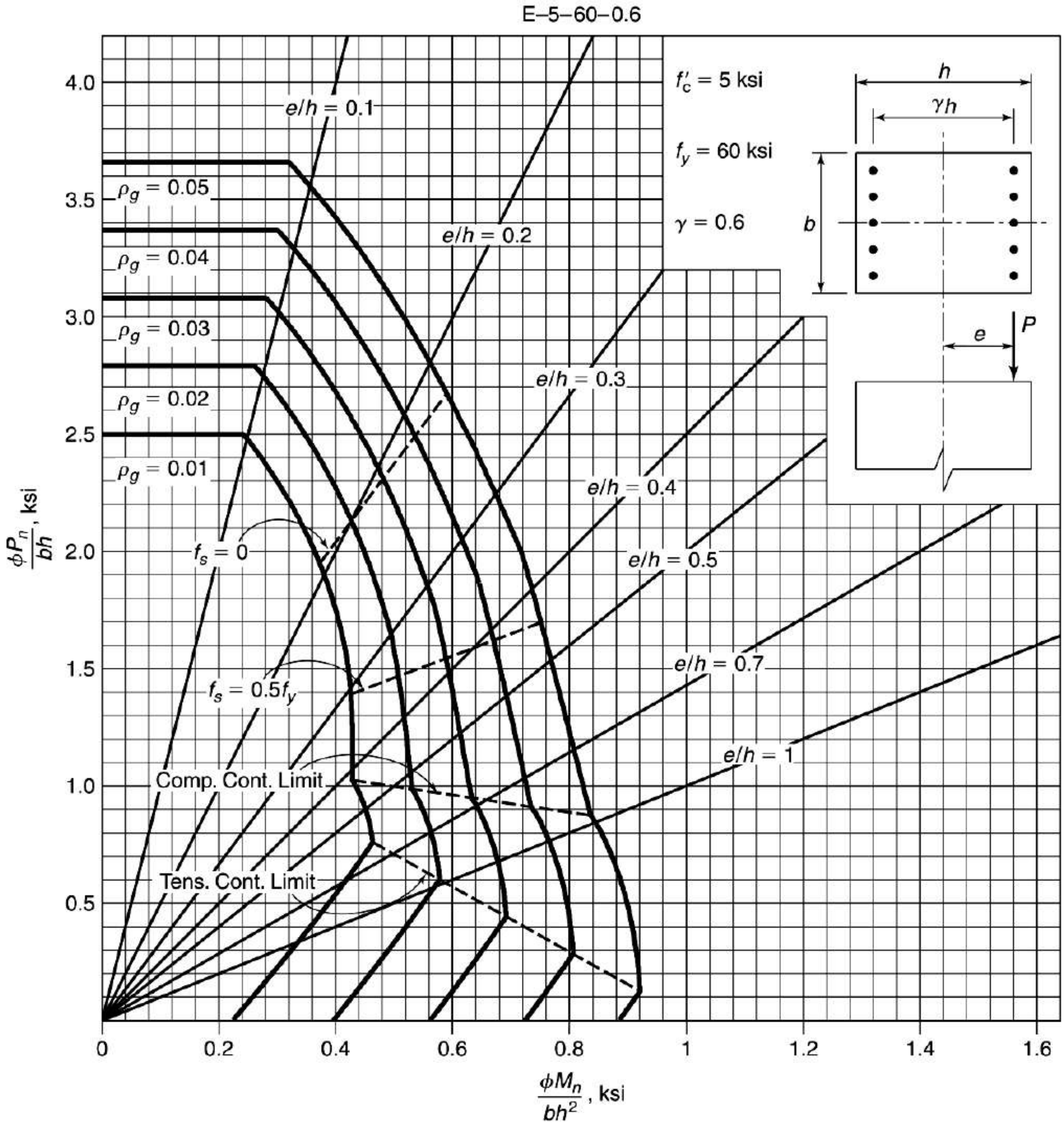



Fig. A-7a
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f_c' = 5000$ psi and $\gamma = 0.60$.

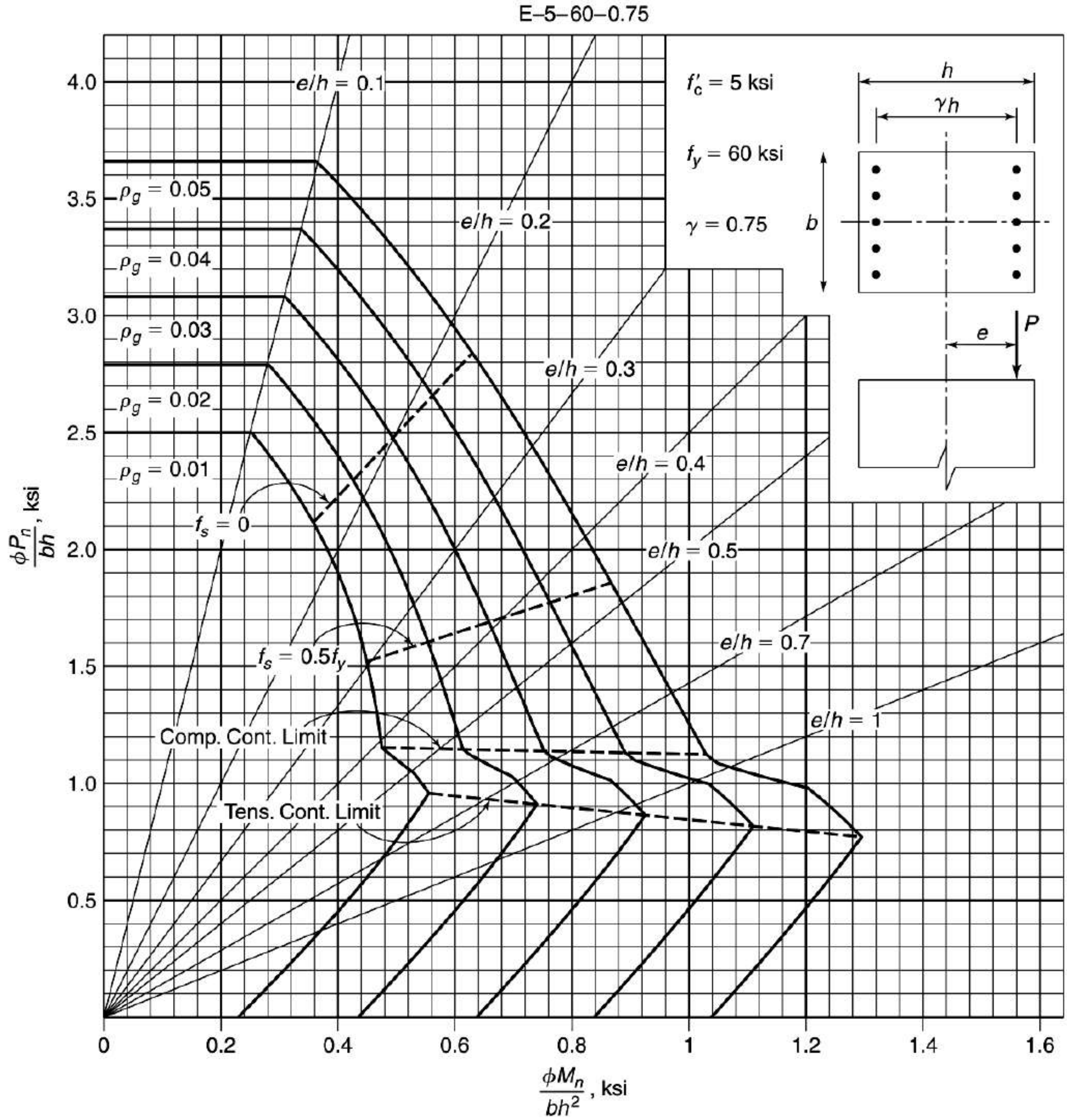



**Fig. A-7b**
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c = 5000$ psi and $\gamma = 0.75$.

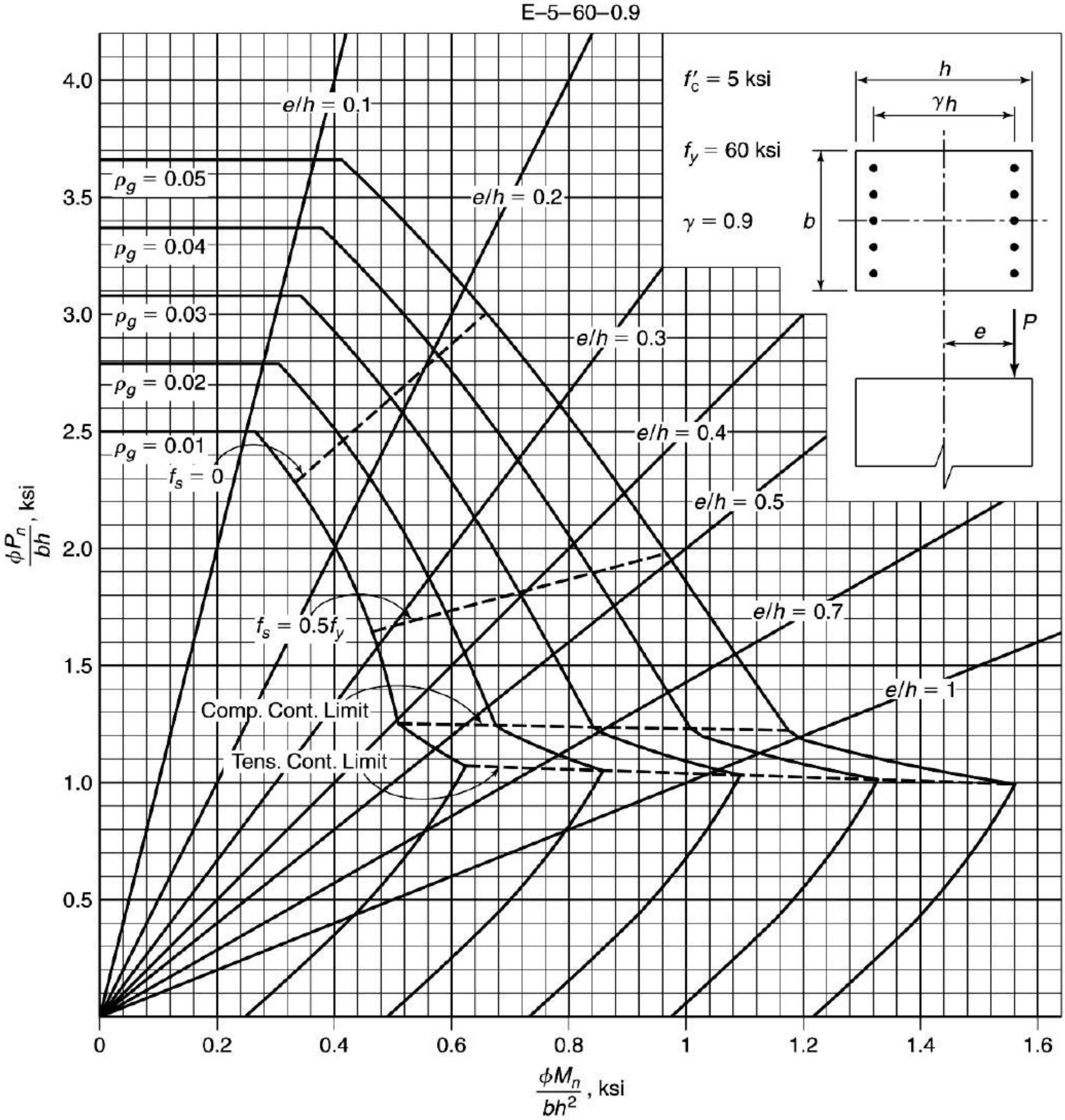



Fig. A-7c
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c$ = 5000 psi and $\gamma$ = 0.90.

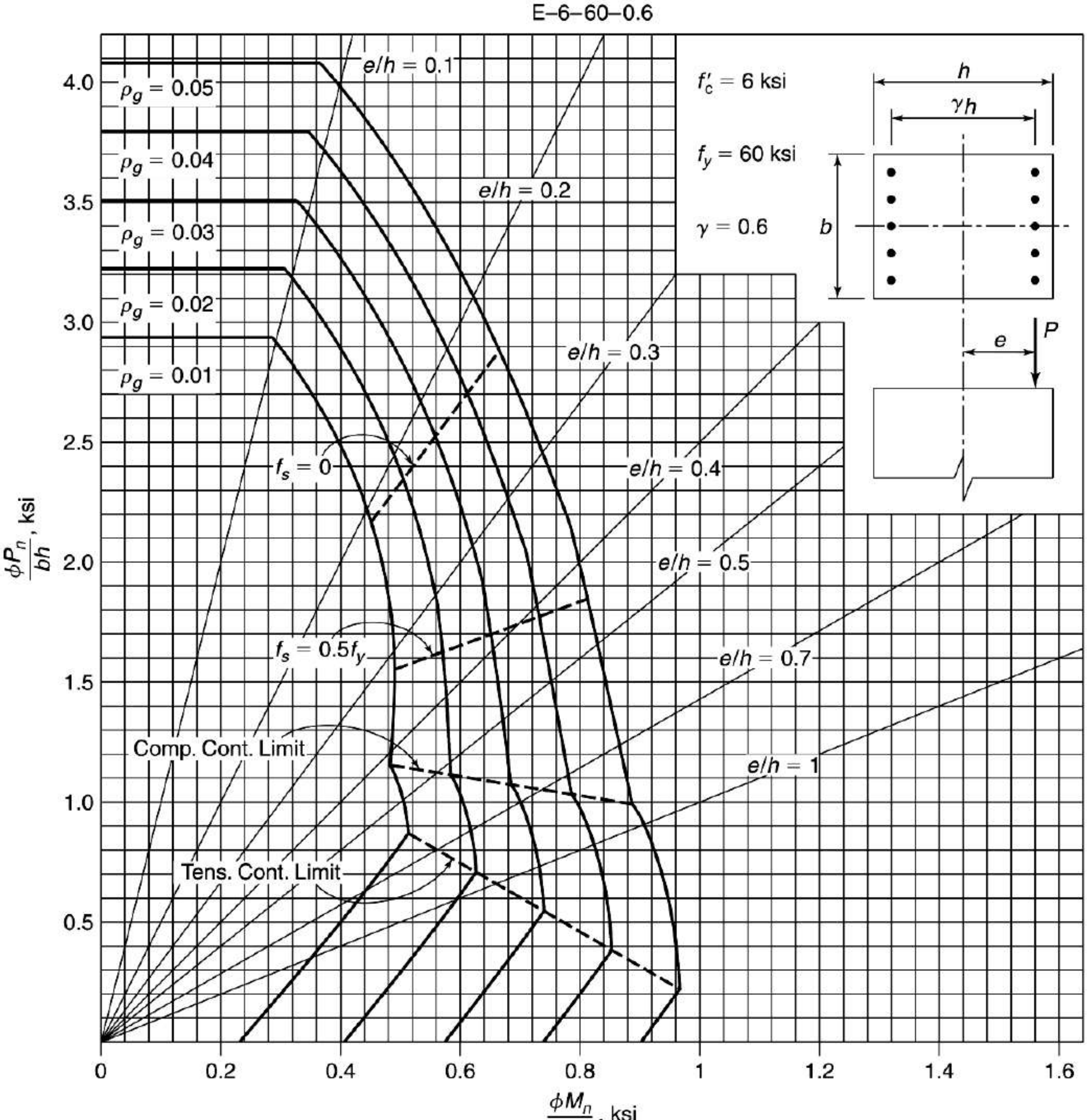



Fig. A-8a
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c = 6000$ psi and $\gamma = 0.60$.

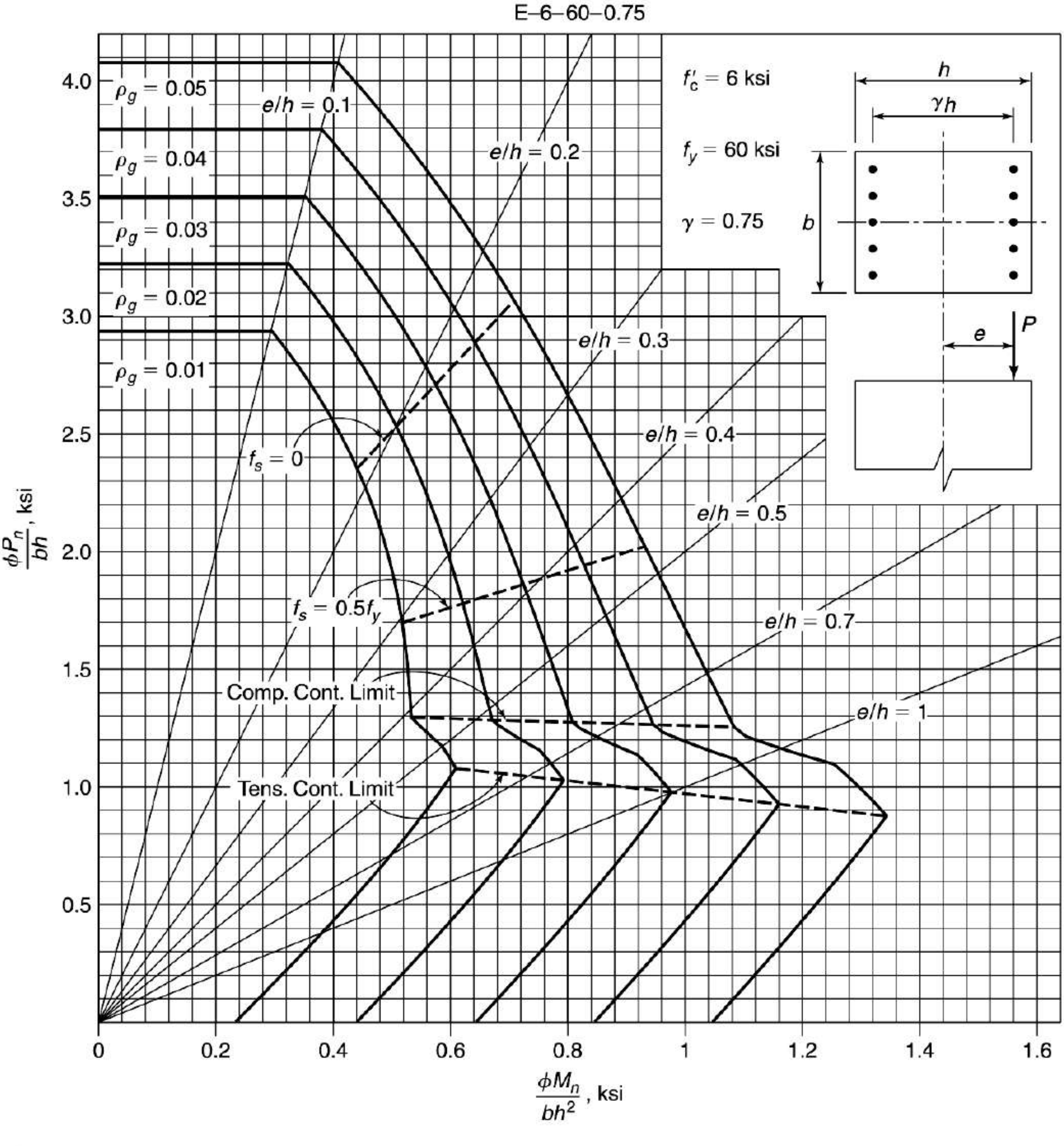



Fig. A-8b
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c$ = 6000 psi and $\gamma$ = 0.75.

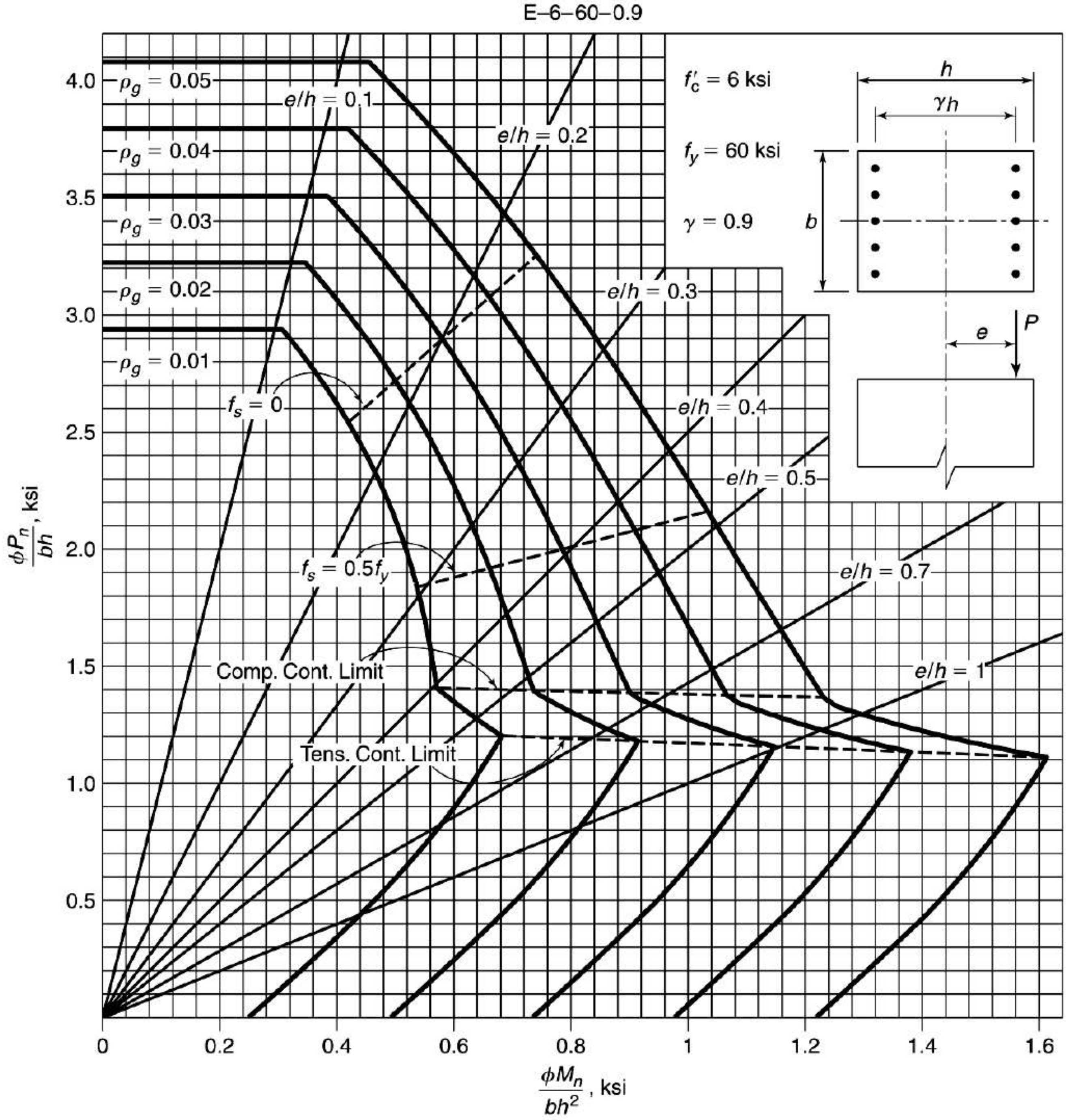



Fig. A-8c
Nondimensional interaction diagram for rectangular tied column with bars in two faces: $f'_c = 6000$ psi and $\gamma = 0.90$.

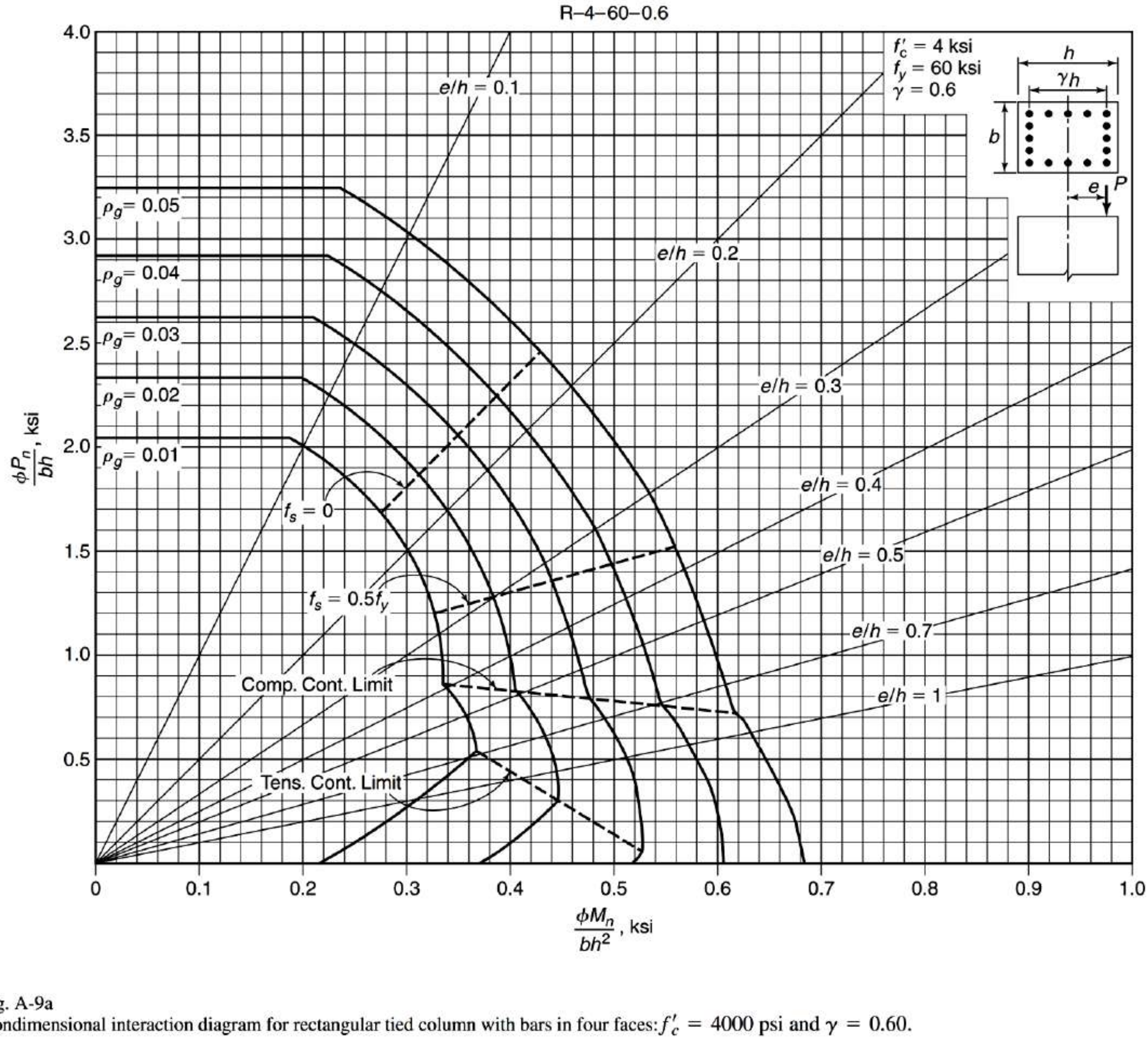



Fig. A-9a
Nondimensional interaction diagram for rectangular tied column with bars in four faces: $f'_c = 4000$ psi and $\gamma = 0.60$.

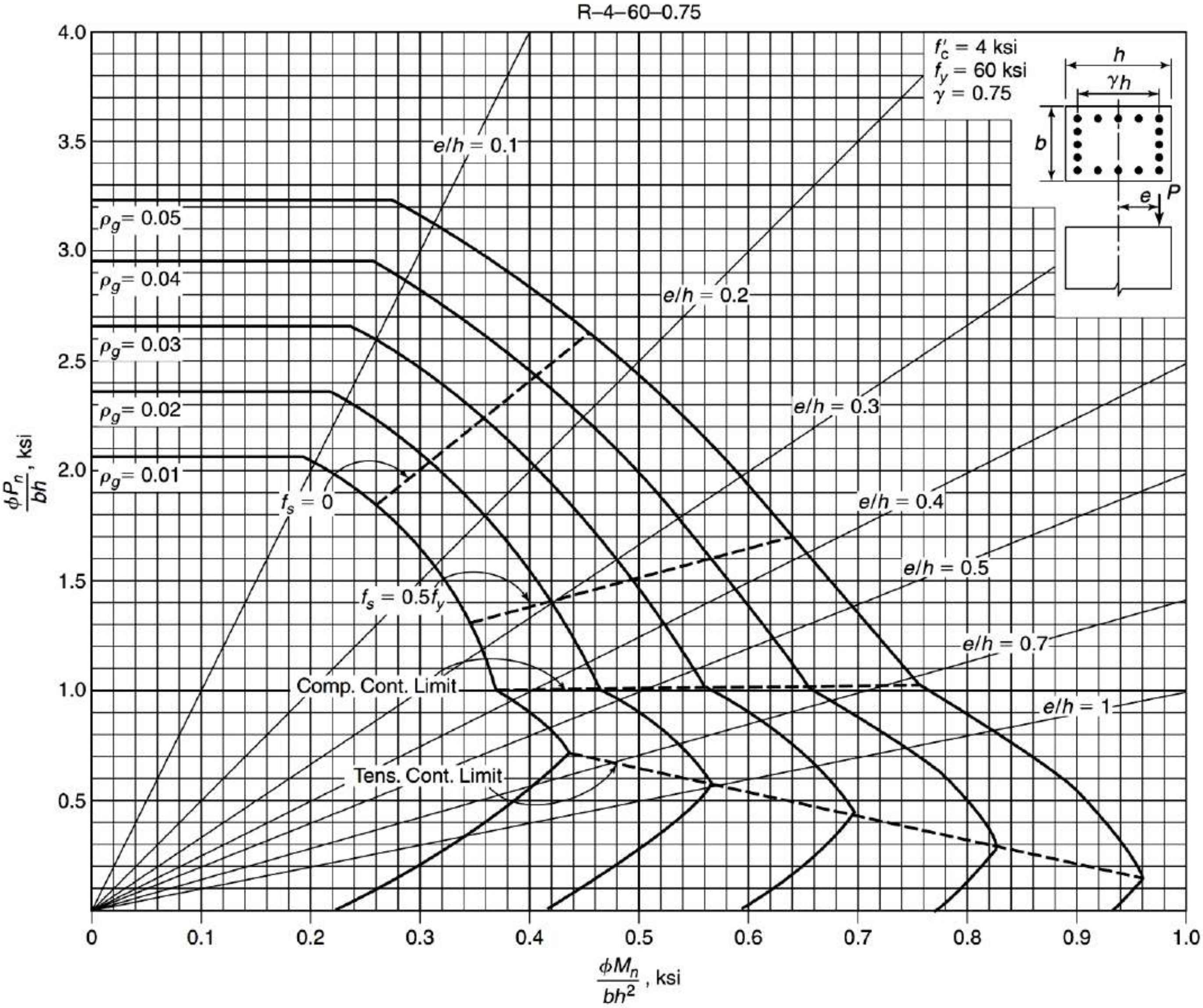



Fig. A-9b
Nondimensional interaction diagram rectangular for tied column with bars in four faces: $f'_c = 4000$ psi and $\gamma = 0.75$.

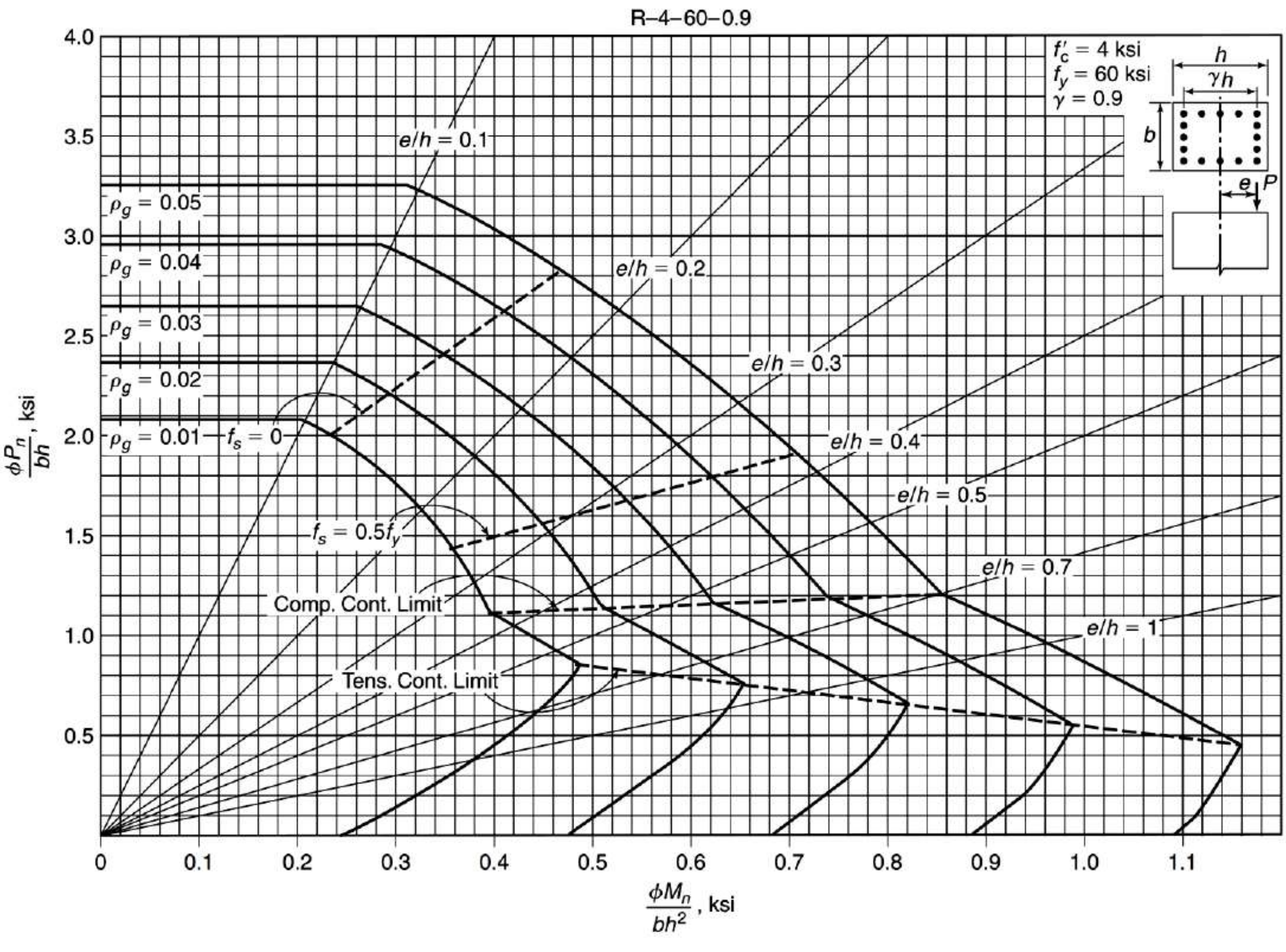



Fig. A-9c
Nondimensional interaction diagram for rectangular tied column with bars in four faces: $f'_c$ = 4000 psi and $\gamma$ = 0.90.

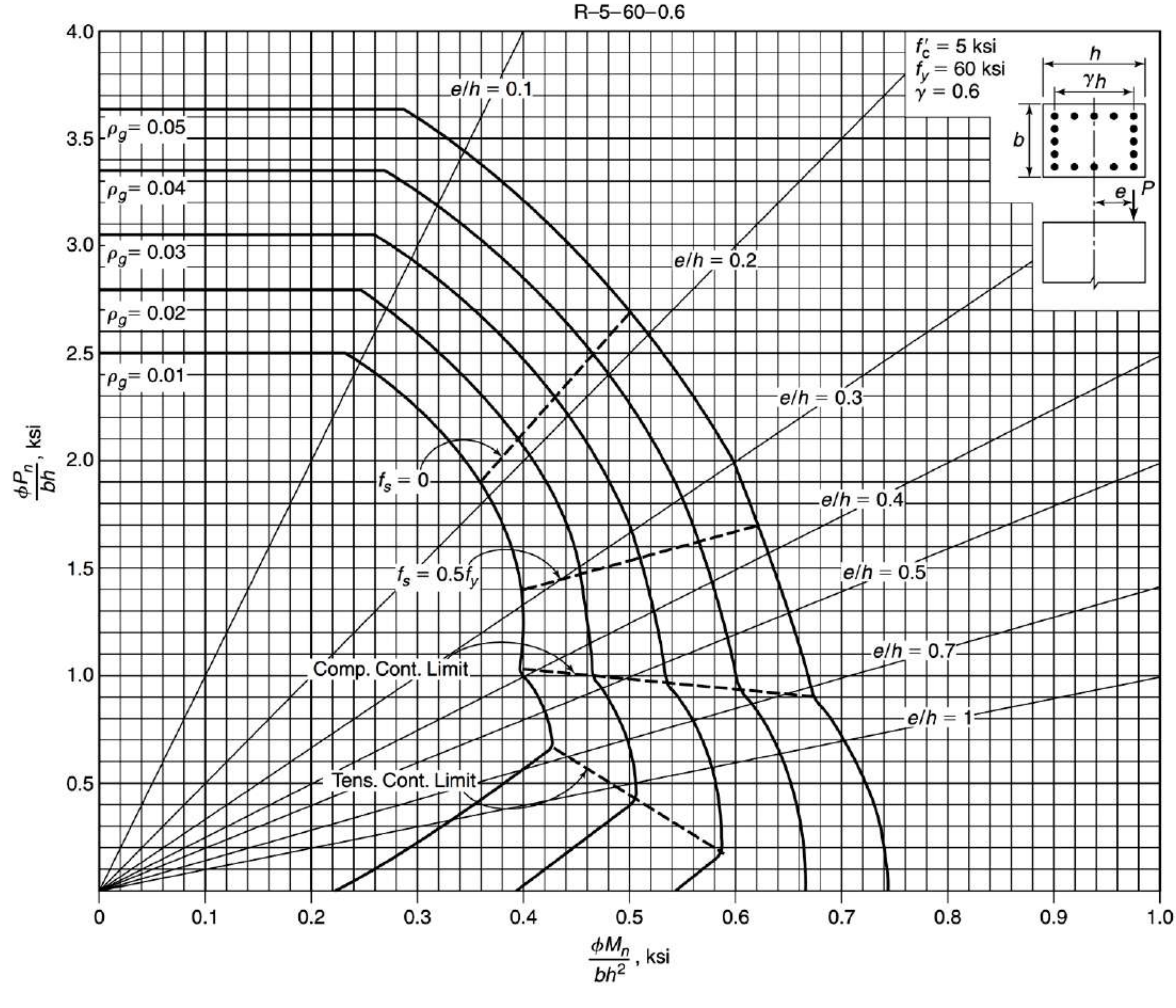



Fig. A-10a
Nondimensional interaction diagram for rectangular tied column with bars in four faces: $f'_c = 5000$ psi and $\gamma = 0.60$.

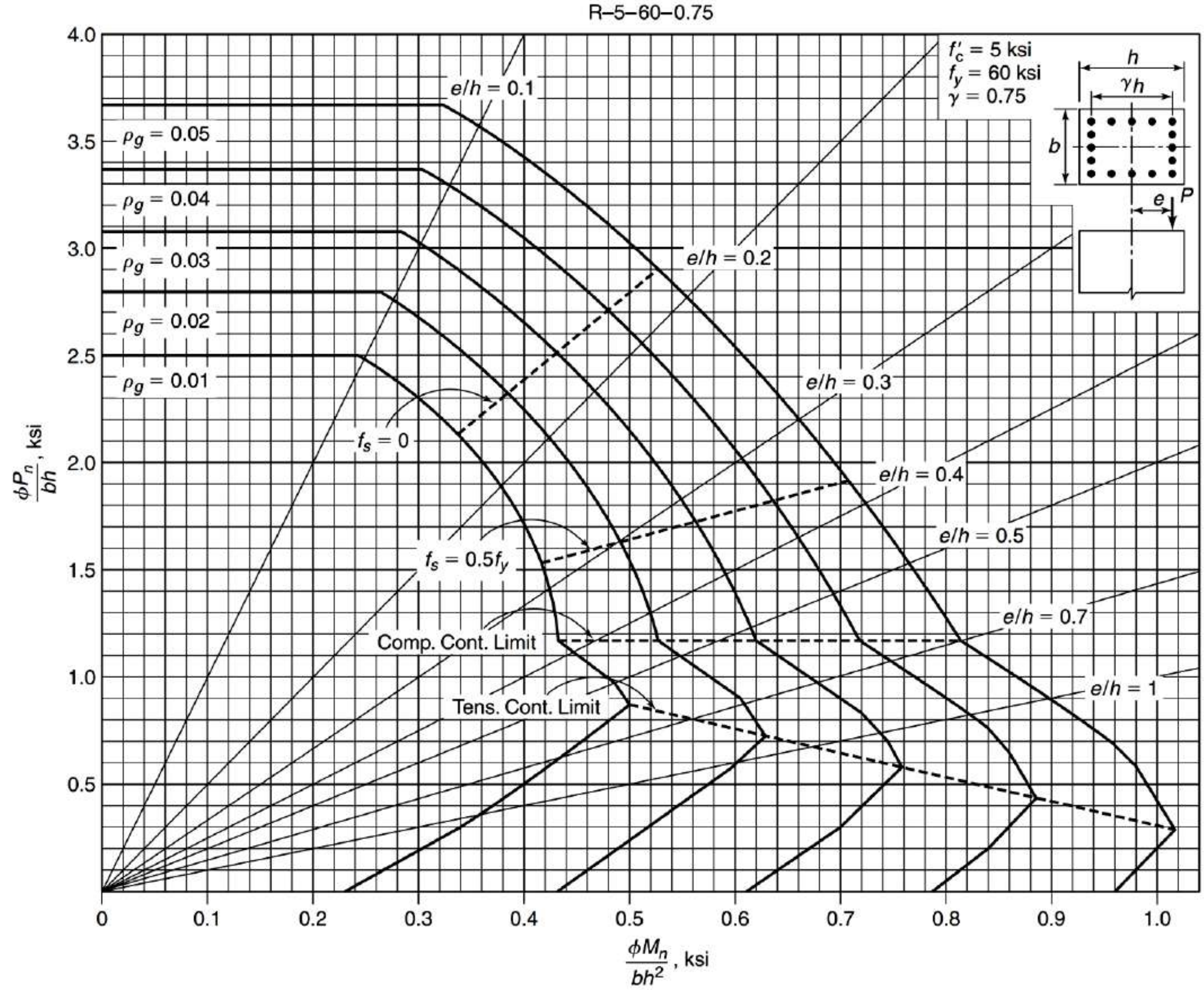



Fig. A-10b
Nondimensional interaction diagram for tied column with bars in four faces: $f'_c = 5000$ psi and $\gamma = 0.75$.

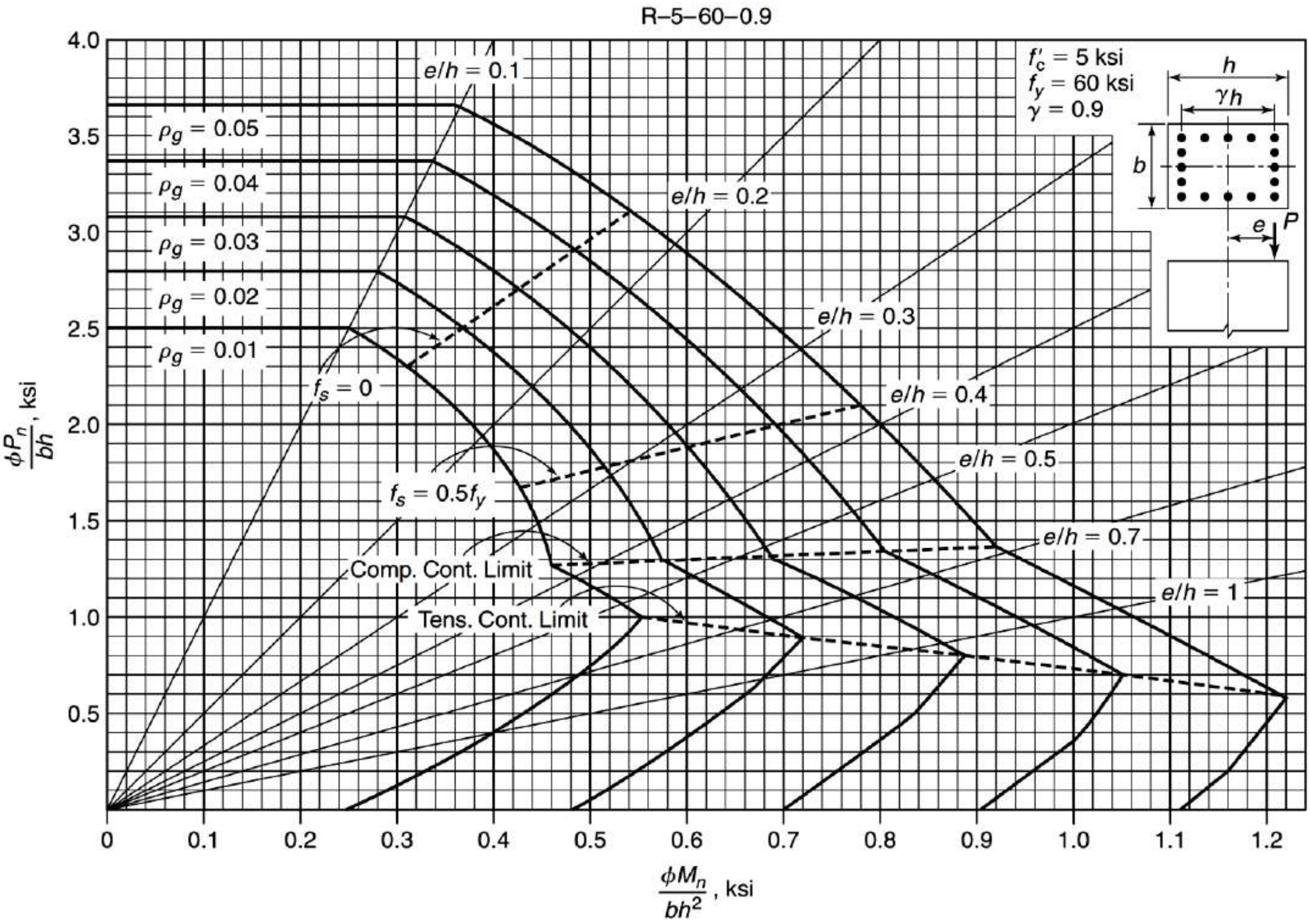



Fig. A-10c
Nondimensional interaction diagram for tied column with bars in four faces: $f'_c = 5000$ psi and $\gamma = 0.90$.

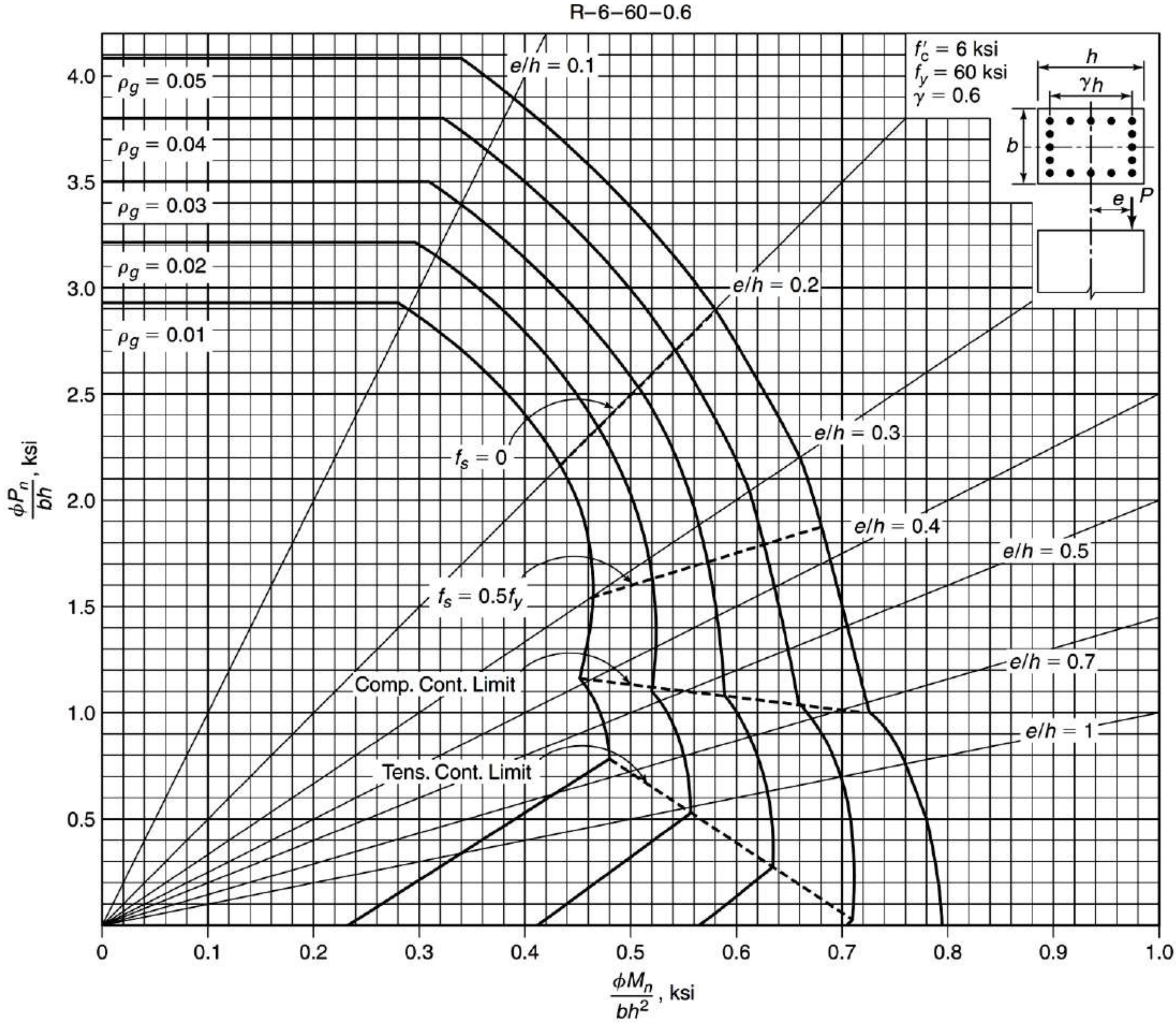



Fig. A-11a
Nondimensional interaction diagram for tied column with bars in four faces: $f'_c = 6000$ psi and $\gamma = 0.60$.

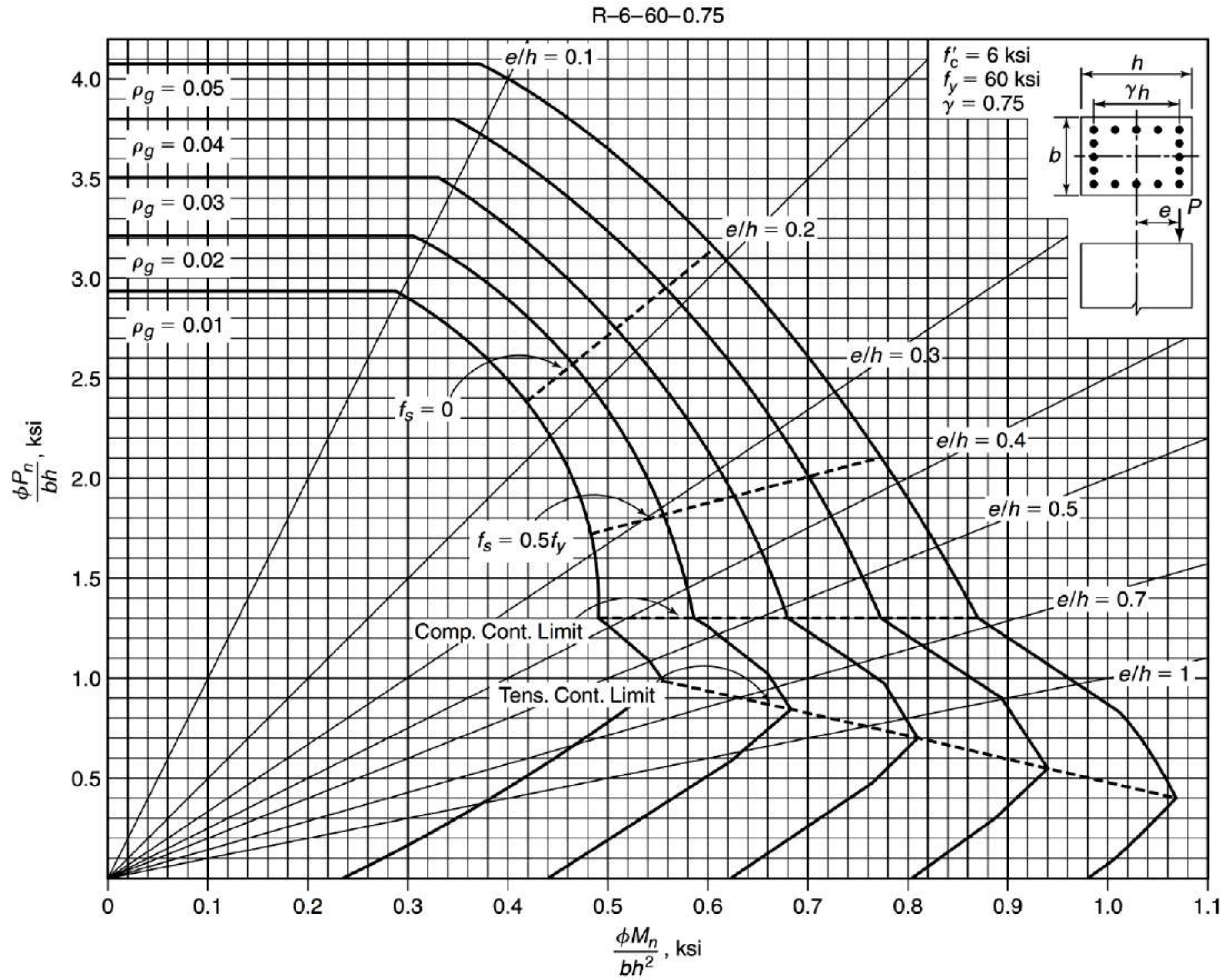



Fig. A-11b
Nondimensional interaction diagram for tied column with bars in four faces: $f'_c = 6000$ psi and $\gamma = 0.75$.

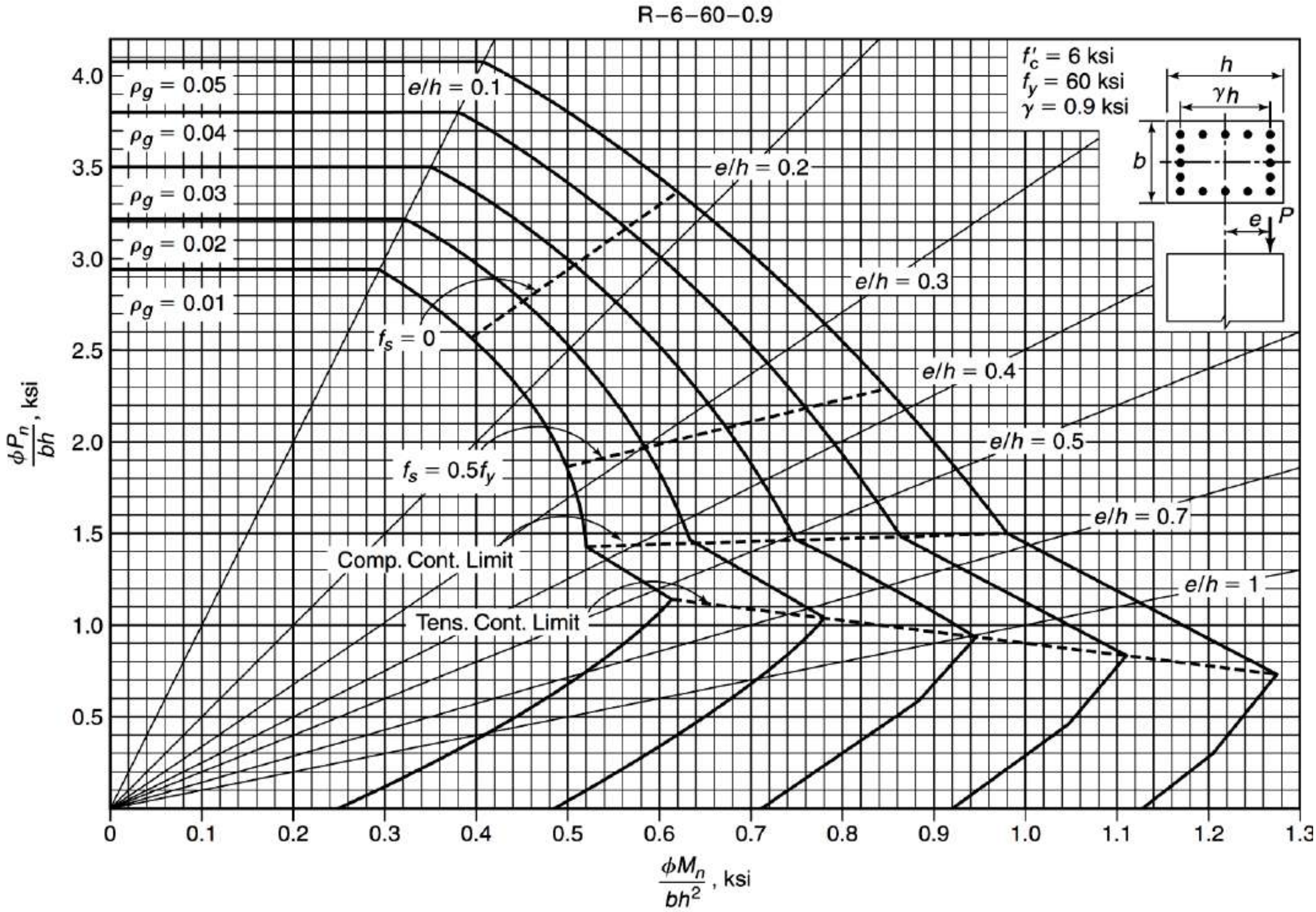



Fig. A-11c
Nondimensional interaction diagram for tied column with bars in four faces: $f'_c = 6000$ psi and $\gamma = 0.90$.

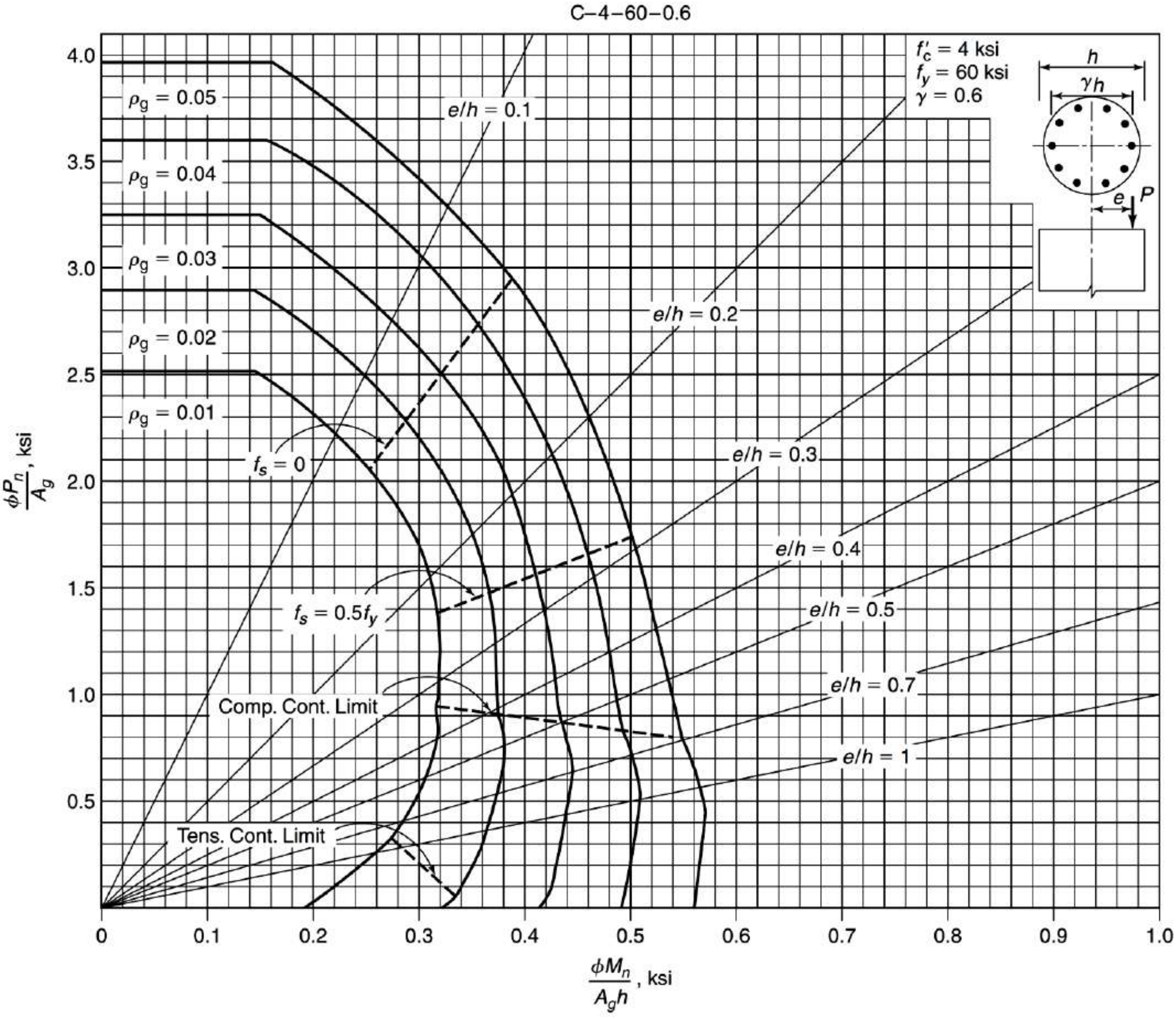



Fig. A-12a
Nondimensional interaction diagram for circular spiral column: $f'_c = 4000$ psi and $\gamma = 0.60$.

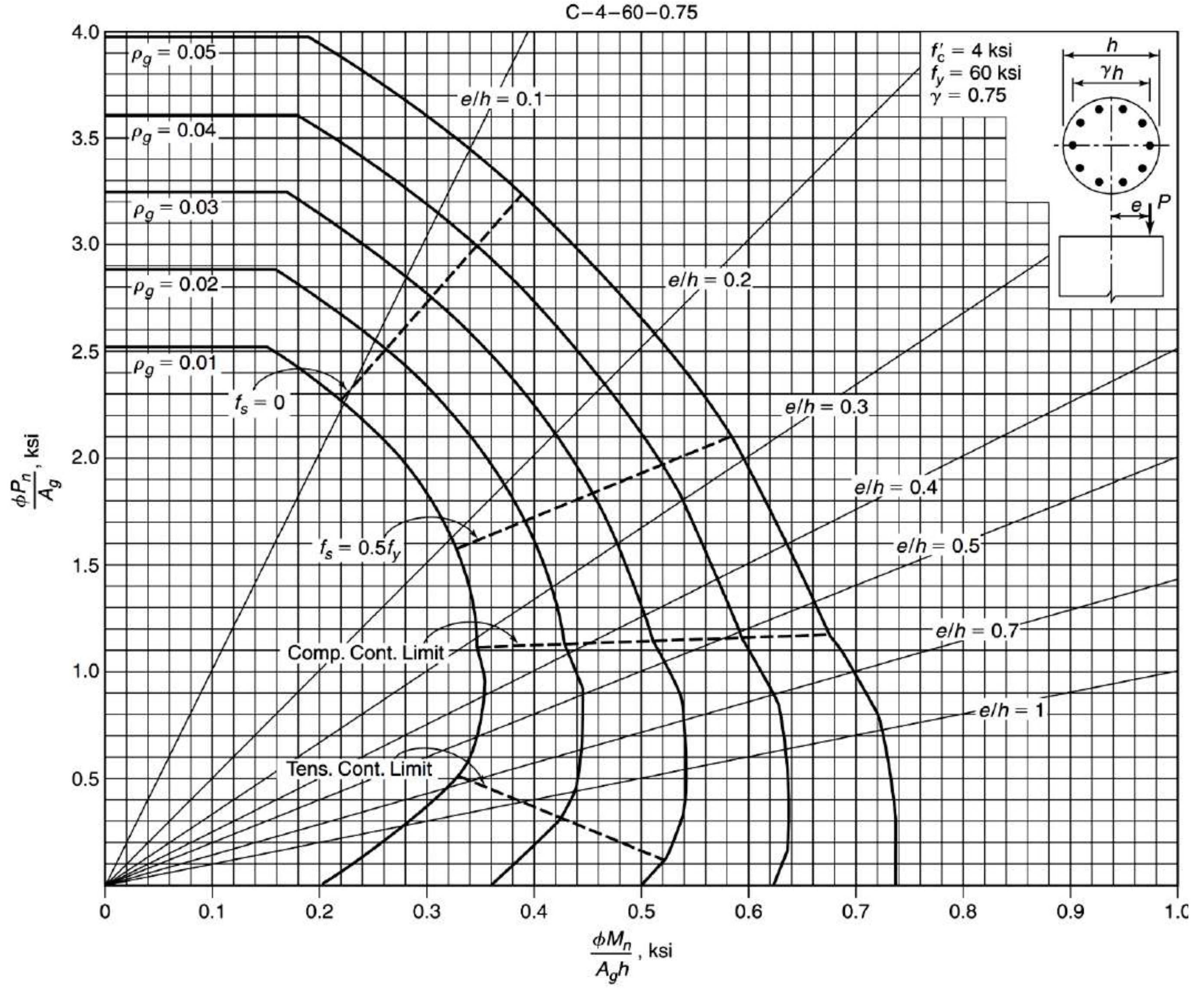



Fig. A-12b
Nondimensional interaction diagram for circular spiral column: $f'_c = 4000$ psi and $\gamma = 0.75$.

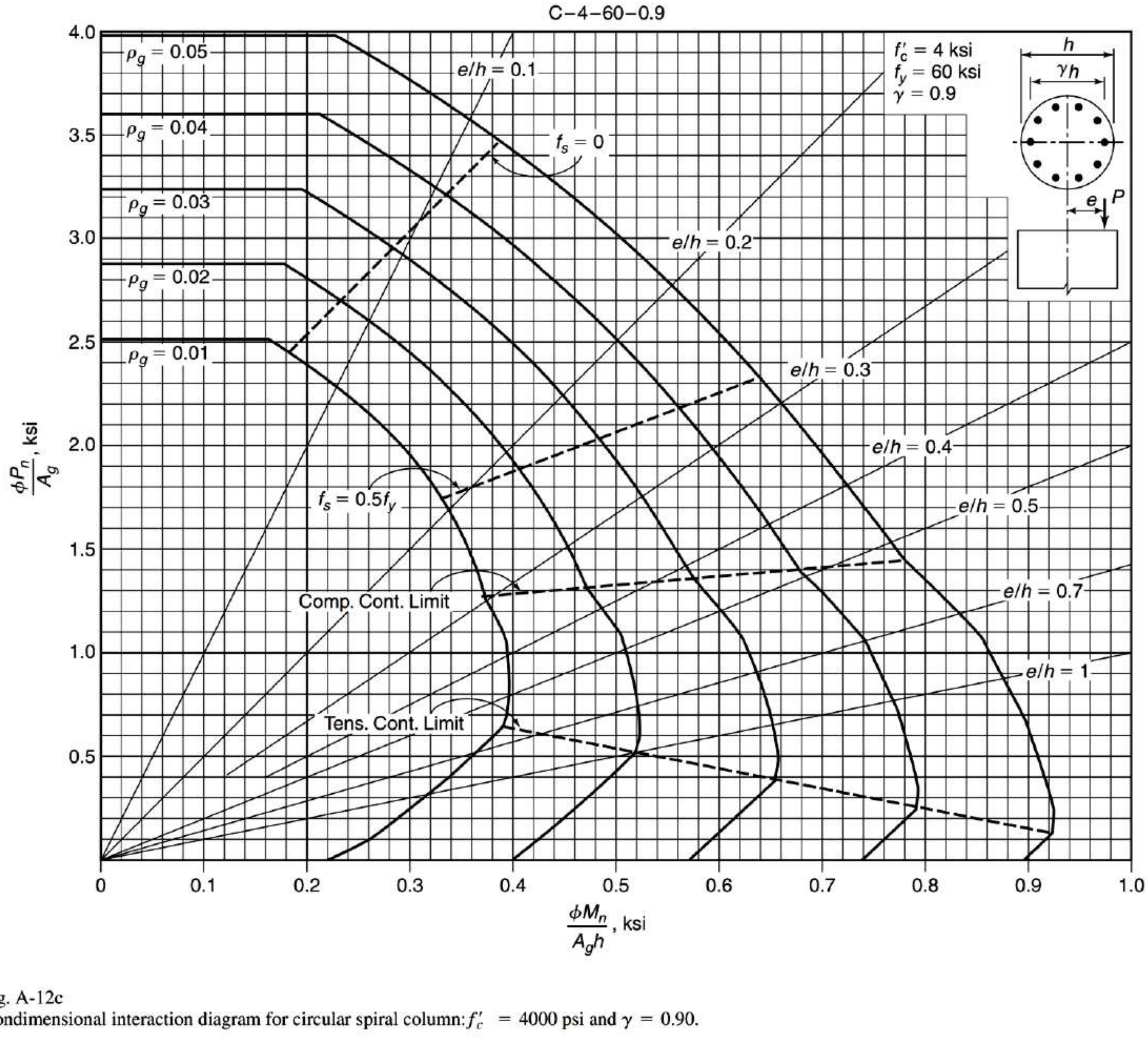



Fig. A-12c
Nondimensional interaction diagram for circular spiral column: $f'_c = 4000$ psi and $\gamma = 0.90$.

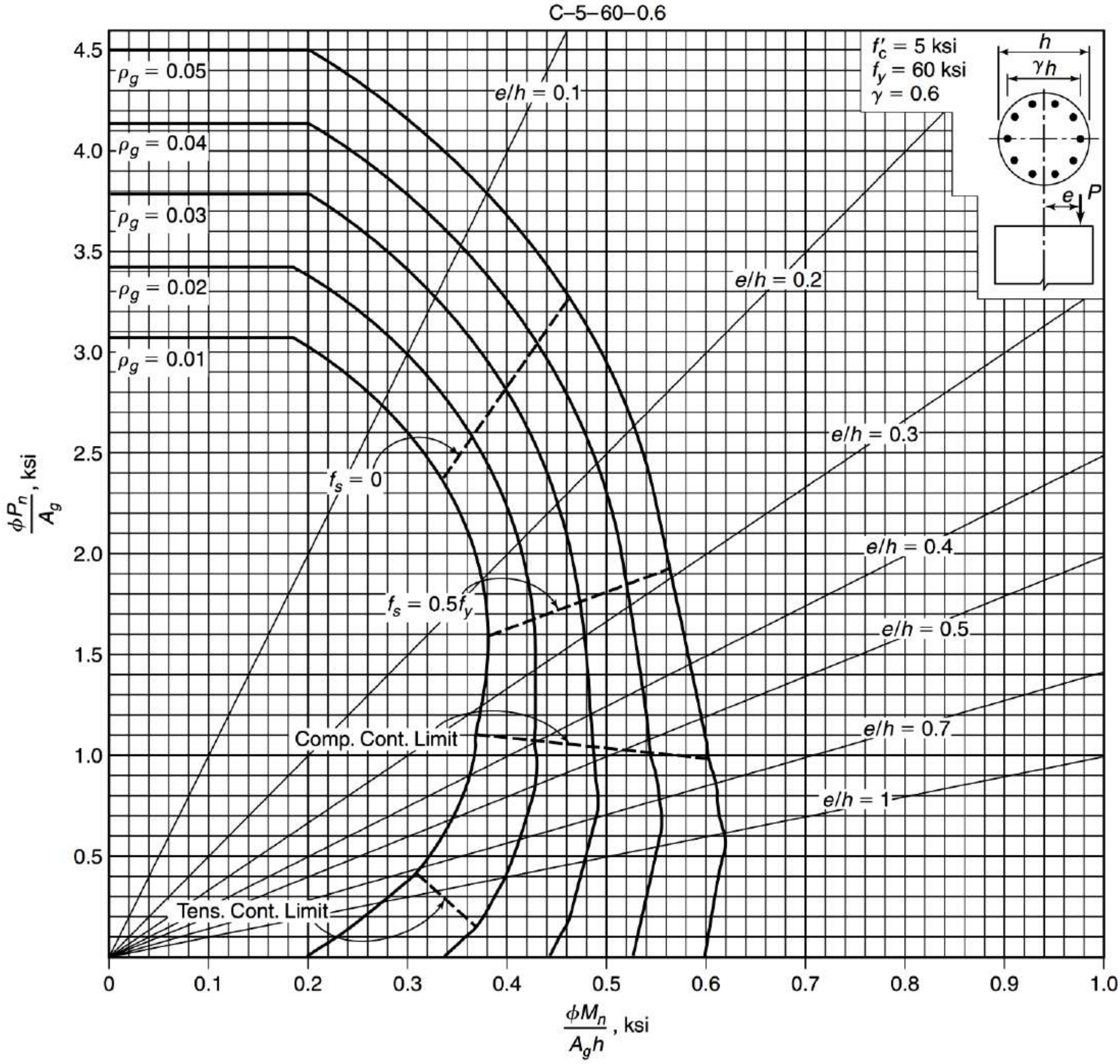



Fig. A-13a
Nondimensional interaction diagram for circular spiral column: $f'_c$ = 5000 psi and $\gamma$ = 0.60.

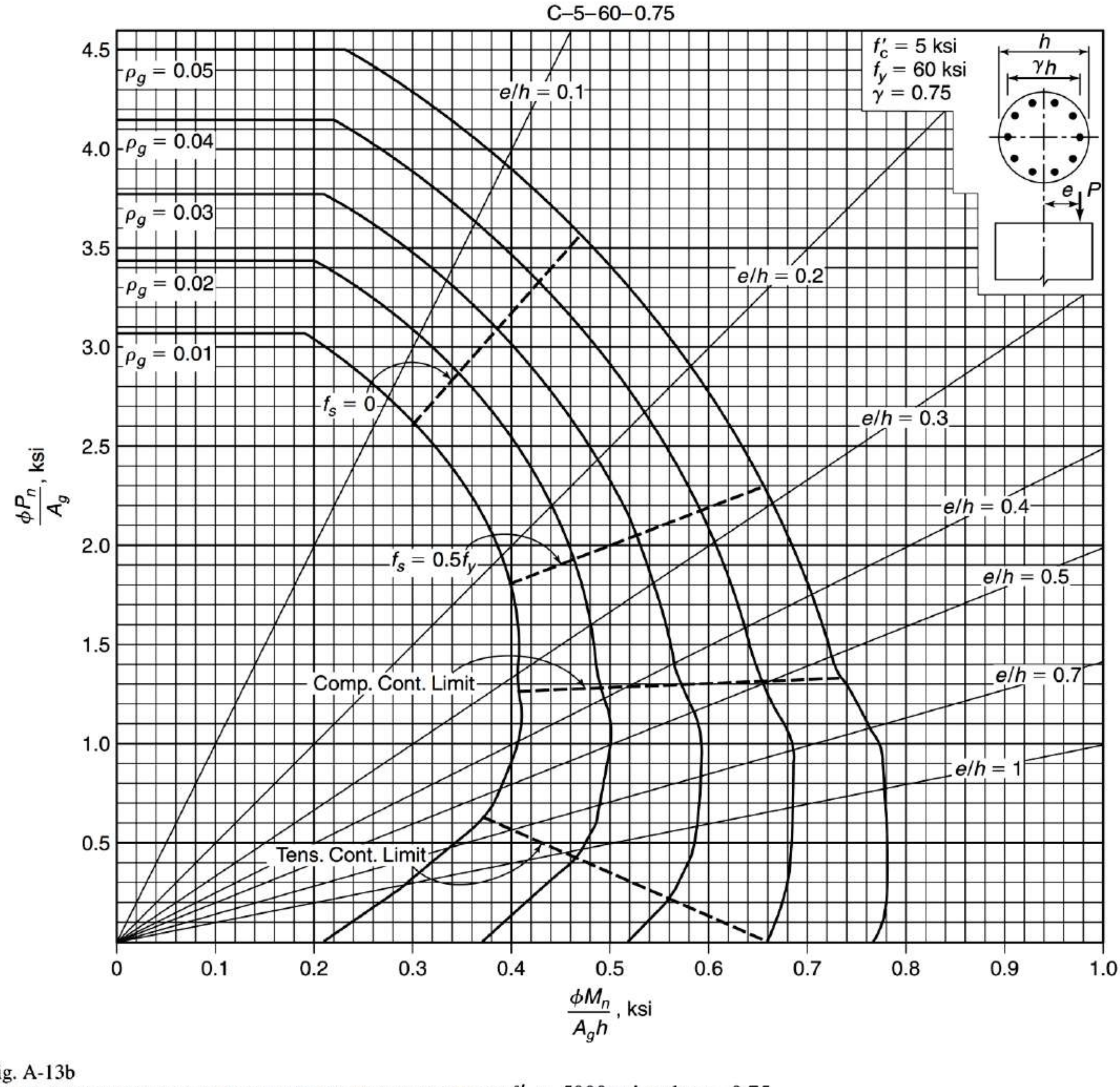



Fig. A-13b
Nondimensional interaction diagram for circular spiral column: $f'_c$ = 5000 psi and $\gamma$ = 0.75.

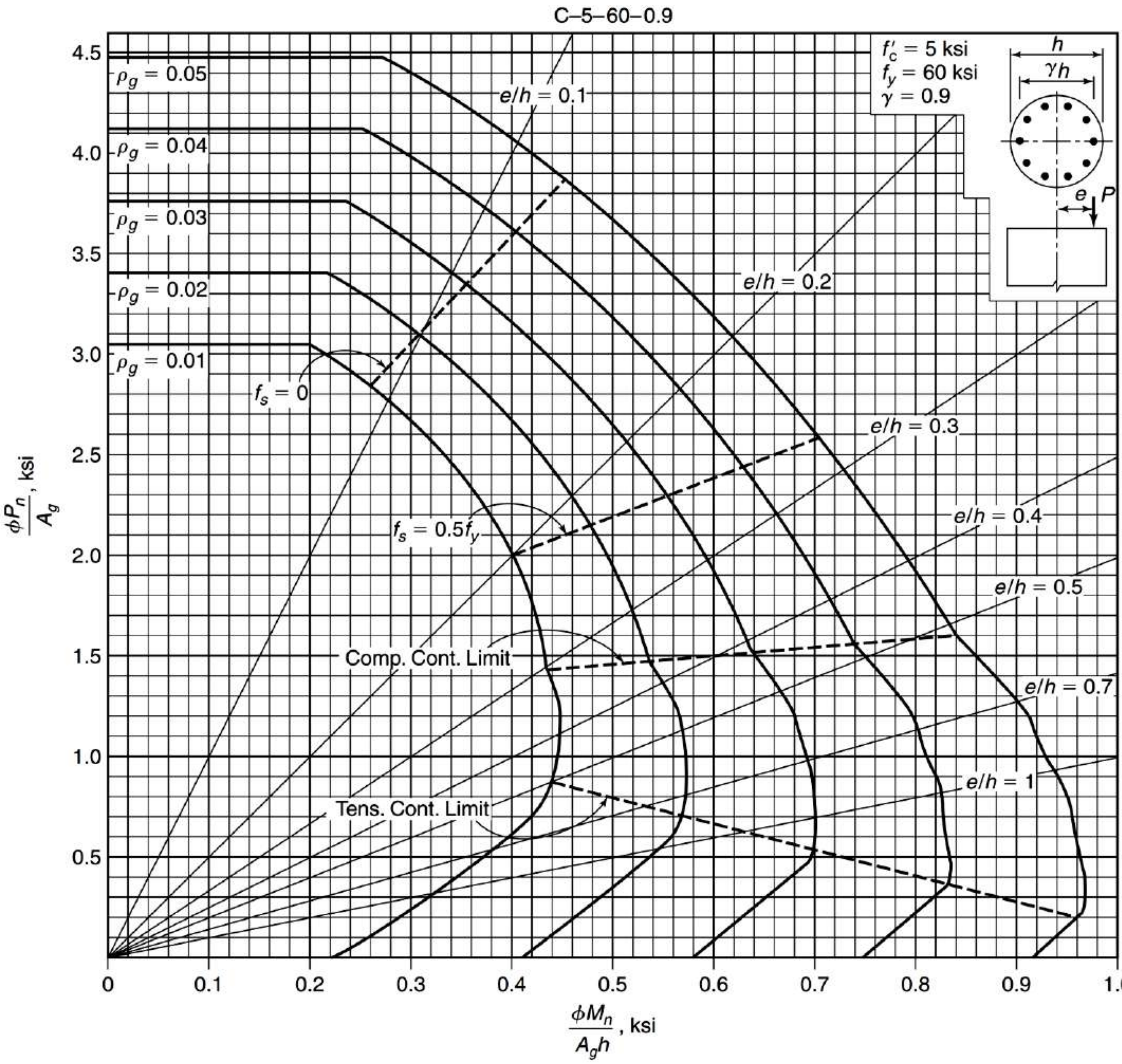



Fig. A-13c

Nondimensional interaction diagram for circular spiral column: $f'_c$ = 5000 psi and $\gamma$ = 0.90.

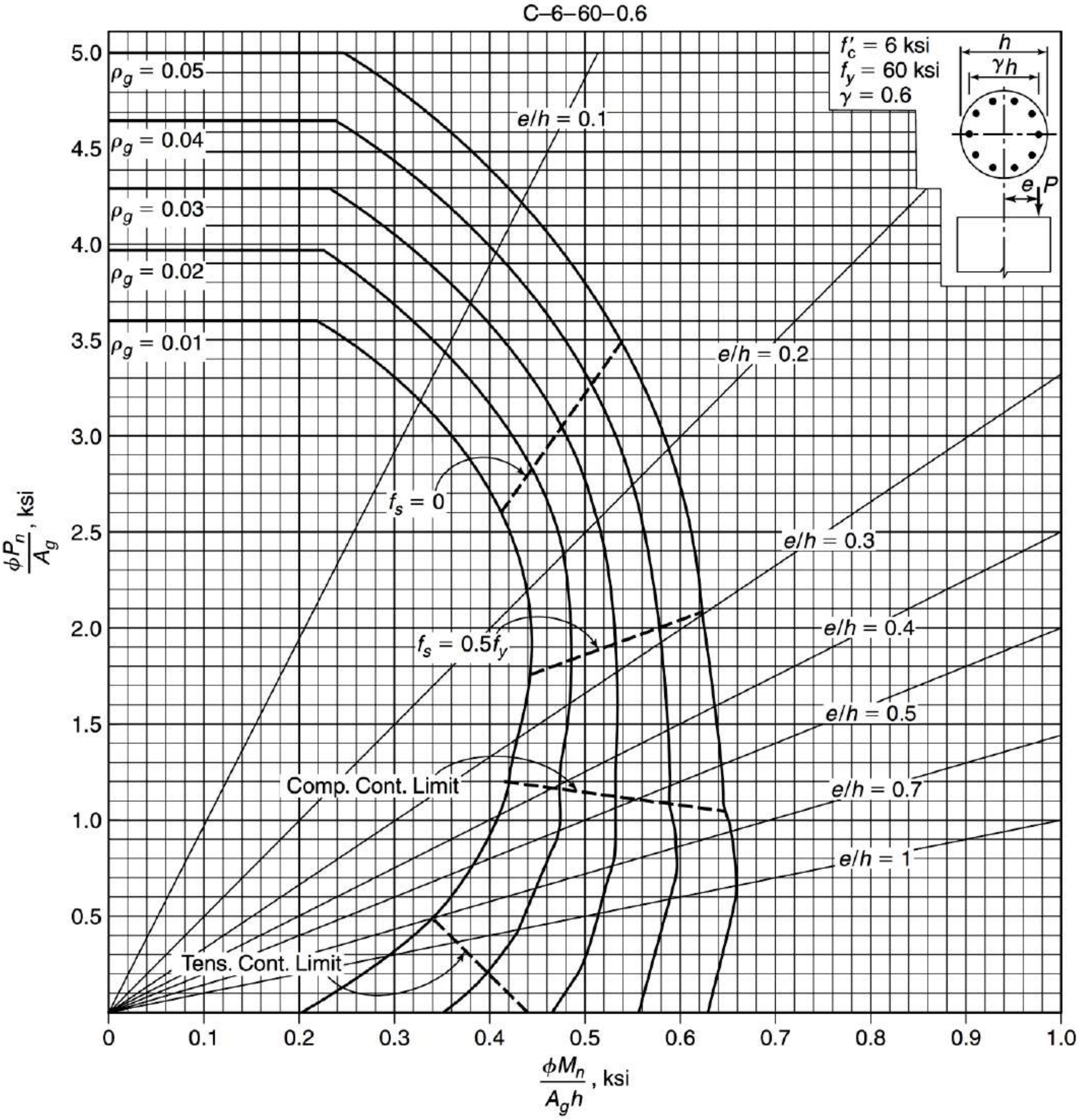



Fig. A-14a
Nondimensional interaction diagram for circular spiral column: $f'_c = 6000$ psi and $\gamma = 0.60$.

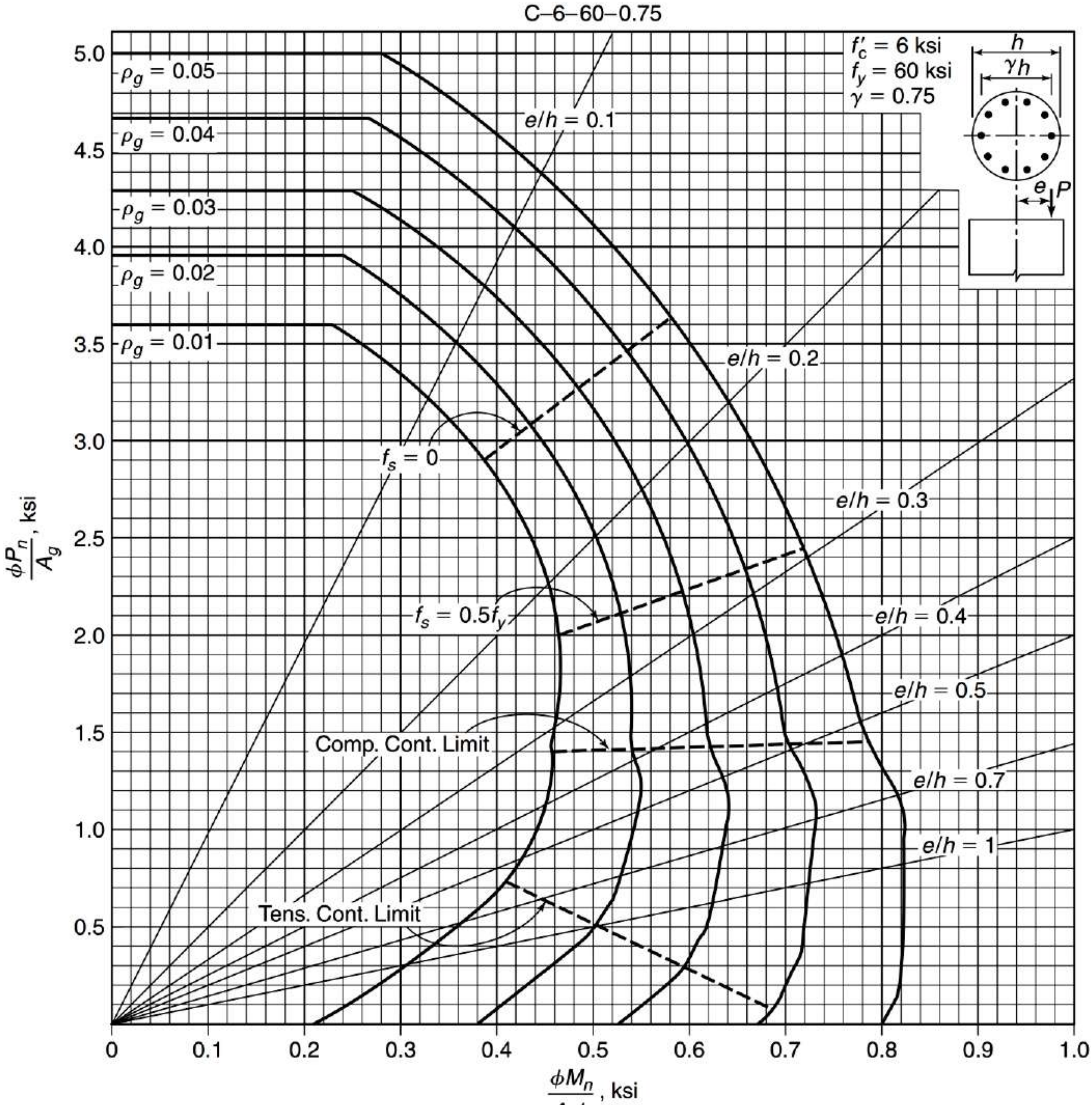



Fig. A-14b
Nondimensional interaction diagram for circular spiral column: $f'_c = 6000$ psi and $\gamma = 0.75$.

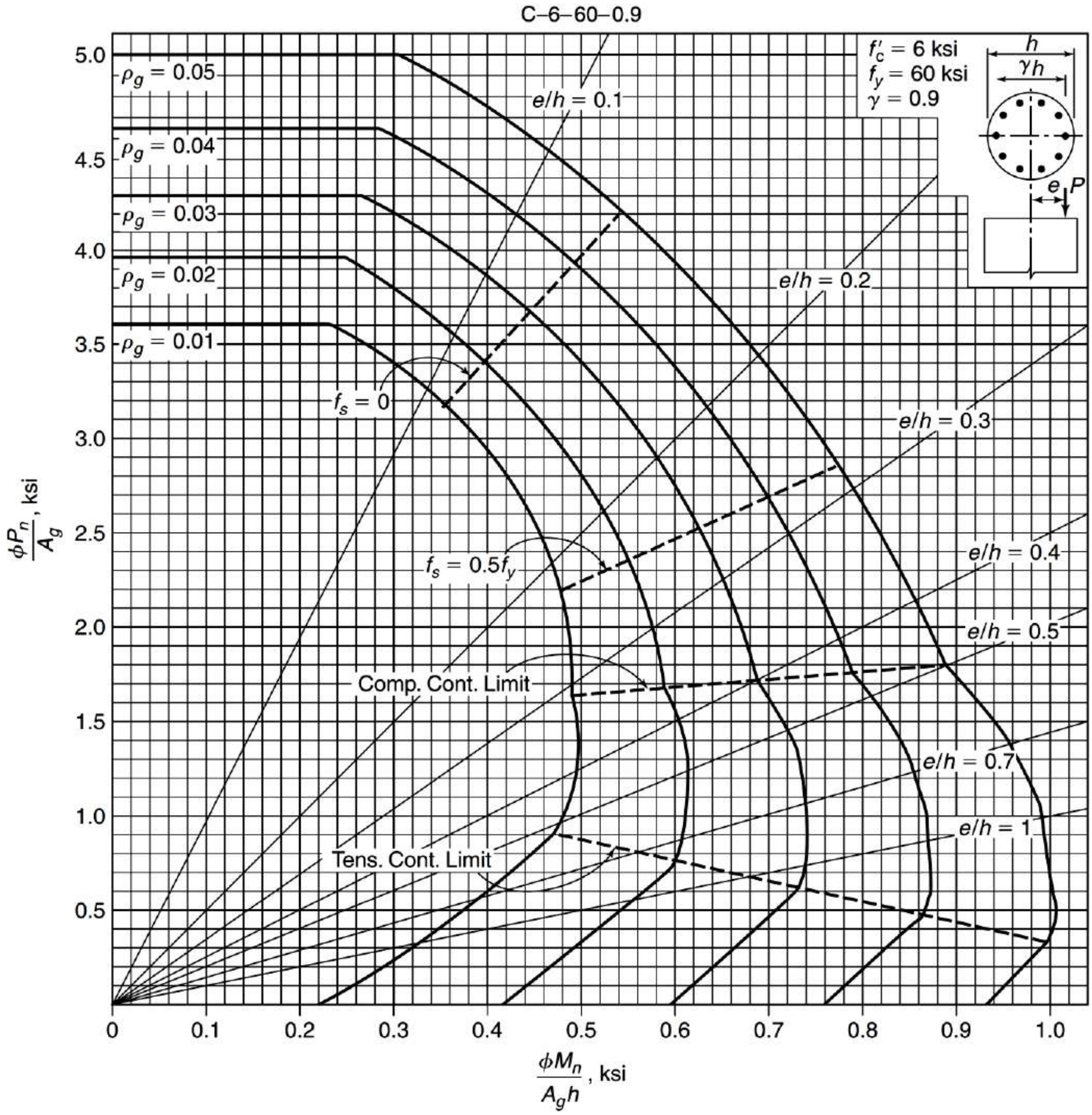



Fig. A-14c
Nondimensional interaction diagram for circular spiral column: $f'_c$ = 6000 psi and $\gamma$ = 0.90.

The design of eccentrically loaded columns using the strain compatibility method of analysis described requires that a trial column be selected. The trial column is then investigated to determine if it is adequate to carry any combination of $P_u$ and $M_u$ that may act on it should the structure be overloaded.

While a simple computer program or spreadsheet can be developed, based on the strain compatibility analysis, to calculate points on the design strength curve, and even to plot the curve, for any trial column, in practice design aids are used such as are available in handbooks and special volumes published by the American Concrete Institute, and the Concrete Reinforcing Steel Institute,. They cover the most frequent practical cases, such as symmetrically reinforced rectangular and square columns and circular spirally reinforced columns. There are also a number of commercially available computer programs (e.g., pcaCOLUMN, PROKON, CSI COLUMN … etc.).

Graphs A.6 through A.14 (pages 106-132) are representative of column design charts, in this case for concrete with $f_c' = (4-6)\ ksi$ and steel with yield strength $f_y = 60\ ksi$, for varying cover distances. Graphs A.6 through A.8 are drawn for rectangular columns with reinforcement along two opposite faces. Graphs A.9 through A. 11 are for rectangular columns with reinforcement distributed around the column perimeter. Circular columns with bars in a circular pattern are shown in Graphs A. 12 through A. 14.

Instead of plotting $P_n$ versus $M_n$, corresponding parameters have been used to make the charts more generally applicable, i.e., load is plotted as

$$\frac{\phi P_n}{A_g},$$

while moment is expressed as

$$\frac{\phi M_n}{A_g h}.$$

Families of curves are drawn for various values of $\rho_g = A_{st}/A_g$ between 0.01 and 0.08. The graphs also include radial lines representing different eccentricity ratios $e/h$, as well as lines representing different ratios of stress $f_s/f_y$ or values of strain $\varepsilon_t = 0.002$ (compression-controlled limit), and $\varepsilon_t = 0.005$ (tension-controlled limit) in the extreme tension steel.

Charts such as these permit the direct design of eccentrically loaded columns throughout the common range of strength and geometric variables. They may be used in one of two ways as follows. For a given factored load $P_u$ and equivalent eccentricity $e = M_u/P_u$:

1. Select trial cross-sectional dimensions $b$ and $h$.
2. Calculate the ratio $\gamma$ based on required cover distances to the bar centroids, and select the corresponding column design chart.
3. Calculate $\phi P_n/A_g$ and $\phi M_n/A_g h$, where $A_g = bh$ (for rectangular section)
4. From the graph, for the values found in (3), read the required reinforcement ratio $\rho_g$.
5. Calculate the total steel area $A_{st} = \rho_g A_g$.
6. Select the reinforcement ratio $\rho_g$.
7. Choose a trial value of $h$ and calculate $e/h$ and $\gamma$ .
8. From the corresponding graph, read $\phi P_n/A_g$ and calculate the required $A_g$.
9. Calculate $b = A_g/h$.

10. Revise the trial value of $h$ if necessary to obtain a well-proportioned section.
11. Calculate the total steel area $A_{st} = \rho_g A_g$.

**Important Note:**

The interaction diagrams are drawn for material's strengths $f_c'$ and $f_y$ in $\boldsymbol{ksi}$ unit, whereas the material's strengths are usually used in Palestine in $\boldsymbol{MPa}$**.**

To convert from $\boldsymbol{MPa}$ *to* $\boldsymbol{ksi}$**:** $\boldsymbol{1\ MPa = 0.145\ ksi}$

**Example**

Design a rectangular tied column with bars in two faces to support the following loads:

$$P_D = 450\ KN, \qquad P_L = 500\ KN,$$
$$M_D = 80\ KN \cdot m, \qquad M_L = 108\ KN \cdot m$$

**Solution:**

$$P_u = 1.2P_D + 1.6P_L = 1.2 \cdot 450 + 1.6 \cdot 500 = 1340\ KN$$
$$M_u = 1.2M_D + 1.6M_L = 1.2 \cdot 80 + 1.6 \cdot 108 = 268.8\ KN \cdot m$$

1. Select the material properties, trial size, and trial reinforcement ratio. Select $f_y = 60\ ksi$, and $f_c' = 4\ ksi$ . The most economical range for $\rho_g$ is (1-2)%. Assume that $\rho_g = 0.015$ for the first trial value:

$$f_c' = 4\ ksi = 4 \cdot \frac{1}{0.145} = 27.6\ MPa$$
$$f_y = 60\ ksi = 60 \cdot \frac{1}{0.145} = 413.8\ MPa$$
$$P_u = \phi\ 0.8\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right], \qquad \phi = 0.65$$

From where

$$A_g = \frac{P_u}{\phi\ 0.8\left[0.85 f_c'\left(1-\rho_g\right) + \rho_g f_y\right]} = \frac{1340 \cdot 10^3}{0.65 \cdot 0.8[0.85 \cdot 27.6\ (1-0.015) + 0.015 \cdot 413.8]} =$$
$$= 87904.3\ mm^2$$

Assume $A_g = bh,\ h = 500\ mm$

$$then \quad b = \frac{A_g}{h} = \frac{87904.3}{500} = 175.8\ mm.$$

Choose $h = 500\ mm, \quad and \quad b = 200\ mm$

**Instead of first trial value for $A_g$, its acceptable to assume directly one dimension such as $h = 500\ mm$ and continue with step 2 and 3 to determine the width $b$ .**

2. Compute the ratio $e/h$:

$$e = \frac{M_u}{P_u} = \frac{268.8}{1340} = 0.2\ m$$
$$\frac{e}{h} = \frac{0.2}{0.5} = 0.4$$

$\gamma$ – the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume $\varnothing$ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{500 - 2 \cdot 40 - 2 \cdot 10 - 25}{500} = 0.75$$

The interaction diagrams are given for $\gamma = 0.6$, $\gamma = 0.75$, and $\gamma = 0.9$. when $\gamma$ is different from these values, then it will be necessary to interpolate. Also, because the diagrams only can be read with limited accuracy, it is recommended to express $\gamma$ with only two significant figures.

3. Use interaction diagram A-6b to determine $\rho_g$. The interaction diagrams are entered with

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{1340 \cdot 10^3}{500 \cdot 200} \times 0.145 = 1.94$$

$$\frac{\phi M_n}{A_g h} = \frac{M_u}{A_g h} = \frac{268.8 \cdot 10^6}{500 \cdot 200 \cdot 500} \times 0.145 = 0.78$$

From the interaction diagram $\rho_g > 0.05$ which is not economic. Change the dimensions of the column. From the interaction diagram, try $\rho_g = 0.02$ then

$$\frac{\phi P_n}{A_g} = 1.27 = \frac{P_u}{A_g} = \frac{1340 \cdot 10^3}{500 \cdot b} 0.145 \quad \Longrightarrow \quad b = 306 \, mm$$

Take column dimensions: $h = 500 \, mm, \quad b = 350 \, mm$

4. Use interaction diagram A-6b to determine $\rho_g$ for the selected dimensions: $h = 500 \, mm, \quad b = 350 \, mm$. The interaction diagrams are entered with

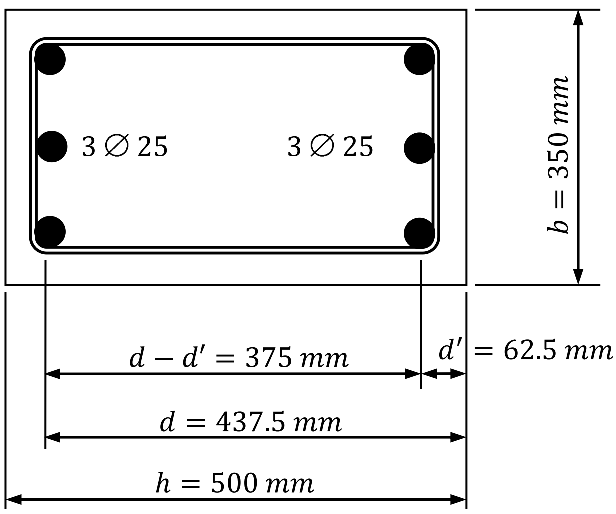
$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{1340 \cdot 10^3}{500 \cdot 350} \times 0.145 = 1.11$$

$$\rho_g = 0.013 > \rho_{min} = 0.01 \; - OK$$

5. Select the reinforcement:

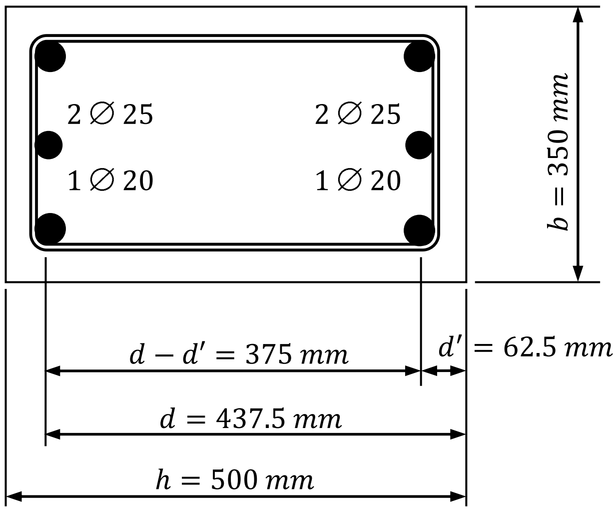
$$A_{st} = \rho_g A_g = 0.013 \cdot 500 \cdot 350 = 2275 \, mm^2$$

Take 6 ∅ 25 with $A_s = 2945.2 \, mm^2 > A_{st} = 2275 \, mm^2$ , three bars in each side.



**OR**

Take $4 \varnothing 25 + 2 \varnothing 20$ with $A_s = 1963.5 + 628.3 = 2591.8\ mm^2 > A_{st} = 2275\ mm^2$, three bars in each side.




**OR**

Take $6 \varnothing 22$ with $A_s = 2280.8\ mm^2 > A_{st} = 2275\ mm^2$, three bars in each side.

**OR**

Take $8 \varnothing 20$ with $A_s = 2513.3\ mm^2 > A_{st} = 2275\ mm^2$, four bars in each side.

**Note** that in last two combination of bars ($6 \varnothing 22$) or ($8 \varnothing 20$) the ratio $\gamma$ will be different when using these bars arrangement, it will be greater than 0.75, which gives less $\rho_g$.

**Check for spaces between bars.**

**Example**

Design a rectangular tied column with bars in four faces to support the following loads :

$$P_D = 450\ KN, \qquad P_L = 500\ KN,$$
$$M_D = 80\ KN \cdot m, \qquad M_L = 108\ KN \cdot m$$

**<u>Solution:</u>**

$$P_u = 1.2P_D + 1.6P_L = 1.2 \cdot 450 + 1.6 \cdot 500 = 1340\ KN$$

$$M_u = 1.2M_D + 1.6M_L = 1.2 \cdot 80 + 1.6 \cdot 108 = 268.8\ KN \cdot m$$

1. Select the material properties, trial size. Select $f_y = 60\ ksi$, and $f_c' = 4\ ksi$ .

Assume $h = 500\ mm$.

2. Compute the ratio $e/h$:

$$e=\frac{M_u}{P_u}=\frac{268.8}{1340}=0.2\ m$$

$$\frac{e}{h}=\frac{0.2}{0.5}=0.4$$

$\gamma -$ the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume $\varnothing$ 25 for bars.

$$\gamma=\frac{d-d'}{h}=\frac{500-2\cdot 40-2\cdot 10-25}{500}=0.75$$

3. Use interaction diagram A-9b to determine $\rho_g$.

From the interaction diagram, try $\rho_g=0.02$ then

$$\frac{\phi P_n}{A_g}=1.12=\frac{P_u}{A_g}=\frac{1340\cdot 10^3}{500\cdot b}0.145 \quad \Rightarrow \quad b=347\ mm$$

Take column dimensions: $h=500\ mm, \quad b=350\ mm$

4. Use interaction diagram A-9b to determine $\rho_g$ for the selected dimensions : $h=500\ mm, \quad b=350\ mm.$ The interaction diagrams are entered with

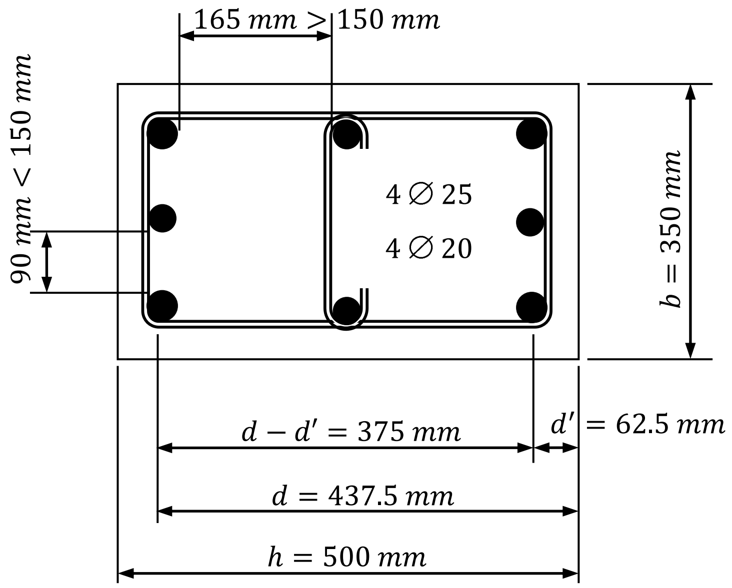
$$\frac{\phi P_n}{A_g}=\frac{P_u}{A_g}=\frac{1340\cdot 10^3}{500\cdot 350}\times 0.145=1.11$$

$$\rho_g=0.018>\rho_{min}=0.01\ -OK$$

5. Select the reinforcement:

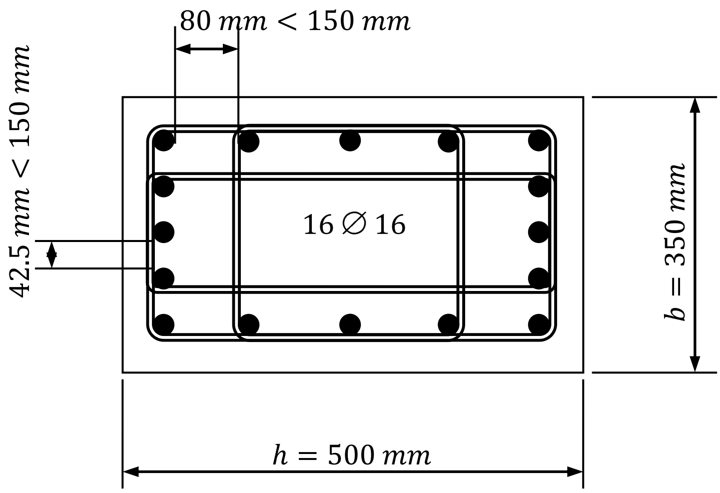
$$A_{st}=\rho_g A_g=0.018\cdot 500\cdot 350=3150\ mm^2$$

Take $4\varnothing 25+4\varnothing 20$ with $A_s=1963.4+1256.6\ mm^2=3220\ mm^2>A_{st}=3150\ mm^2$.



**OR**

Take 16 ∅ 16 with $A_s = 3217\ mm^2 > A_{st} = 3150\ mm^2$.




**Note** that in last combination of bars (16 ∅ 16) the ratio γ will be different when using this bars arrangement, it will be greater than 0.75, which gives less $\rho_g$.

6. Try another section taking $h = 500, \quad b = 400\ mm.$
   Use interaction diagram A-9b to determine $\rho_g$ for the selected dimensions. The interaction diagrams are entered with

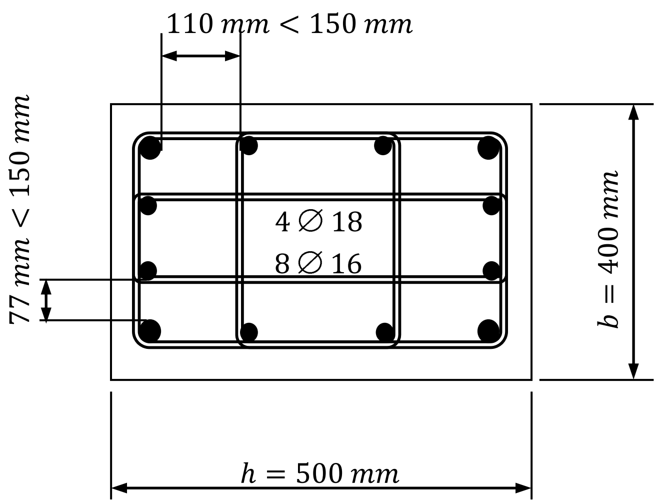
$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{1340 \cdot 10^3}{500 \cdot 400} \times 0.145 = 0.97$$

$$\rho_g = 0.013 > \rho_{min} = 0.01 \ - OK$$

7. Select the reinforcement:

$$A_{st} = \rho_g A_g = 0.013 \cdot 500 \cdot 400 = 2600\ mm^2$$

Take 4 ∅ 18 + 4 ∅ 16 with $A_s = 1017.8 + 1608.4\ mm^2 = 2626\ mm^2 > A_{st} = 2600\ mm^2$.



**Example**

Design a rectangular tied column with bars in four faces to support the following loads:

$$P_D = 750\ KN, \qquad P_L = 1000\ KN,$$
$$M_D = 100\ KN \cdot m, \qquad M_L = 150\ KN \cdot m$$

Take $f_y = 414\ MPa \approx 60\ ksi$, $\quad f_c' = 28\ MPa \approx 4\ ksi$

The dimension $h = 650\ mm$.

**Solution:**

$$P_u = 1.2P_D + 1.6P_L = 1.2 \cdot 750 + 1.6 \cdot 1000 = 2500\ KN$$
$$M_u = 1.2M_D + 1.6M_L = 1.2 \cdot 100 + 1.6 \cdot 150 = 360\ KN \cdot m$$

1. Compute the ratio $e/h$:

$$e = \frac{M_u}{P_u} = \frac{360}{2500} = 0.144\ m$$
$$\frac{e}{h} = \frac{144}{650} = 0.22$$

To construct the $\frac{e}{h}$ line, take value 0.22 on $\frac{\phi M_n}{bh^2}$ axis and value 1.0 on $\frac{\phi P_n}{bh}$ axis.

$\gamma$ – the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume $\varnothing$ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{650 - 2 \cdot 40 - 2 \cdot 10 - 25}{650} = 0.81$$

Because the interaction diagrams are given for $\gamma = 0.75$ and $\gamma = 0.9$ it will be necessary to interpolate. Also, because the diagrams only can be read with limited accuracy, it is recommended to express with only two significant figures.

2. Use interaction diagrams trying $\rho_g = 0.02$:

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 1.7$$

Diagram A-9c (for $\gamma = 0.9$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 1.8$$

Use linear interpolation to compute the value for $\gamma = 0.81$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.81) = 1.7 + \left(\frac{1.8 - 1.7}{0.9 - 0.75}\right)(0.81 - 0.75) = 1.74$$

$$Or \qquad \frac{\phi P_n}{A_g}\ for\ (\gamma = 0.81) = 1.8 - \left(\frac{1.8 - 1.7}{0.9 - 0.75}\right)(0.9 - 0.81) = 1.74$$

$$\frac{\phi P_n}{A_g} = 1.74 = \frac{P_u}{A_g} = \frac{2500 \cdot 10^3}{650 \cdot b} 0.145 \quad \Rightarrow \quad b = 321\ mm$$

Take column dimensions: $h = 650\ mm$, $\quad b = 300\ mm$

3. Use interaction diagram A-9b and A-9c to determine $\rho_g$ for the selected dimensions : $h = 650\ mm, \quad b = 300\ mm.$

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{2500 \cdot 10^3}{650 \cdot 300} \times 0.145 = 1.86$$

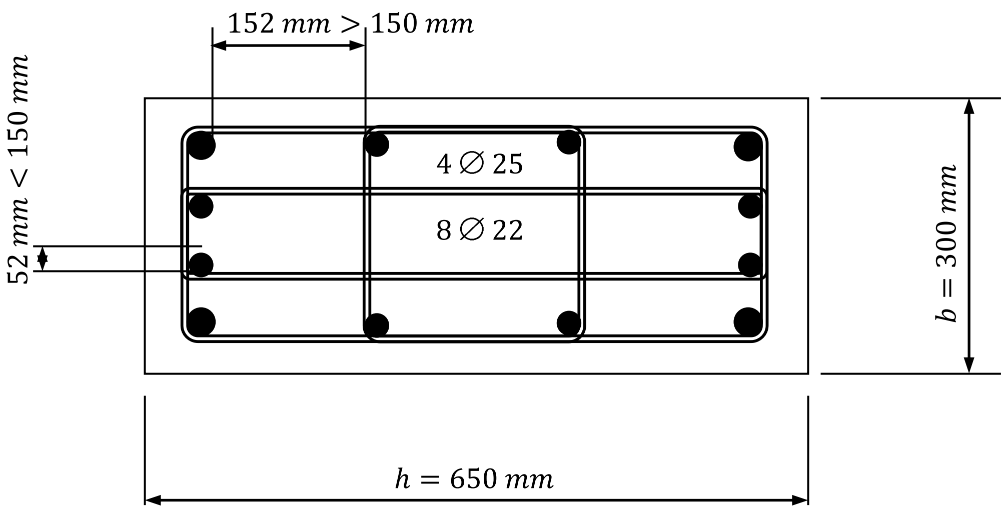
Diagram A-9b (for $\gamma = 0.75$ ), $\rho_g = 0.027$
Diagram A-9c (for $\gamma = 0.9$ ), $\rho_g = 0.022$

$$\rho_g\ (\gamma = 0.81\ ) = 0.027 - \left(\frac{0.027 - 0.022}{0.9 - 0.75}\right)(0.81 - 0.75) = 0.025 > \rho_{min} = 0.01\ - OK$$

4. Select the reinforcement:

$$A_{st} = \rho_g A_g = 0.025 \cdot 650 \cdot 300 = 4875\ mm^2$$

Take $4 \varnothing 25 + 8 \varnothing 22$ with $A_s = 1963.5 + 3041.1 = 5004.6\ \ mm^2 > A_{st} = 4875\ mm^2$.




**Example**

Design a circular spiral column section to support the following loads:

$$P_u = 3000\ KN$$
$$M_u = 360\ KN \cdot m$$

**Solution:**

1. Try $D = 600\ mm$
2. Compute the ratio $e/h$:

$$e = \frac{M_u}{P_u} = \frac{360}{3000} = 0.12\ m$$
$$\frac{e}{h} = \frac{120}{600} = 0.2$$

γ − the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume ∅ 25 for bars.

$$\gamma = \frac{d-d'}{h} = \frac{600-2\cdot 40-2\cdot 10-25}{600} = 0.79$$

Because the interaction diagrams are given for $\gamma = 0.75$ and $\gamma = 0.9$ it will be necessary to interpolate.

3. Use interaction diagram A-12b and A-12c to determine $\rho_g$ for the selected diameter: $D = h = 600\ mm$.

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{3000\cdot 10^3}{\frac{\pi 600^2}{4}} \times 0.145 = 1.54$$

$$\frac{\phi M_n}{A_g h} = \frac{M_u}{A_g h} = \frac{360\cdot 10^6}{\frac{\pi 600^2}{4} 600} \times 0.145 = 0.31$$

Diagram A-12b (for $\gamma = 0.75$ ), $\rho_g < 0.01 - Not\ OK$

Diagram A-12c (for $\gamma = 0.9$ ), $\rho_g < 0.01 - Not\ OK$.

4. Try $D = h = 550\ mm$

$$\frac{e}{h} = \frac{120}{550} = 0.22$$

To construct the $\frac{e}{h}$ line, take value 0.22 on $\frac{\phi M_n}{bh^2}$ axis and value 1.0 on $\frac{\phi P_n}{bh}$ axis.

$$\gamma = \frac{d-d'}{h} = \frac{550-2\cdot 40-2\cdot 10-25}{550} = 0.77$$

Because the interaction diagrams are given for $\gamma = 0.75$ and $\gamma = 0.9$ it will be necessary to interpolate.

5. Use interaction diagram A-12b and A-12c to determine $\rho_g$ for the selected diameter: $D = h = 550\ mm$.

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{3000\cdot 10^3}{\frac{\pi 550^2}{4}} \times 0.145 = 1.83$$

$$\frac{\phi M_n}{A_g h} = \frac{M_u}{A_g h} = \frac{360\cdot 10^6}{\frac{\pi 550^2}{4} 550} \times 0.145 = 0.4$$

Diagram A-12b (for $\gamma = 0.75$ ), $\rho_g = 0.0232 > 0.01 - OK$.

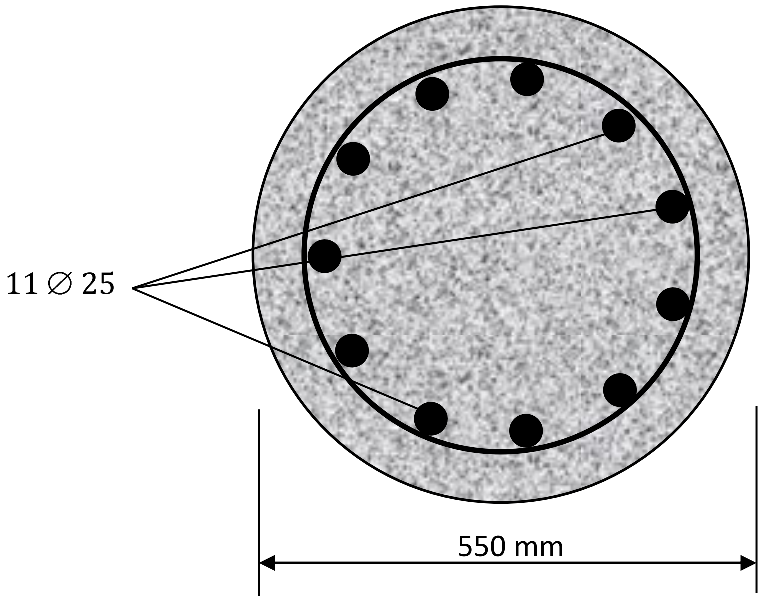
Diagram A-12c (for $\gamma = 0.9$ ), $\rho_g = 0.0185 > 0.01 - OK$.

$$\rho_g\ (\gamma = 0.77) = 0.0232 - \left(\frac{0.0232-0.0185}{0.9-0.75}\right)(0.77-0.75) = 0.0226 > \rho_{min} - OK$$

6. Select the reinforcement:

$$A_{st} = \rho_g A_g = 0.0226 \cdot \frac{\pi 550^2}{4} = 5369.4\ mm^2$$

Take 11 ∅ 25 with $A_s = 5400\ mm^2 > A_{st} = 5369.4\ mm^2$.




For all previous examples, the ties design and the check for ACI requirements must be provided as in examples for concentrically loaded columns (pages 81-84):

- Design of Ties:

Use ties $\varnothing 10$ with spacing of ties shall not exceed the smallest of:

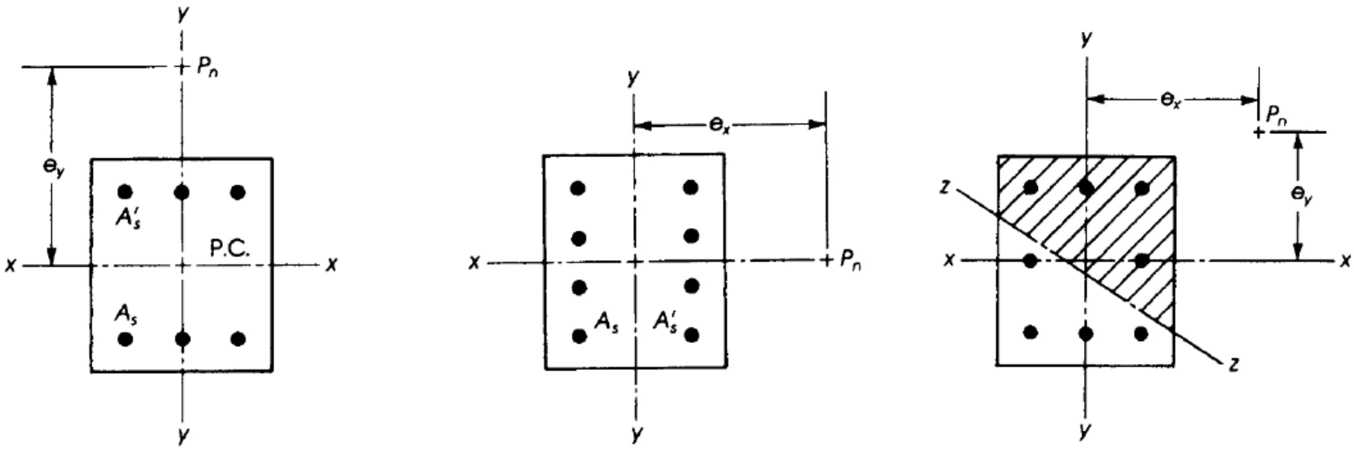
1. 48 times the tie diameter,
2. 16 times the longitudinal bar diameter,
3. the least dimension of the column.

- Check for code requirements:

1. Clear spacing between longitudinal bars:
2. Gross reinforcement ratio, $0.01 < \rho_g < 0.08$
3. Number of bars.
4. Minimum tie diameter.
5. Spacing of ties.
6. Arrangement of ties.

**6.11 BIAXIALLY LOADED COLUMNS**.

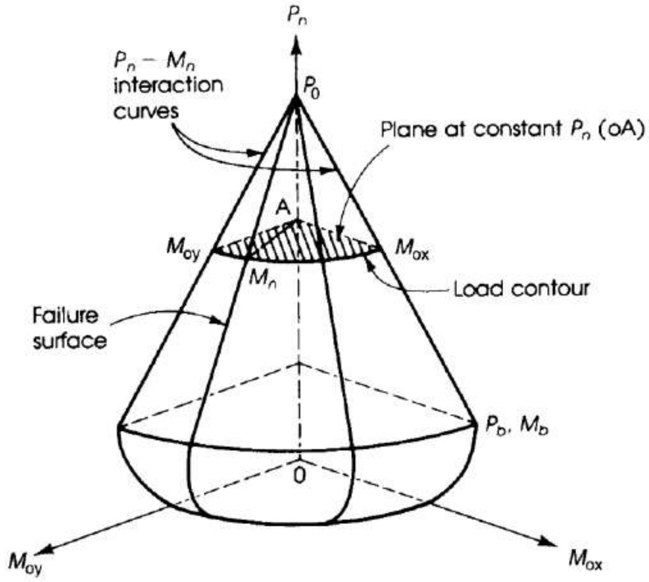
The analysis and design of columns under eccentric loading was discussed earlier in this chapter, considering a uniaxial case. This means that the load $P_n$ was acting along the $y-axis$, causing a combination of axial load $P_n$ and a moment about the $x-axis$ equal to $M_{nx}=P_n e_y$ or acting along the $x-axis$ with an eccentricity $e_x$, causing a combination of an axial load $P_n$ and a moment $M_{ny}=P_n e_x$.




Uniaxial bending with load $P_n$ along the $y$-axis with eccentricity $e_y$. Uniaxial bending with load $P_n$ along the $x$-axis, with eccentricity $e_x$. Biaxial bending.

If the load $P_n$ is acting anywhere such that its distance from the $x-axis$ is $e_y$ and its distance from the $y-axis$ is $e_x$, then the column section will be subjected to a combination of forces: an axial load $P_n$ and a moment about the $x-axis=M_{nx}=P_n e_y$ and a moment about the $y-axis=M_{ny}=P_n e_x$ .The column section in this case is said to be subjected to **biaxial bending**. The analysis and design of columns under this combination of forces is not simple when the principles of statics are used. The neutral axis is at an angle with respect to both axes, and lengthy calculations are needed to determine the location of the neutral axis, strains, concrete compression area, and internal forces and their point of application. Therefore, it was necessary to develop practical solutions to estimate the strength of columns under axial load and biaxial bending. The formulas developed relate the response of the column in biaxial bending to its uniaxial strength about each major axis.

The biaxial bending strength of an axially loaded column can be represented by a three-dimensional interaction curve, as shown in figure. The surface is formed by a series of uniaxial interaction curves drawn radially from the $P_n-axis$. The




Biaxial interaction surface.

curve $M_{ox}$ represents the interaction curve in uniaxial bending about the $x - axis$, and the curve $M_{oy}$ represents the curve in uniaxial bending about the $y - axis$. The plane at constant axial load $P_n$ represents the contour of the bending moment $M_n$ about any axis.
Different shapes of columns may be used to resist axial loads and biaxial bending. Circular, square, or rectangular column cross-sections may be used with equal or unequal bending capacities in the $x -$ and $y -$directions.

## 6.12 SQUARE AND RECTANGULAR COLUMNS UNDER BIAXIAL BENDING. BRESLER RECIPROCAL METHOD.

Square or rectangular columns with unequal bending moments about their major axes will require a different amount of reinforcement in each direction. An approximate method of analysis of such sections was developed by Boris Bresler and is called the **Bresler reciprocal method**. According to this method, the load capacity of the column under biaxial bending can be determined by using the following expression (**Bresler equation**):

$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_{uo}}$$

$or$

$$\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{no}}$$

where
$P_u -$ factored load under biaxial bending,
$P_{ux} -$ factored uniaxial load when the load acts at an eccentricity $e_y$ and $e_x = 0$,
$P_{uy} -$ factored uniaxial load when the load acts at an eccentricity $e_x$ and $e_y = 0$,
$P_{uo} -$ factored axial load when $e_x = e_y = 0$

$$P_n = \frac{P_u}{\phi}, \qquad P_{nx} = \frac{P_{ux}}{\phi}, \qquad P_{ny} = \frac{P_{uy}}{\phi}, \qquad P_{no} = \frac{P_{uo}}{\phi}$$

The uniaxial load strengths $P_{nx}$, $P_{ny}$, and $P_{no}$ can be calculated according to the equations and method given earlier in this chapter. After that, they are substituted into the above **Bresler equation** to calculate $P_n$. The Bresler equation is valid for all cases when $P_n$ is equal to or greater than $0.1P_{no}$. When $P_n$ is less than $0.1P_{no}$., the axial force may be neglected and the section can be designed as a member subjected to pure biaxial bending according to the following equations:

$$\frac{M_{ux}}{M_x} + \frac{M_{uy}}{M_y} \leq 1$$

$or$

$$\frac{M_{nx}}{M_{ox}} + \frac{M_{ny}}{M_{oy}} \leq 1$$

where

$M_{ux} = P_u e_y -$ design moment about the $x - axis$
$M_{uy} = P_u e_x -$ design moment about the $y - axis$
$M_x$ and $M_y -$ uniaxial moment strengths about the $x -$ and $y -$ axes

$$M_{nx} = \frac{M_{ux}}{\phi}, \qquad M_{ny} = \frac{M_{uy}}{\phi}, \qquad M_{ox} = \frac{M_x}{\phi}, \qquad M_{oy} = \frac{M_y}{\phi}$$

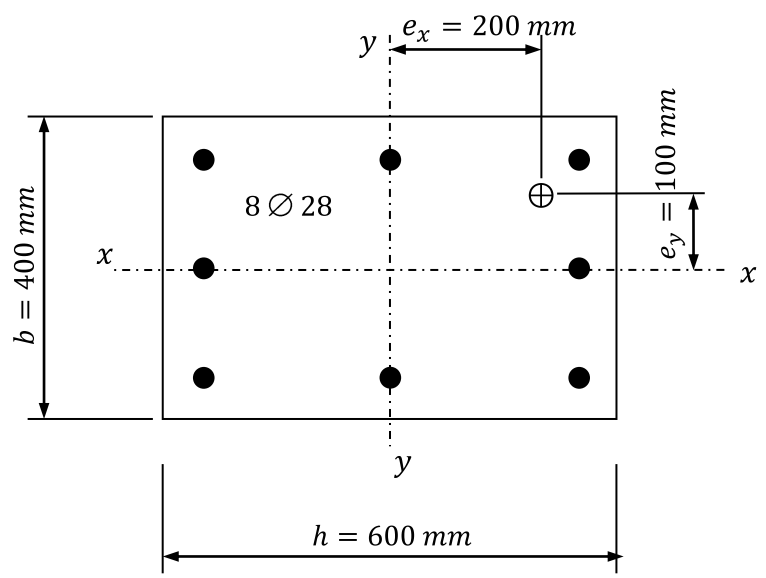
The Bresler equation is not recommended when the section is subjected to axial tension loads.

**Example**

Determine the nominal compressive strength $P_n$ of the short tied column, which is subjected to biaxial bending.

$$e_x = 200\ mm, \qquad \text{and} \qquad e_y = 100\ mm$$

Take $f_y = 414\ MPa \approx 60\ ksi,$ $\qquad f_c' = 28\ MPa \approx 4\ ksi$




**Solution:**

1. Calculate $\rho_g$:

$$\rho_g = \frac{A_{st}}{A_g} = \frac{4926}{600 \cdot 400} = 0.0205$$

2. For $x$ direction (bending about $y$ –axis):

- compute the ratio $e_x/h$:

$$\frac{e_x}{h} = \frac{200}{600} = 0.33$$

- compute the ratio γ:

$$\gamma = \frac{d - d'}{h} = \frac{600 - 2 \cdot 40 - 2 \cdot 10 - 28}{600} = 0.79$$

- draw line of $\frac{e_x}{h} = 0.33$ on diagram A-9b (for $\gamma = 0.75$ ) and diagram A-9c (for $\gamma = 0.9$).

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 1.32$$

Diagram A-9c (for $\gamma = 0.9$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 1.45$$

Use linear interpolation to compute the value for $\gamma = 0.79$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.79) = 1.32 + \left(\frac{1.45 - 1.32}{0.9 - 0.75}\right)(0.79 - 0.75) = 1.35\ \ ksi$$

$$\frac{\phi P_n}{A_g} = \frac{P_u}{600 \cdot 400} 0.145 = 1.35 \qquad \Longrightarrow\ P_{uy} = \frac{1.35 \cdot 600 \cdot 400}{0.145} 10^{-3} = 2234.5\ KN$$

$$P_{ny} = \frac{P_{uy}}{\phi} = \frac{2234.5}{0.65} = 3437.7\ KN$$

**Note** that $\phi = 0.65 -$ compression-controlled section (see interaction diagrams).

3. For $y$ direction (bending about $x$ −axis):

- compute the ratio $e_y/h$:

$$\frac{e_y}{h} = \frac{100}{400} = 0.25$$

- compute the ratio $\gamma$:

$$\gamma = \frac{d - d'}{h} = \frac{400 - 2 \cdot 40 - 2 \cdot 10 - 28}{400} = 0.68$$

- line of $\frac{e_y}{h} = 0.25$ on diagram A-9a (for $\gamma = 0.6$ ) and diagram A-9b (for $\gamma = 0.75$).

Diagram A-9a (for $\gamma = 0.6$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 1.49$$

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 1.6$$

Use linear interpolation to compute the value for $\gamma = 0.68$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.68) = 1.49 + \left(\frac{1.6 - 1.49}{0.75 - 0.6}\right)(0.68 - 0.6) = 1.55\ \ ksi$$

$$\frac{\phi P_n}{A_g} = \frac{P_u}{600 \cdot 400} 0.145 = 1.55 \qquad \Rightarrow\ P_{ux} = \frac{1.55 \cdot 600 \cdot 400}{0.145} 10^{-3} = 2565.5\, KN$$

$$P_{nx} = \frac{P_{ux}}{\phi} = \frac{2565.5}{0.65} = 3946.9\, KN$$

**Note** that $\phi = 0.65 -$ compression-controlled section (see interaction diagrams).

4. Determine $P_{no}$ for the section dimensions : $h = 600\, mm,\ \ b = 400\, mm,$ and $\rho_g = 0.0205$:

$$P_{no} = A_g\left[0.85 f_c'\left(1 - \rho_g\right) + \rho_g f_y\right] =$$

$$= 600 \cdot 400[0.85 \cdot 28(1 - 0.0205) + 0.0205 \cdot 414]10^{-3} = 7631.8\, KN$$

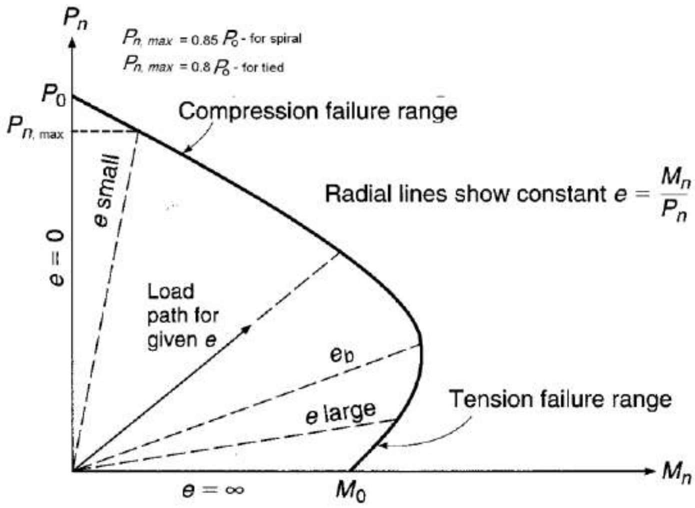
**Note** that $P_{no}$ can be defined directly from interaction diagrams when $e_x = e_y = 0$

From diagrams A-9 for any value of $\gamma$

$$\frac{\phi P_n}{A_g} = 3$$

$$P_{no} = 3\ \frac{A_g}{\phi} = \frac{3}{0.145} \times \frac{600 \cdot 400}{0.65} 10^{-3} =$$

$$= 7639\, KN$$




5. Substituting $P_{nx},\ P_{ny},\ P_{no}$ in Bresler equation:

$$\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{no}}$$

$$\frac{1}{P_n} = \frac{1}{3946.9} + \frac{1}{3437.7} - \frac{1}{7631.8} \qquad \Rightarrow \quad P_n = 2420\, KN$$

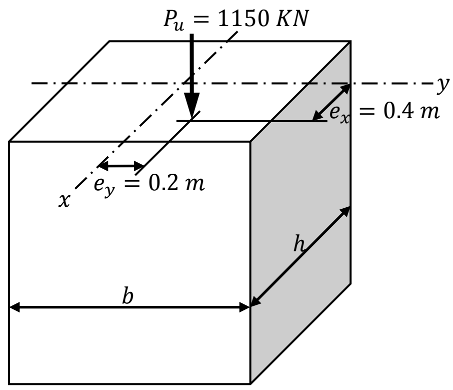
$$P_u = \phi P_n = 0.65 \cdot 2420 = 1573\, KN$$

**Example**

Design a rectangular tied column subjected to:

$$P_u = 1150\ KN \quad \text{with} \quad e_x = 0.4\ m, \text{ and } \quad e_y = 0.2\ m$$

Take $f_y = 414\ MPa \approx 60\ ksi, \qquad f_c' = 28\ MPa \approx 4\ ksi$




**Solution:**

1. Select trial size along $x-$ axis, $e_x = 0.4\ m$

Assume $h = 650\ mm$.

2. Compute the ratio $e/h$:

$$\frac{e_x}{h} = \frac{400}{650} = 0.62$$

Assume ∅ 25 for bars.

$$\gamma = \frac{d-d'}{h} = \frac{650 - 2\cdot 40 - 2\cdot 10 - 25}{650} = 0.81$$

3. Use interaction diagrams trying $\rho_g = 0.01$:

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.72$$

Diagram A-9c (for $\gamma = 0.9$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.78$$

Use linear interpolation to compute the value for $\gamma = 0.81$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.81) = 0.72 + \left(\frac{0.78 - 0.72}{0.9 - 0.75}\right)(0.81 - 0.75) = 0.744$$

$$\frac{\phi P_n}{A_g} = 0.744 = \frac{P_u}{A_g} = \frac{1150\cdot 10^3}{650\cdot b}0.145 \quad \Rightarrow \quad b = 344.8\ mm$$

Take column dimensions: $h = 650\ mm, \quad b = 400\ mm$ and $\rho_g = 0.01$

4. Use interaction diagram A-9 to determine $P_{ux}$ and $P_{uy}$ for the selected dimensions :

$h = 650\ mm, \quad b = 400\ mm,$ and $\rho_g = 0.01$:

- For $x$ direction $h = 650\ mm, \quad e_x = 400\ mm.$

$$\frac{e_x}{h} = \frac{400}{650} = 0.62, \qquad \gamma = 0.81$$

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.72$$

Diagram A-9c (for $\gamma = 0.9$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.78$$

Use linear interpolation to compute the value for $\gamma = 0.81$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.81) = 0.72 + \left(\frac{0.78 - 0.72}{0.9 - 0.75}\right)(0.81 - 0.75) = 0.744\ \ ksi$$

$$\frac{\phi P_n}{A_g} = 0.744\ , \qquad P_{uy} = \frac{0.744 \cdot 650 \cdot 400}{0.145} 10^{-3} = 1334.1\ KN$$

- For $y$ direction $b = 400\ mm,\ \ e_y = 200\ mm.$

$$\frac{e_y}{h} = \frac{200}{400} = 0.5$$

Assume $\varnothing$ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{400 - 2 \cdot 40 - 2 \cdot 10 - 25}{400} = 0.688$$

Diagram A-9a (for $\gamma = 0.6$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.71$$

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.83$$

Use linear interpolation to compute the value for $\gamma = 0.688$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.688) = 0.71 + \left(\frac{0.83 - 0.71}{0.75 - 0.6}\right)(0.688 - 0.6) = 0.78\ \ ksi$$

$$\frac{\phi P_n}{A_g} = 0.78\ , \qquad P_{ux} = \frac{0.78 \cdot 650 \cdot 400}{0.145} 10^{-3} = 1398.6\ KN$$

5. Determine $P_{uo}$ for the selected dimensions : $h = 650\ mm,\ \ b = 400\ mm,$ and $\rho_g = 0.01$:

$$P_{uo} = \phi P_{no} = \phi A_g\left[0.85 f_c'\left(1 - \rho_g\right) + \rho_g f_y\right] =$$

$$= 0.65 \cdot 650 \cdot 400[0.85 \cdot 28(1 - 0.01) + 0.01 \cdot 414]10^{-3} = 4681.6\ KN$$

6. Substituting $P_{ux},\ P_{uy},\ P_{uo}$ in Bresler equation:

$$\frac{1}{\phi P_n} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_{uo}}$$

$$\frac{1}{\phi P_n} = \frac{1}{1398.6} + \frac{1}{1334.1} - \frac{1}{4681.6}$$

$$\phi P_n = 799.4\ KN < P_u = 1150\ KN - Not\ OK.$$

7. Try new dimensions $h = 700\ mm, \quad b = 450\ mm$ and $\rho_g = 0.015$
8. Repeat the calculations using interaction diagram A-9 to determine $P_{ux}$ and $P_{uy}$ for the selected dimensions : $h = 700\ mm, \quad b = 450\ mm,$ and $\rho_g = 0.015$:

- For $x$ direction $h = 700\ mm, \quad e_x = 400\ mm.$

$$\frac{e_x}{h} = \frac{400}{700} = 0.57$$

Assume ∅ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{700 - 2 \cdot 40 - 2 \cdot 10 - 25}{700} = 0.82$$

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.83$$

Diagram A-9c (for $\gamma = 0.9$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.93$$

Use linear interpolation to compute the value for $\gamma = 0.82$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.82) = 0.83 + \left(\frac{0.93 - 0.83}{0.9 - 0.75}\right)(0.82 - 0.75) = 0.877\ \ ksi$$

$$\frac{\phi P_n}{A_g} = 0.877\ , \qquad P_{uy} = \frac{0.877 \cdot 700 \cdot 450}{0.145} 10^{-3} = 1905.2\ KN$$

- For $y$ direction $b = 450\ mm, \quad e_y = 200\ mm.$

$$\frac{e_y}{h} = \frac{200}{450} = 0.44$$

Assume ∅ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{450 - 2 \cdot 40 - 2 \cdot 10 - 25}{450} = 0.72$$

Diagram A-9a (for $\gamma = 0.6$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.85$$

Diagram A-9b (for $\gamma = 0.75$ ).

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = 0.98$$

Use linear interpolation to compute the value for $\gamma = 0.72$

$$\frac{\phi P_n}{A_g}\ for\ (\gamma = 0.72) = 0.85 + \left(\frac{0.98 - 0.85}{0.75 - 0.6}\right)(0.72 - 0.6) = 0.954\ \ ksi$$

$$\frac{\phi P_n}{A_g} = 0.954\,, \qquad P_{ux} = \frac{0.954 \cdot 700 \cdot 450}{0.145} 10^{-3} = 2072.5\ KN$$

9. Determine $P_{uo}$ for the selected dimensions : $h = 700\ mm$, $b = 450\ mm$, and $\rho_g = 0.015$:

$$P_{uo} = \phi P_{no} = \phi A_g\left[0.85 f_c'\left(1 - \rho_g\right) + \rho_g f_y\right] =$$

$$= 0.65 \cdot 700 \cdot 450[0.85 \cdot 28(1 - 0.015) + 0.015 \cdot 414]10^{-3} = 6071.5\ KN$$

10. Substituting $P_{ux}$, $P_{uy}$, $P_{uo}$ in Bresler equation:

$$\frac{1}{\phi P_n} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_{uo}}$$

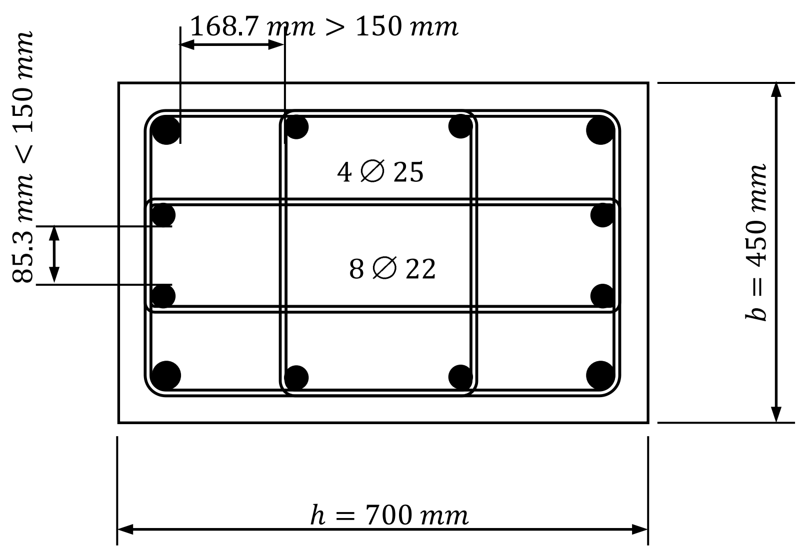
$$\frac{1}{\phi P_n} = \frac{1}{2072.5} + \frac{1}{1905.2} - \frac{1}{6071.5}$$

$$\phi P_n = 1186.7\ KN > P_u = 1150\ KN - OK.$$

11. Select the reinforcement:

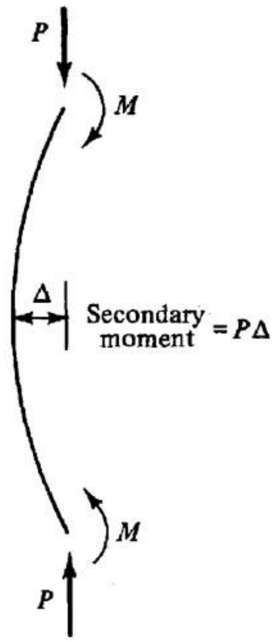
$$A_{st} = \rho_g A_g = 0.015 \cdot 700 \cdot 450 = 4725\ mm^2$$

Take 4 ∅ 25 + 8 ∅ 22 with $A_s = 1963.5 + 3041.1 = 5004.6\ \ mm^2 > A_{st} = 4725\ mm^2$



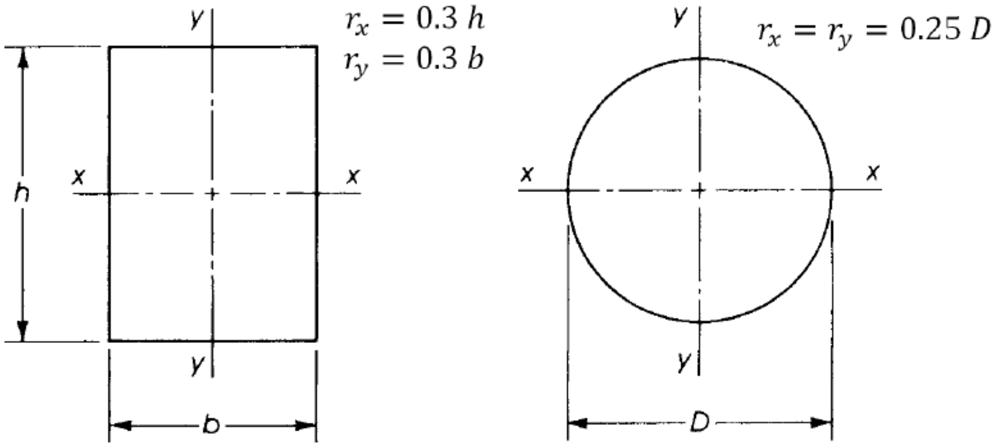
# CHAPTER 7 SLENDER COLUMNS

## 7.1 INTRODUCTION




When a column bends or deflects laterally an amount $\Delta$, its axial load will cause an increased column moment equal to $P\Delta$. This moment will be superimposed onto any moments already in the column. Should this $P\Delta$ moment be of such magnitude as to reduce the axial load capacity of the column significantly, the column will be referred to as a **slender column**.

A column is said to be slender if its cross-sectional dimensions are small compared with its length. The degree of slenderness is generally expressed in terms of the slenderness ratio $kl_u/r$, where $l_u$ is the unsupported length of the member and $r$ is the radius of gyration of its cross section, equal to $\sqrt{I/A}$. For square or circular members, the value of $r$ is the same about either axis; for other shapes $r$ is smallest about the minor principal axis, and it is generally this value that must be used in determining the slenderness ratio of a freestanding column.




Rectangular and circular sections of columns, with radius of gyration $r$.

For a rectangular section of width $b$ and depth $h$, $I_x = bh^3/12$ and $A = bh$. Therefore, $r_x = \sqrt{I/A} = 0.288\,h$ (or, approximately, $r_x = 0.3\,h$). Similarly, $I_y = hb^3/12$ and $r_y = 0.288\,b$ (or, approximately, $r_y = 0.3\,b$). For a circular column with diameter $D$, $I_x = I_y = \pi D^4/64$ and $A = \pi D^2/4$; therefore, $r_x = r_y = 0.25\,D$.

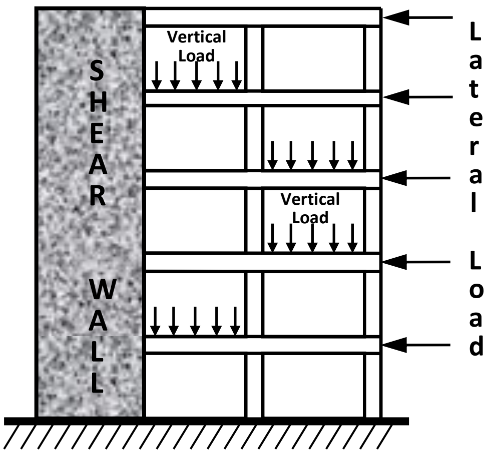
It has long been known that a member of great slenderness will collapse under a smaller compression load than a stocky member with the same cross-sectional dimensions. When a stocky member, say with $kl_u/r = 10$ (e.g., a square column of length equal to about 3 times its cross-sectional dimension $h$), is loaded in axial compression, it will fail at the load

$P_o$, $\left(P_o = 0.85f_c'A_n + A_{st}f_y\right)$, because at that load both concrete and steel are stressed to their maximum carrying capacity and give way, respectively, by crushing and by yielding. If a member with the same cross section has a slenderness ratio $kl_u/r = 100$ (e.g., a square column hinged at both ends and of length equal to about 30 times its section dimension), it may fail under an axial load equal to one-half or less of $P_o$. In this case, collapse is caused by buckling, i.e., by sudden lateral displacement of the member between its ends, with consequent over-stressing of steel and concrete by the bending stresses that are superimposed on the axial compressive stresses.

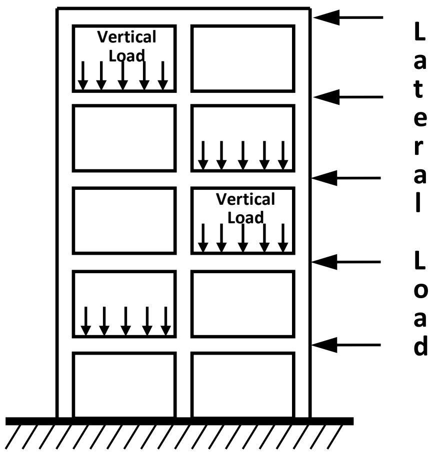
Most columns in practice are subjected to bending moments as well as axial loads, as was made clear in Chapter 6. These moments produce lateral deflection of a member between its ends and may also result in relative lateral displacement of joints. Associated with these lateral displacements are secondary moments that add to the primary moments and that may become very large for slender columns, leading to failure. A practical definition of a slender column is one for which there is a significant reduction in axial load capacity because of these secondary moments. In the development of ACI Code column provisions, for example, any reduction greater than about 5% is considered significant, requiring consideration of slenderness effects.

### 7.2 NONSWAY AND SWAY FRAMES.

For this discussion it is necessary to distinguish between frames without sidesway and those with sidesway. In the ACI Code these are referred to respectively as nonsway (braced) frames and sway (unbraced) frames.




Braced (Nonsway) Frame




Unbraced (sway) Frame

You must realize that you will rarely find a frame that is completely braced against swaying or one that is completely unbraced against swaying. Therefore, you are going to have to decide which way to handle it.

The question may possibly be resolved by examining the lateral stiffness of the bracing elements for the story in question. You may observe that a particular column is located in a story where there is such substantial lateral stiffness provided by bracing members, shear walls, shear trusses, and so on that any lateral deflections occurring will be too small to affect the strength of the column appreciably. You should realize while examining a particular structure that there may be some nonsway stories and some sway stories.

If we cannot tell by inspection whether we have a nonsway frame or a sway frame, the Code provides two ways of making a decision. First, in ACI Section 10.10.5.1, a story in a frame is said to be a nonsway one if the increase in column end moments due to second-order effects is 5% or less of the first-order end moments.

The second method presented by the Code for determining whether a particular frame is braced or unbraced is given in the Code (10.10.5.2). If the value of the so-called **stability index** which follows is $\leq 0.05$, the Commentary states that the frame may be classified as a nonsway one.

$$Q = \frac{\sum P_u \, \Delta_o}{V_{us} l_c} \leq 0.05$$

where

$\sum P_u$ – total factored vertical load for all of the columns on the story in question,

$\Delta_o$ – the elastically determined first-order lateral deflection due to $V_{us}$ at the top of the story in question with respect to the bottom of that story,

$V_{us}$ – the total factored horizontal shear for the story in question,

$l_c$ – the height of a compression member in a frame measured from center to center of the frame joints.

Despite these suggestions from the ACI, the individual designer is going to have to make decisions as to what is adequate bracing and what is not, depending on the presence of structural walls and other bracing items. For the average size reinforced concrete building, load eccentricities and slenderness values will be small and frames will be considered to be braced.

## 7.3 BUCKLING OF AXIALLY LOADED ELASTIC COLUMNS. THE EFFECTIVE COLUMN LENGTH.

The basic information on the behavior of straight, concentrically loaded slender columns was developed by Euler more than 200 years ago. In generalized form, it states that such a member will fail by buckling at the critical load

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

where

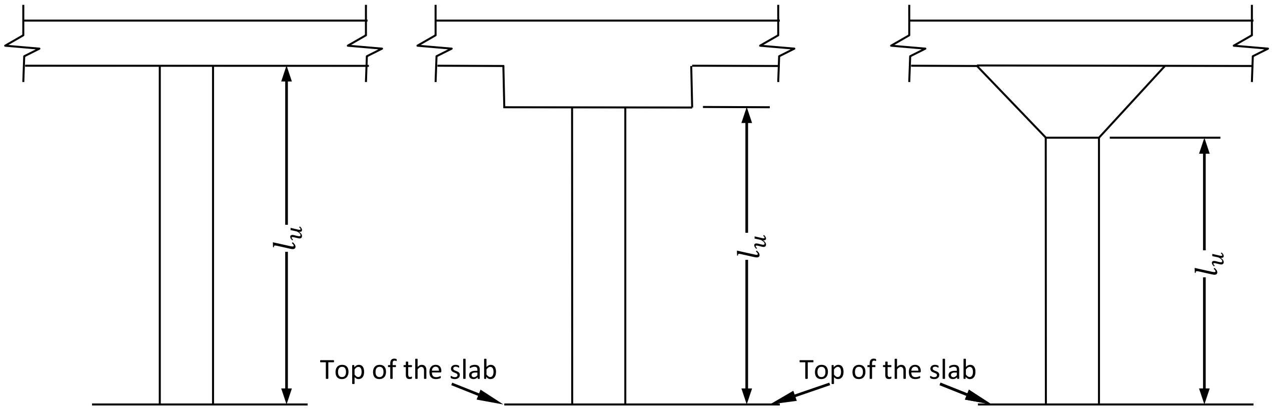
$EI$ – flexural rigidity of column cross section,

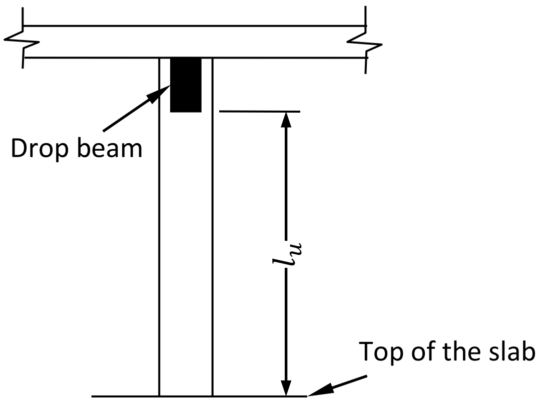
$(kl_u)$ – effective length of the column.

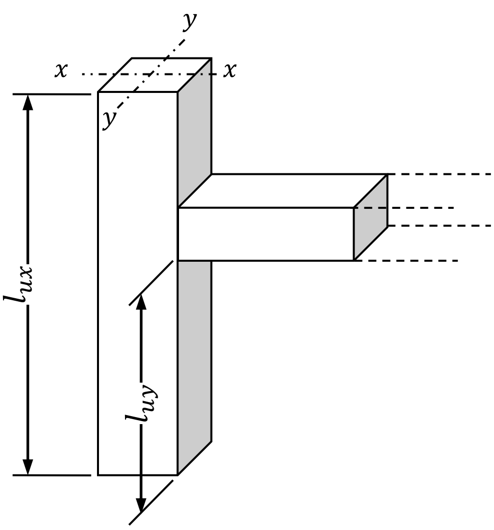
It is seen that the buckling load decreases rapidly with increasing slenderness ratio $\frac{kl_u}{r}$.

The slenderness of columns is based on their geometry and on their lateral bracing. As their slenderness increases, their bending stresses increase, and thus buckling may occur.

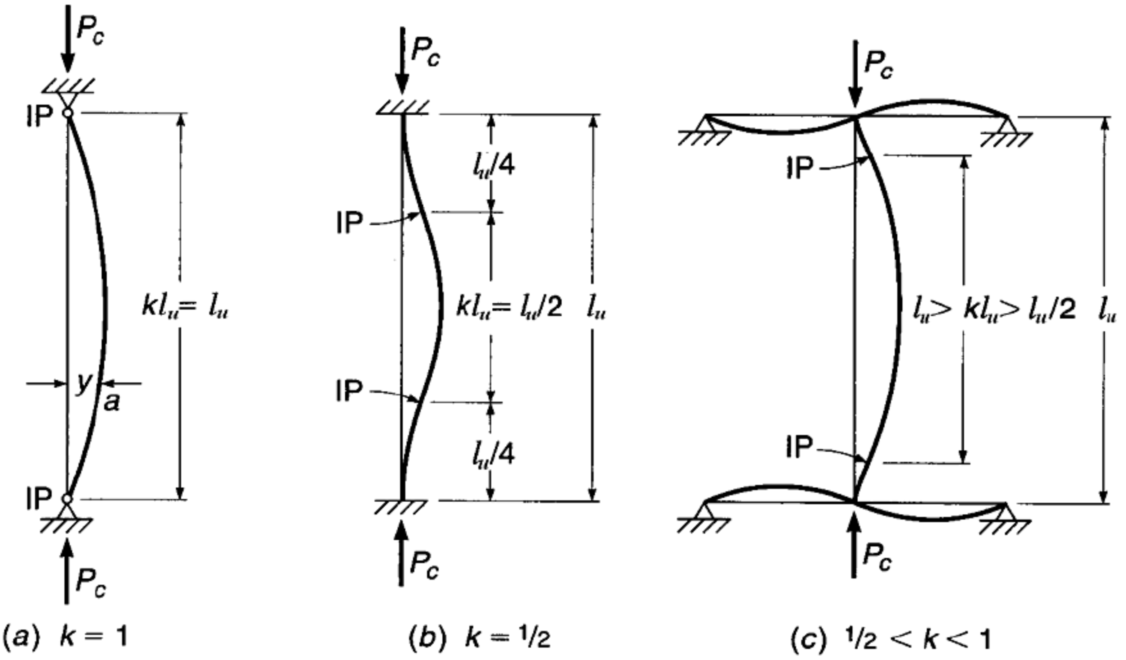
Reinforced concrete columns generally have small slenderness ratios. As a result, they can usually be designed as short columns without strength reductions due to slenderness.
The length used for calculating the slenderness ratio of a column, $l_u$, is its unsupported length. This length is considered to be equal to the clear distance between slabs, beams, or other members that provide lateral support to the column. If haunches or capitals are present, the clear distance is measured from the bottoms of the capitals or haunches.



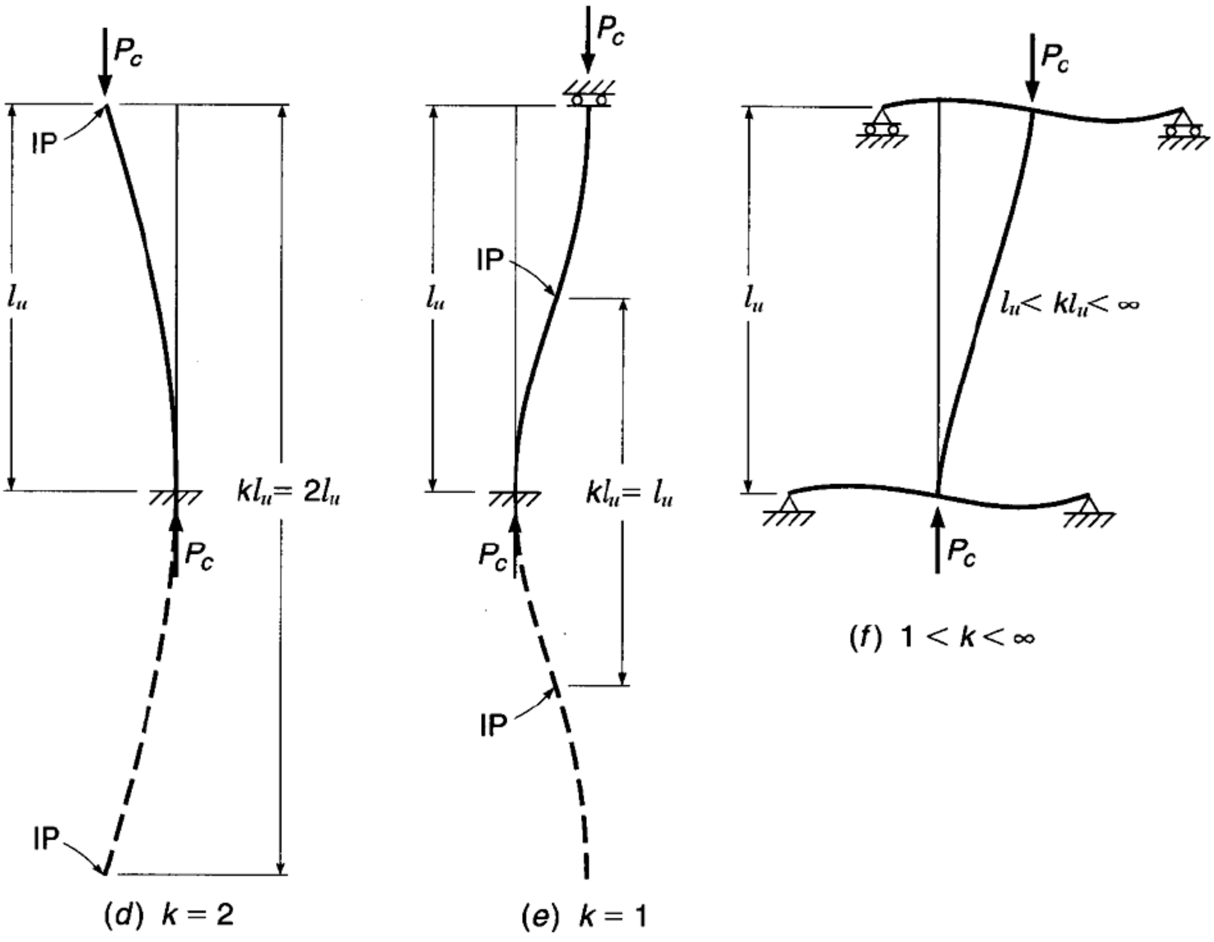




To calculate the slenderness ratio of a particular column, it is necessary to estimate its effective length. This is the distance between points of zero moment in the column (The inflection points - IP). For this initial discussion it is assumed that no sidesway or joint translation is possible. Sidesway or joint translation means that one or both ends of a column can move laterally with respect to each other.
If there were such a thing as a perfectly pinned end column, its effective length would be its supported length, as shown in Figure (a). The effective length factor $k$ is the number that must be multiplied by the column's unsupported length to obtain its effective length. For a perfectly pinned end column, $k = 1.0$.
Columns with different end conditions have entirely different effective lengths. For instance, if there were such a thing as a perfectly fixed end column, its points of inflection (or points of zero moment) would occur at its one-fourth points, and its effective length would be $l_u/2$, as shown in Figure (b). As a result, its $k$ value would equal $0.5$.




Buckling and effective length of columns in braced frames




Buckling and effective length of columns in unbraced frames

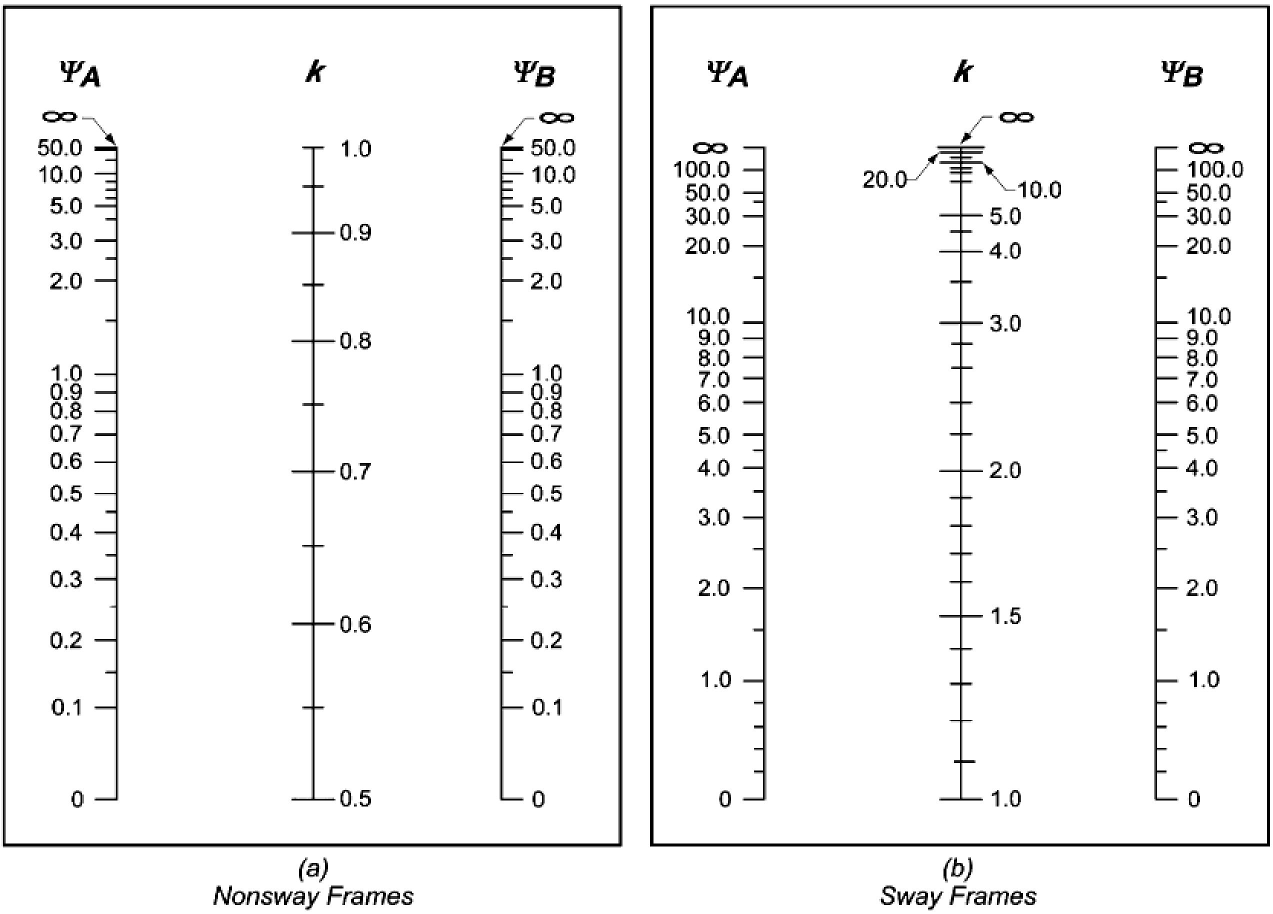
### 7.4 EFFECTIVE LENGTH FACTOR $-\ k$

The preliminary procedure used for estimating effective lengths involves the use of the alignment charts shown below. The chart of part (a) of the figure is applicable to braced frames, whereas the one of part (b) is applicable to unbraced frames.

To use the alignment charts for a particular column, $\psi$ factors are computed at each end of the column. The $\psi$ factor at one end of the column equals

$$\psi = \frac{\sum E_c I_c / l_c \ \ of\ columns\ at\ joint}{\sum E_b I_b / l_b \ \ of\ beams\ at\ joint}$$

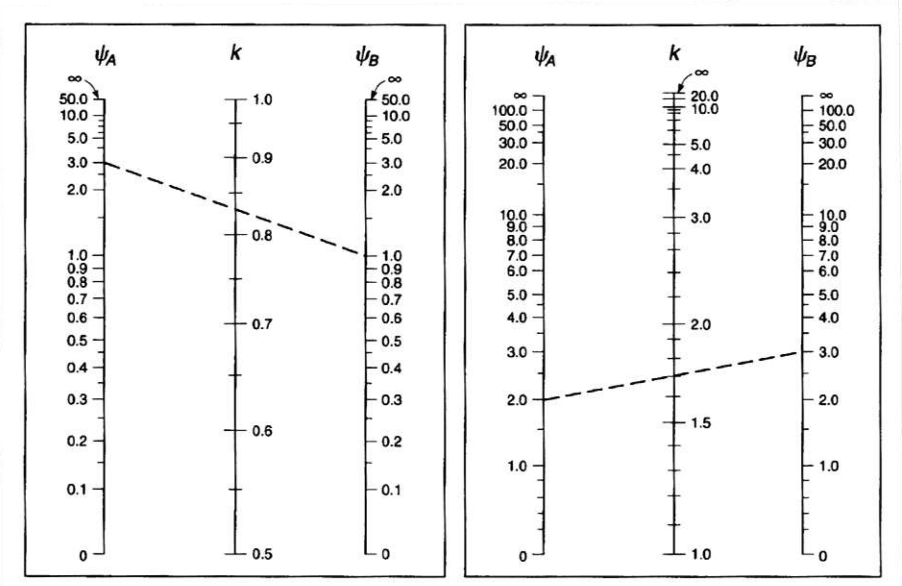
(both in the plane of bending) where the lengths $l_c$ and $l_b$ are measured center-to-center of the joints. Should one end of the column be pinned, $\psi$ is theoretically equal to $\infty$, and if fixed, $\psi = 0$. Since a perfectly fixed end is practically impossible to have, $\psi$ is usually taken as 1.0 instead of 0 for assumed fixed ends. When column ends are supported by, but not rigidly connected to a footing, $\psi$ is theoretically infinity, but usually is taken as about 10 for practical design. One of the two $\psi$ values is called $\psi_A$ and the other is called $\psi_B$. After these values are computed, the effective length factor $k$ is obtained by placing a straightedge between $\psi_A$and $\psi_B$. The point where the straightedge crosses the middle nomograph is $k$.



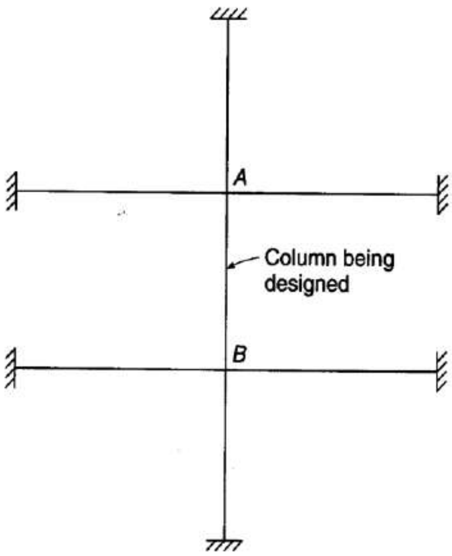

$\Psi$ = ratio of $\Sigma(\boldsymbol{EI}/\ell_c)$ of compression members to $\Sigma(\boldsymbol{EI}/\ell)$ of flexural members in a plane at one end of a compression member

$\ell$ = span length of flexural member measured center to center of joints

Effective Length factors - $k$



(a) Nonsway frames (b) Sway frames




The values of $E_cI_c$ and $E_bI_b$ should be realistic for the state of loading immediately prior to failure of the columns. Generally, at this stage of loading, the beams are extensively cracked and the columns are uncracked or slightly cracked. Ideally, the values of $EI$ should reflect the degree of cracking and the actual reinforcement present. This is not practical, however, because this information is not known at this stage of design. ACI Commentary Section R10.10.6.3 states that the calculation of $k$ shall be based on a $\psi$ based on the $E$ and $I$ values given in ACI Code Section 10.10.4. In most cases the $I$ values given in ACI Code Section 10.10.4.1(b) are used for the evaluation of $\psi$.

The stiffness of a structural member is equal to $EI$. The values of $E$ and $I$ for reinforced concrete members can be estimated as follows:

- $E_c = 0.043 w_c^{1.5} \sqrt{f_c'}$ , $\qquad E_s = 200\ 000\ MPa$

For normalweight concrete, $E_c$ shall be permitted to be taken as $E_c = 4700\sqrt{f_c'}$,
Where, $1440 \le w_c \le 2560\ kg/m^3 \quad and \quad f_c'\ in\ MPa$ .

- For reinforced concrete members, the moment of inertia $I$ varies along the member, depending on the degree of cracking and the percentage of reinforcement in the section considered. To evaluate the factor $\psi$, $EI$ must be calculated for beams and columns. For this purpose, $I$ can be estimated as follows (10.10.4.1(b)):

**Compression members:**

| | |
|---|---|
| Columns | $0.70\ I_g$ |
| Walls – Uncracked | $0.70\ I_g$ |
| – Cracked | $0.35\ I_g$ |

**Flexural members:**

| | |
|---|---|
| Beams | $0.35\ I_g$ |
| Flat plates and flat slabs | $0.25\ I_g$ |

Alternatively, the moments of inertia of compression and flexural members, $I$, shall be permitted to be computed as follows:

**Compression members:**

$$I = \left(0.8 + 25\frac{A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_u h} - 0.5\frac{P_u}{P_o}\right) I_g \leq 0.875\, I_g$$

where $P_u$ and $M_u$ shall be determined from the particular load combination under consideration, or the combination of $P_u$ and $M_u$ determined in the smallest value of $I$. $I$ need not be taken less than $0.35 I_g$.

**Flexural members:**

$$I = (0.1 + 25\rho)\left(1.2 - 0.2\frac{b_w}{d}\right) I_g \leq 0.5\, I_g$$

where $I_g -$ the moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement.

$\rho -$ ratio of $A_s/bd$ in cross section.

- The moment of inertia of T-beams should be based on the effective flange width defined in Section 8.12.2. It is generally sufficiently accurate to take $I_g$ of a T-beam as two times the $I_g$ of the web, $2(b_w h^3/12)$.
- If the factored moments and shears from an analysis based on the moment of inertia of a wall, taken equal to $0.70\, I_g$, indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with $I = 0.35\, I_g$ in those stories where cracking is predicted using factored loads.
- For continuous flexural members, $I$ shall be permitted to be taken as the average of values obtained from the above equation for the critical positive and negative moment sections. $I$ need not be taken less than $0.25\, I_g$.
- The cross-sectional dimensions and reinforcement ratio used in the above formulas shall be within 10% of the dimensions and reinforcement ratio shown on the design drawings or the stiffness evaluation shall be repeated.
- When sustained lateral loads are present, $I$ for compression members shall be divided by $(1 + \beta_{ds})$. The term $\beta_{ds}$ shall be taken as the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination, but shall not be taken greater than 1.0.

## 7.5 LIMITATION OF THE SLENDERNESS RATIO $- \left(\frac{kl_u}{r}\right)$

### 7.5.1. Nonsway (braced) frames:

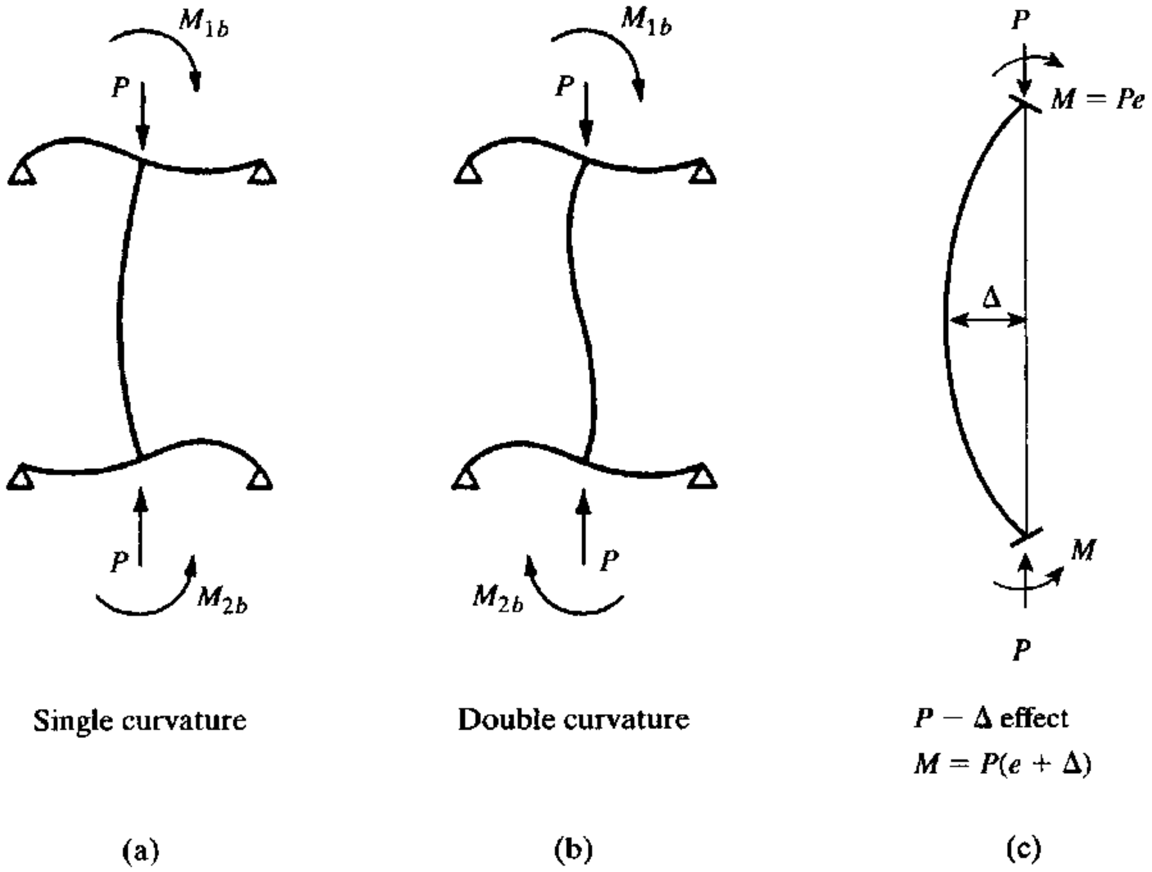
The ACI Code, Section 10.10.1 recommends the following limitations between short and long columns in braced (nonsway) frames:

1. The effect of slenderness may be neglected and the column may be designed as a short column when

$$\frac{kl_u}{r} \leq 34 - 12\left(\frac{M_1}{M_2}\right) \quad \leq 40$$

where $M_1$ and $M_2$ are the factored end moments of the column and $M_2 > M_1$.

2. The ratio $M_1/M_2$ is considered positive if the member is bent in single curvature and negative for double curvature (see next figure).




Single and double curvatures.

3. The term $\left[34 - 12\left(\frac{M_1}{M_2}\right)\right]$ shall not be taken greater than 40.
4. If the factored column moments are zero or $e = M_u/P_u < e_{min}$, the value of $M$ should be calculated using the minimum eccentricity given by ACI Code Section 10.10.6.5:

$$e_{min} = (15 + 0.03h) \ \ where \quad h \ \ in\, mm$$

$$M_{min} = P_u(15 + 0.03h)$$

The moment $M_{min}$ shall be considered about each axis of the column separately. The value of $k$ may be assumed to be equal to 1.0 for a braced frame unless it is calculated on the basis of $EI$ analysis.

5. It shall be permitted to consider compression members braced against sidesway when bracing elements have a total stiffness, resisting lateral movement of that story, of at least 12 times the gross stiffness of the columns within the story.

**7.5.2. Sway (unbraced) frames:**

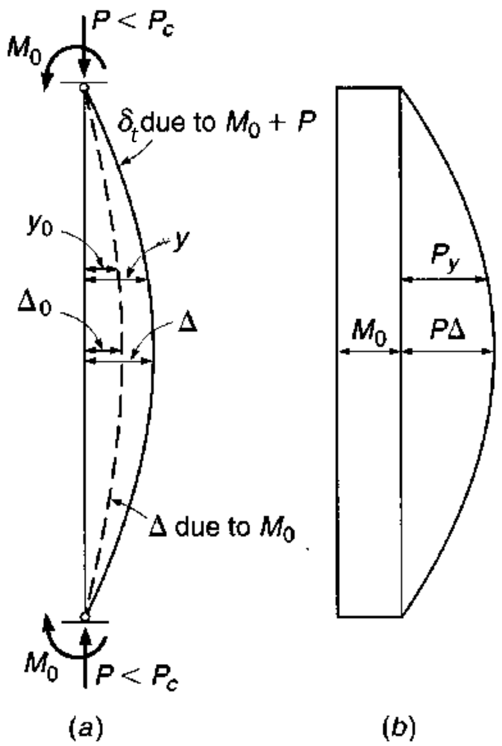
In compression members not braced against sidesway the effect of the slenderness ratio may be neglected when

$$\frac{kl_u}{r} \leq 22$$

### 7.6 MOMENT-MAGNIFIER DESIGN METHOD

The first step in determining the design moments in a long column is to determine whether the frame is braced or unbraced against sidesway. If lateral bracing elements, such as shear walls and shear trusses, are provided or the columns have substantial lateral stiffness, then the lateral deflections produced are relatively small and their effect on the column strength is substantially low. It can be assumed that a story within a structure is nonsway if the stability index

$$Q = \frac{\sum P_u \Delta_o}{V_{us} l_c} \leq 0.05$$




In general, compression members may be subjected to lateral deflections that cause secondary moments. If the secondary moment, $M'$, is added to the applied moment on the column, $M_o$, the final moment is $M = M_o + M'$. An approximate method for estimating the final moment $M$ is to multiply the applied moment $M_o$ by a factor called the **magnifying moment factor** $\delta$, which must be equal to or greater than 1.0, or

$$M_{max} = \delta M_o \quad and \quad \delta \geq 1.0.$$

The moment $M_o$ is obtained from the elastic structural analysis using factored loads, and it is the maximum moment that acts on the column at either end or within the column if transverse loadings are present.

If the $P - \Delta$ effect is taken into consideration, it becomes necessary to use a second-order analysis to account for the nonlinear relationship between the load, lateral displacement, and the moment. This is normally performed using computer programs. The ACI Code permits the use of first-order analysis of columns. The ACI Code moment-magnifier design method is a simplified approach for calculating the moment-magnifier factor in both braced and unbraced frames.

### 7.6.1. Magnified Moments in Nonsway Frames

The effect of slenderness ratio $\left(\frac{kl_u}{r}\right)$ in a compression member of a braced frame may be ignored if $\frac{kl_u}{r} \leq 34 - 12\left(\frac{M_1}{M_2}\right)$, as given in Section 7.5.1. If $\left(\frac{kl_u}{r}\right)$ is greater than $\left[34 - 12\left(\frac{M_1}{M_2}\right)\right]$, then slenderness effect must be considered. The procedure for determining the magnification factor $\delta_{ns}$ in nonsway frames can be summarized as follows (ACI Code, Section 10.10.6):

1. Determine if the frame is braced against sidesway and find the unsupported length, $l_u$, and the effective length factor, $k$ (ACI Commentary Section R10.10.1 states that the effective length factor, $k$, can be taken conservatively as 1.0 for columns in nonsway frames).

2. Calculate the member stiffness $EI$, using the reasonably approximate equation

$$EI = \frac{\left(0.2E_cI_g + E_sI_{se}\right)}{1 + \beta_{dns}}$$

or the more simplified approximate equation

$$EI = \frac{0.4E_cI_g}{1 + \beta_{dns}}$$

$I_g$ – gross moment of inertia of the section about the axis considered, neglecting $A_s$.
$I_{se}$ – moment of inertia of the reinforcing steel

The term $\beta_{dns}$ shall be taken as the ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination, but shall not be taken greater than 1.0.

$$\beta_{dns} = \frac{1.2\ D\ (sustained)}{1.2\ D + 1.6\ L} \leq 1$$

Alternatively, $EI$ shall be permitted to be computed using the value of $I$ from equation for compression members divided by $(1 + \beta_{dns})$.

$$I = \left(0.8 + 25\frac{A_{st}}{A_g}\right)\left(1 - \frac{M_u}{P_uh} - 0.5\frac{P_u}{P_o}\right)I_g \leq 0.875\ I_g$$

$I$ need not be taken less than $0.35I_g$.
where
$A_{st}$ – Total area of longitudinal reinforcement ($mm^2$),
$P_o$ – Nominal axial strenght at zero eccentricity ($N$),
$P_u$ – Factored axial force (+ve for compression) ($N$),
$M_u$ – Factored moment at section ($N \cdot mm$),
$h$ – thickness of member perpendicular to the axis of bending ($mm$).

3. Determine the Euler buckling load, $P_c$:

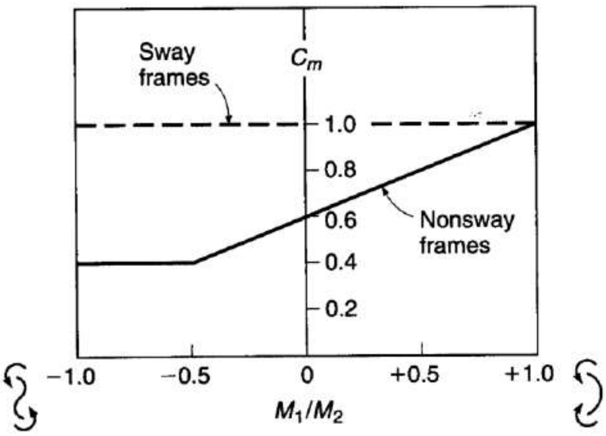
$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

Use the values of $EI, k$ and $l_u$ as calculated from steps 1 and 2.
However, the $EI$ values given in ACI Code Sections 10.10.4 **should not be used** to compute $EI$ for use in Euler equation because those values are assumed to be average values for an entire story in a frame and are intended for use in first- or second-order frame analyses.

4. Calculate the value of the factor $C_m$ to be used in the equation of the moment-magnifier factor. For braced members without transverse loads,

$$C_m = 0.6 + 0.4\frac{M_1}{M_2}$$

where $M_1/M_2$ is positive if the column is bent in single curvature, and negative if the member is bent in double curvature. For members with transverse loads between supports, $C_m$ shall be taken as 1.0.




5. Calculate the moment magnifier factor $\delta_{ns}$:

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0$$

where $P_u$ is the applied factored load and $P_c$ and $C_m$ are as calculated previously.
If $P_u$ exceeds $0.75P_c$ , $\delta_{ns}$ will be negative, and the column would be unstable. Hence, if $P_u$ exceeds $0.75P_c$ the column cross section must be enlarged. Further, if $\delta_{ns}$ exceeds 2.0, strong consideration should be given to enlarging the column cross section, because calculations for such columns are very sensitive to the assumptions being made.

6. Design the compression member using the axial factored load, $P_u$, from the conventional frame analysis and a magnified moment, $M_c$, computed as follows:

$$M_c = \delta_{ns}M_2$$

where $M_2$ is the larger factored end moment due to loads that result in no sidesway and shall not be taken less than

$$M_{2,min} = P_u(15 + 0.03h)$$

about each axis separately, where 15 and $h$ are in mm. For members in which $M_{2,min}$ exceeds $M_2$, the value of $C_m$ shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments, $M_1/M_2$.

### 7.6.2. Magnified Moments in Sway Frames

The effect of slenderness may be ignored in sway (unbraced) frames when $\frac{kl_u}{r} \leq 22$. The procedure for determining the magnification factor, $\delta_s$, in sway (unbraced) frames may be summarized as follows (ACI Code, Section 10.10.7):

1. Determine if the frame is unbraced against sidesway and find the unsupported length $l_u$ and $k$, which can be obtained from the alignment charts (page 157).

2-4. Calculate $EI$, $P_c$, and $C_m$ as given in section 7.6.1. Note that here will be used $\beta_{ds}$ instead of $\beta_{dns}$ (to calculate $I$) which is the ratio of maximum factored sustained shear within a story to the total factored shear in that story.

$$\beta_{ds} = \frac{maximum\ factored\ sustained\ shear\ in\ the\ story}{total\ factored\ shear\ in\ the\ story}$$

5. Calculate the moment-magnifier factor, $\delta_s$ using one of the following methods:

**a. Direct $P - \Delta$ analysis:**

As part of the moment magnification method for sway frames, ACI Code Section 10.10.7 permits the use of a direct calculation of moments using $\delta_s$ in the form

$$\delta_s = \frac{1}{1 - Q} \geq 1.0$$

where

$$Q = \frac{\sum P_u \Delta_o}{V_{us} l_c}$$

If $\delta_s$ calculated here exceeds 1.5, $\delta_s$ shall be calculated using second-order elastic analysis or using method (b).

**b. Moment Magnifier method**

ACI Code Section 10.10.7 also allows the use of the traditional sway-frame moment magnifier for computing the magnified sway moments

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1.0$$

where $\sum P_u$ is the summation for all the factored vertical loads in a story and $\sum P_c$ is the summation for all sway-resisting columns in a story.

In most sway frames, the story shear is due to wind or seismic loads and hence is not sustained, resulting in $\beta_{ds} = 0$. The use of the summation terms in the previous equation for $\delta_s$ accounts for the fact that sway instability involves all the columns and bracing members in the story.

If $\left(1 - \frac{\sum P_u}{0.75 \sum P_c}\right)$ is negative, the load on the frame, $\sum P_u$, exceeds the buckling load for the story, $\sum P_c$, indicating that the frame is unstable. A stiffer frame is required.

6. Calculate the magnified end moments $M_1$ and $M_2$ at the ends of an individual compression member, as follows:

$$M_1 = M_{1ns} + \delta_s M_{1s}$$

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

where $M_{1ns}$ and $M_{2ns}$ are the moments obtained from the **no-sway** condition, whereas $M_{1s}$ and $M_{2s}$ are the moments obtained from the sway condition. If $M_2$ is greater than $M_1$ from structural analysis, then the design magnified moment is

$$M_{\mathrm{c}} = M_{2ns} + \delta_s M_{2s}$$

A separate stability check for sway frames, which was required in prior editions of the ACI Code, is now covered by ACI Code Section 10.10.2.1: Total moment including second-order effects in compression members, restraining beams, or other structural members shall not exceed 1.4 times the moment due to first-order effects.

Analytical studies have shown that the probability of a stability failure increases when the stability index, $Q$, exceeds 0.2. This is similar to the limit of 0.25 set by ASCE/SEI 7-10. Using a value of 0.25 results in a secondary-to-primary moment ratio of 1.33, which is the basis for the ACI Code limit of 1.4 for that ratio. If that limit is satisfied, a separate stability check is not required.

The Commentary Section R10.10.2.1 gives the structural designer indications where to consider stiffening the building sides for winds blowing, and thus, reduce the story drift below 1/500 when needed.

**Maximum Moment between the Ends of the Column.** In most columns in sway frames, the maximum moment will occur at one end of the column and will have the value given by $M_1$ and $M_2$. Occasionally, for very slender, highly loaded columns, the deflections of the column can cause the maximum column moment to exceed the moment at one or both ends of the column. The ACI Code Section 10.10.2.2 calls attention to this potential problem but does not offer guidance. The ACI Commentary Section R.10.10.2.2 does suggest that this can be accounted for in structural analysis by adding nodes along the length of the column.

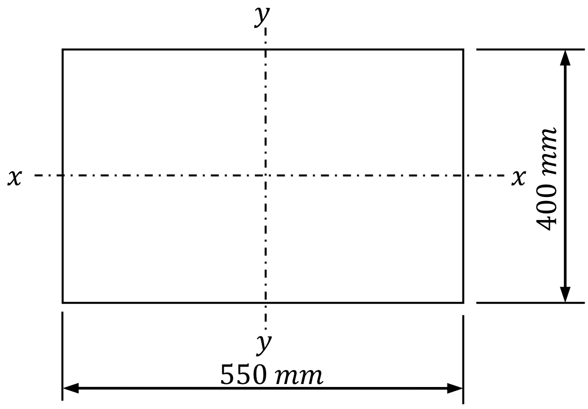
This is a rare occurrence and prior editions of the ACI Code used the following equation to identify columns that may have moments between the ends of the column that exceed the moments at the ends.

$$\frac{l_u}{r} > \frac{35}{\sqrt{\dfrac{P_u}{f_c' A_g}}}$$

If $\frac{l_u}{r}$ exceeds the value given by $35/\sqrt{\frac{P_u}{f_c' A_g}}$, there is a chance that the maximum moment on the column will exceed the larger end moment, $M_2$. This would occur if $M_c$ was larger than the end moments $M_1$ and $M_2$. If $M_c < M_2$, the maximum design moment is at the end of the column and is equal to $M_2$. If $M_c \geq M_2$, the maximum design moment occurs between the ends of the column and is equal to $M_c$.

**Example:**

Design a tied column in a nonsway story to support the following loads:
$P_D = 1000\ KN$ and $P_L = 1250\ KN$.
The column's dimensions as shown and its length in both directions is $3.5\ m$
Take $f_c' = 28\ MPa, \quad f_y = 414\ MPa.$




**Solution:**

1. Check for slenderness:

$$\frac{kl_u}{r} \le 34 - 12\left(\frac{M_1}{M_2}\right) \quad \le 40$$

$\left(\frac{M_1}{M_2}\right) = 1.0 -$ braced frame with $M_{min}$

$k = 1.0 -$ for columns in nonsway frames.

$$\frac{kl_u}{r} \le 34 - 12 \cdot 1.0 = 22 \ < 40$$

$$\frac{kl_u}{r_x} = \frac{1.0 \cdot 3.5}{0.3 \cdot 0.4} = 29.17 > 22 \ - long\ column\ for\ bending\ about\ x - axis$$

$$\frac{kl_u}{r_y} = \frac{1.0 \cdot 3.5}{0.3 \cdot 0.55} = 21.21 < 22 - \ short\ column\ for\ bending\ about\ y - axis$$

2. Calculate the minimum eccentricity $e_{min}$ and the minimum moment $M_{min}$:

$$e_{min} = (15 + 0.03h) = 15 + 0.03 \cdot 400 = 27\ mm$$

$$P_u = 1.2 \cdot P_D + 1.6\ P_L = 1.2 \cdot 1000 + 1.6 \cdot 1250 = 3200\ KN$$

$$M_{min} = P_u \cdot e_{min} = 3200 \cdot 0.027 = 86.4\ KN \cdot m$$

3. Compute $EI$.

At this stage, the area of reinforcement is not known. Additional calculations are needed before it is possible to compute $EI = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}}$, but $EI = \frac{0.4E_cI_g}{1+\beta_{dns}}$ can be used.

$$E_c = 4700\sqrt{f_c'} = 4700\sqrt{28} = 24870\ MPa$$

$$I_g = \frac{bh^3}{12} = \frac{550 \cdot 400^3}{12} = 2.933 \cdot 10^9\ mm^4$$

$$\beta_{dns} = \frac{1.2\ D\ (sustained)}{1.2\ D + 1.6\ L} = \frac{1.2 \cdot 1000}{3200} = 0.375$$

$$EI = \frac{0.4E_cI_g}{1 + \beta_{dns}} = \frac{0.4 \cdot 24870 \cdot 2.933}{1 + 0.375} = 21220\ KN \cdot m^2$$

4. Determine the Euler buckling load, $P_c$:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot 21220}{(1.0 \cdot 3.5)^2} = 17096.6\ KN$$

5. Calculate the moment magnifier factor $\delta_{ns}$:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4 \cdot 1.0 = 1.0$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} = \frac{1.0}{1 - \frac{3200}{0.75 \cdot 17096.6}} = 1.33 > 1.0$$

Normally, if $\delta_{ns}$ exceeds 1.75 to 2.0, a larger cross section should be selected.

The magnified eccentricity and moment:

$$e = e_{min} \cdot \delta_{ns} = 27 \cdot 1.33 = 35.91\ mm$$

$$M_c = \delta_{ns} M_2 = 1.33 \cdot 86.4 = 114.91\ KN \cdot m$$

$where \qquad M_2 = M_{min} = 86.4\ KN \cdot m$

The magnified moments are less than 1.4 times the first-order moments, as required by ACI Code Section 10.10.2.1.

6. Select the column reinforcement. We will use the tied-column interaction diagrams with bars in four faces (A-9).
   - Compute the ratio $e/h$:

$$\frac{e}{h} = \frac{35.91}{400} = 0.09$$

To construct the $\frac{e}{h}$ line, take value 0.09 on $\frac{\phi M_n}{bh^2}$ axis and value 1.0 on $\frac{\phi P_n}{bh}$ axis.

   - Compute the ratio γ:

γ − the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume ∅ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{400 - 2 \cdot 40 - 2 \cdot 10 - 25}{400} = 0.688$$

   - Use interaction diagrams A-9a and A-9b to determine $\rho_g$ for the selected dimensions : $h = 550\ mm$, $b = 400\ mm$. The interaction diagrams are entered with

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{3200 \cdot 10^3}{550 \cdot 400} \times 0.145 = 2.11\ ksi$$

Diagram A-9a (for $\gamma = 0.6$ ), $\rho_g = 0.013$

Diagram A-9b (for $\gamma = 0.75$ ), $\rho_g = 0.012$

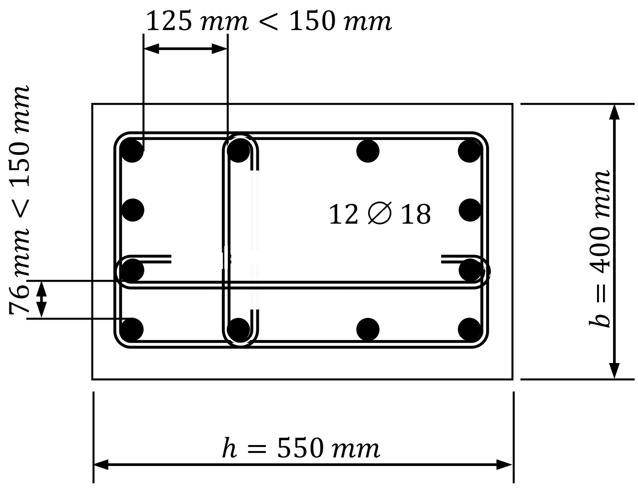
$$\rho_g\ (\gamma = 0.688\ ) = 0.013 - \left(\frac{0.013 - 0.012}{0.75 - 0.6}\right)(0.688 - 0.6) = 0.0124 > \rho_{min} = 0.01\ - OK$$

- Select the reinforcement:

$$A_{st} = \rho_g A_g = 0.0124 \cdot 550 \cdot 400 = 2728\, mm^2$$

Take 12 $\varnothing$ 18 with $A_s = 3054\ mm^2 > A_{st} = 2728\, mm^2$.

Note that $e = 0.09h < 0.1h$ –the limit for concentrically loaded short column. Here we can solve this problem using equations for the concentrically loaded short tied column to calculate area of steel reinforcement $A_{st}$:




$$P_{n,max} = 0.8\left[0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right]$$

$$\phi = 0.65 - for\ tied\ colum$$

$$3200 \times 10^3 = 0.65 \cdot 0.8[0.85 \cdot 28(550 \cdot 400 - A_{st}) + A_{st} \cdot 414]$$

$$A_{st} = \left[\frac{3200 \times 10^3}{0.65 \cdot 0.8} - 4675000\right]\frac{1}{390.2} = 3790\, mm^2.$$

$$\rho_g = \frac{A_{st}}{A_g} = \frac{3790}{550 \cdot 400} = 0.0172 > \rho_{req} = 0.0124 \qquad - as\ it\ is\ expected.$$

**Example:**

Design a 6 m tall column to support an unfactored dead load of 400 KN and an unfactored live load of 334 KN. The loads act at an eccentricity of 75 mm at the top and 50 mm at the bottom, as shown.

Take $f_c' = 28\, MPa$ and $f_y = 414\, MPa$.

**Solution:**

1. Compute the factored loads and moments.

$P_D = 400\, KN,\qquad P_L = 334\, KN$

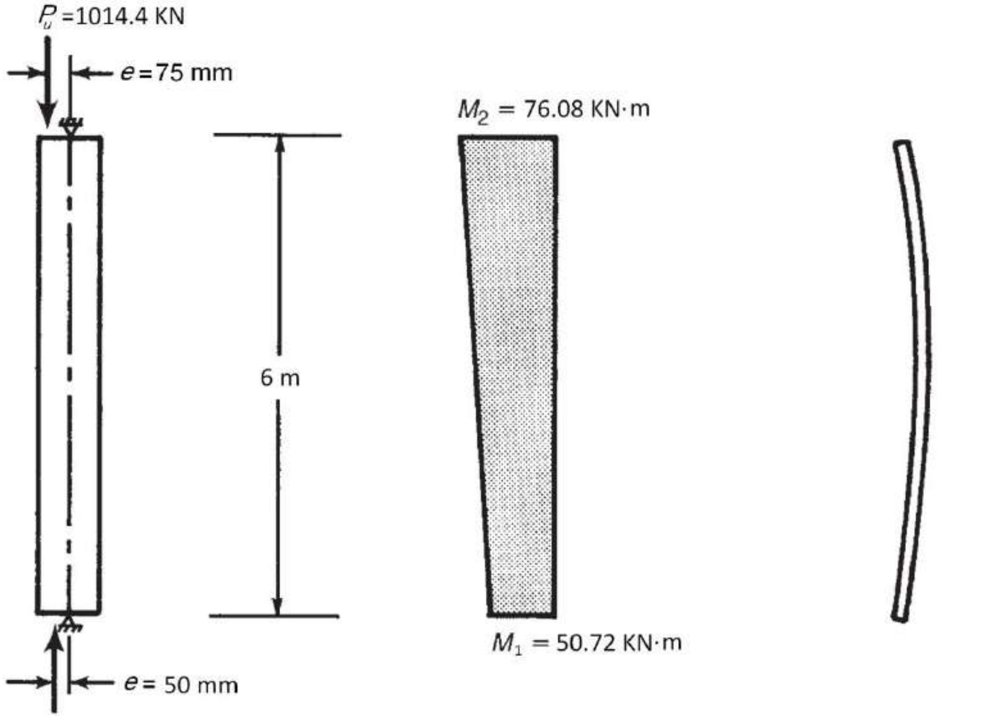
$P_u = 1.2 \cdot P_D + 1.6\, P_L = 1.2 \cdot 400 + 1.6 \cdot 334 = 1014.4\, KN$

$$M_{top} = P_u \cdot e = 1014.4 \cdot 0.075 = 76.08\, KN \cdot m$$

$$M_{bot} = P_u \cdot e = 1014.4 \cdot 0.05 = 50.72\, KN \cdot m$$

By definition, $M_2$ is the larger end moment in the column. Therefore, $M_2 = 76.08\, KN \cdot m$, and $M_1 = 50.72\, KN \cdot m$. The ratio $\left(\frac{M_1}{M_2}\right)$ is taken to be positive, because the column is bent in single curvature. Thus

$$\left(\frac{M_1}{M_2}\right) = \frac{50.72}{76.08} = 0.667$$




(a) Column. (b) Moment diagram. (c) Deflected shape.

2. Estimate the column size:

Assume the section as a square tied column $400 \times 400\ mm$.

**Instead of assumed value for column dimension (400 mm), it's acceptable to assume directly one dimension such as $h = 500\ mm$ and then determine the width $b$ as was done before in short column design. It should be noted that the section dimensions which were derived for short columns will underestimate the required sizes of slender columns.**

3. Check for slenderness: a column in a nonsway frame is short if

$$\frac{kl_u}{r} \leq 34 - 12\left(\frac{M_1}{M_2}\right) \quad \leq 40$$

$k = 1.0 -$ because the column is pinended.

$r = 0.3h$

$$\frac{kl_u}{r} \leq 34 - 12 \cdot 0.667 = 26 < 40$$

$$\frac{kl_u}{r} = \frac{1.0 \cdot 6}{0.3 \cdot 0.4} = 50 > 26 - long\ column\ in\ both\ directions$$

4. Check whether the moments are less than the minimum

The minimum eccentricity $e_{min}$:

$$e_{min} = (15 + 0.03h) = 15 + 0.03 \cdot 400 = 27\ mm$$

$$\left.\begin{matrix} e_{top} = 75\ mm \\ e_{bot} = 50\ mm \end{matrix}\right\} > e_{min} = 27\ mm$$

Because the maximum end eccentricity exceeds this, design for the moments from step 1.

5. Compute $EI$.

At this stage, the area of reinforcement is not known. Additional calculations are needed before it is possible to compute $EI = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}}$, but $EI = \frac{0.4E_cI_g}{1+\beta_{dns}}$ can be used.

$$E_c = 4700\sqrt{f_c'} = 4700\sqrt{28} = 24870\ MPa$$

$$I_g = \frac{bh^3}{12} = \frac{400^4}{12} = 2.133 \cdot 10^9\ mm^4$$

$$\beta_{dns} = \frac{1.2\ D\ (sustained)}{1.2\ D + 1.6\ L} = \frac{1.2 \cdot 400}{1014.4} = 0.473$$

$$EI = \frac{0.4E_cI_g}{1+\beta_{dns}} = \frac{0.4 \cdot 24870 \cdot 2.133}{1+0.473} = 14405\ KN \cdot m^2$$

6. Determine the Euler buckling load, $P_c$:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot 14405}{(1.0 \cdot 6)^2} = 3949.2\ KN$$

7. Calculate the moment magnifier factor $\delta_{ns}$:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4 \cdot 0.667 = 0.867$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} = \frac{0.867}{1 - \frac{1014.4}{0.75 \cdot 3949.2}} = 1.32 > 1.0$$

Normally, if $\delta_{ns}$ exceeds 1.75 to 2.0, a larger cross section should be selected.

The magnified eccentricity and moment:

$$e_{c,top} = \delta_{ns} \cdot e_{top} = 1.32 \cdot 75 = 99\ mm$$

$$M_c = \delta_{ns}M_2 = 1.32 \cdot 76.08 = 100.43\ KN \cdot m$$

The magnified moments are less than 1.4 times the first-order moments, as required by ACI Code Section 10.10.2.1.

8. Select the column reinforcement. We will use the tied-column interaction diagrams with bars in two faces (A-6).
   - Compute the ratio $e/h$:

$$\frac{e}{h} = \frac{99}{400} = 0.248 \approx 0.25$$

To construct the $\frac{e}{h}$ line, take value 0.25 on $\frac{\phi M_n}{bh^2}$ axis and value 1.0 on $\frac{\phi P_n}{bh}$ axis.

   - Compute the ratio $\gamma$:

$\gamma$ – the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume $\varnothing$ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{400 - 2 \cdot 40 - 2 \cdot 10 - 25}{400} = 0.688$$

- Use interaction diagrams A-6a and A-6b to determine $\rho_g$ for the selected dimensions : $h = b = 400\ mm$. The interaction diagrams are entered with

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{1014.4 \cdot 10^3}{400^2} \times 0.145 = 0.919 \quad ksi$$

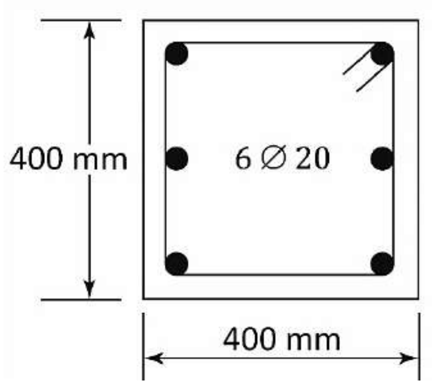
Diagram A-6a (for $\gamma = 0.6$ ), $\rho_g < \rho_{min} = 0.01$
Diagram A-6b (for $\gamma = 0.75$ ), $\rho_g < \rho_{min} = 0.01$

From both diagrams A-6a and A-6b the required value for $\rho_g$ is less than 0.01. Therefore, to satisfy the minimum column longitudinal-reinforcement ratio, use $\rho_g = \rho_{min} = 0.01$.

- Select the reinforcement:

$$A_{st} = \rho_g A_g = 0.01 \cdot 400 \cdot 400 = 1600\ mm^2$$

Take 6 ∅ 20 with $A_s = 1885\ \ mm^2 > A_{st} = 1600\ mm^2$.




This section design would be very conservative if we were designing a short column, but the slenderness of the column has required the use of this larger section.

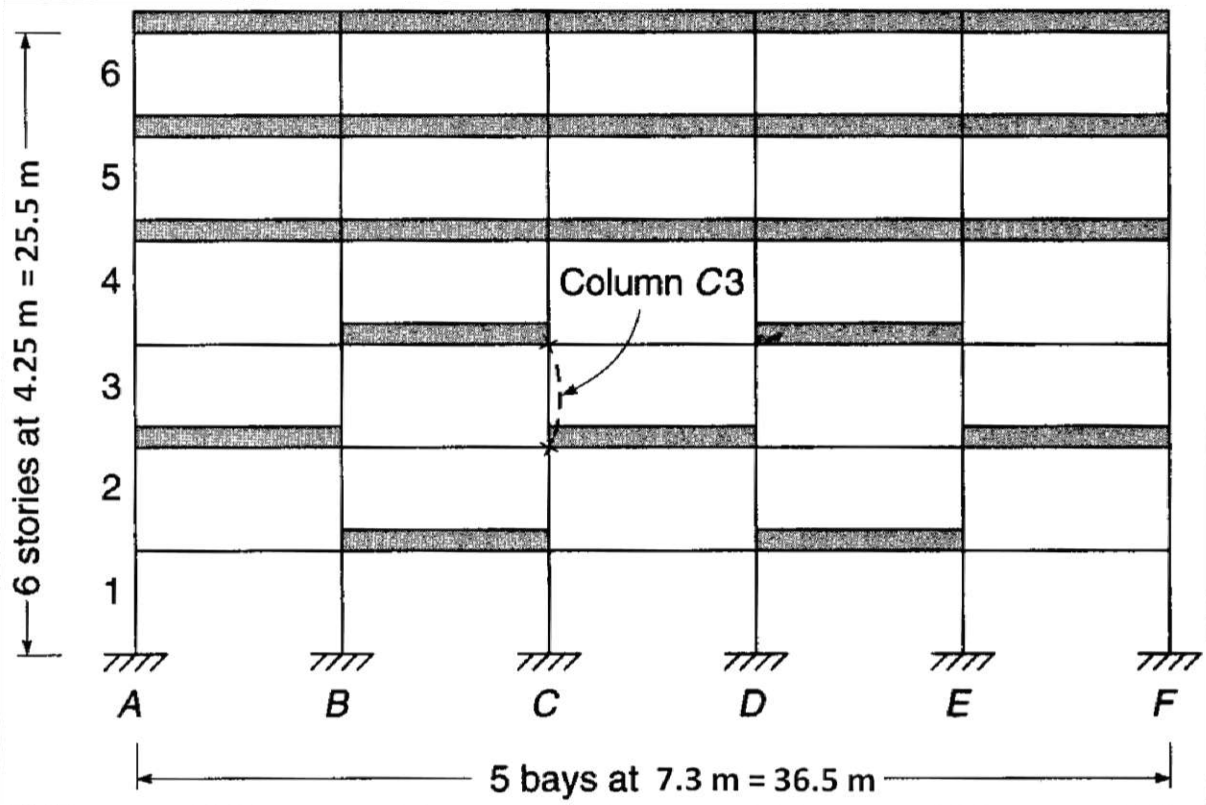
**Example:**

Design of a slender column in a nonsway frame. The figure below shows an elevation view of a multistory concrete frame building, with $120\ cm\ wide \times\ 30\ cm\ deep$ beams on all column lines, carrying two-way solid slab floors and roof. The clear height of the columns is $3.95\ m$. Interior columns are tentatively dimensioned at $450\ \times\ 450\ mm$, and exterior columns at $400\ \times\ 400\ mm$. The frame is effectively braced against sway by stair and elevator shafts having concrete walls that are monolithic with the floors, located in the building corners (not shown in the figure). The structure will be subjected to vertical dead and live loads. Trial calculations by first-order analysis indicate that the pattern of live loading shown in the figure, with full load distribution on roof and upper floors and a checkerboard pattern adjacent to column C3, produces maximum moments with single curvature in that column, at nearly maximum axial load. Dead loads act on all spans. Service load values of dead and live load axial force and moments for the typical interior column C3 are as follows:

| Dead load | Live load |
|---|---|
| $P\ =\ 1000\ KN$ | $P\ =\ 750\ KN$ |
| $M_{top}\ =\ 3\ KN \cdot m$ | $M_{top}\ =\ 145\ KN \cdot m$ |
| $M_{bot}\ =\ -3\ KN \cdot m$ | $M_{bot}\ =\ 135\ KN \cdot m$ |

The column is subjected to double curvature under dead load alone and single curvature under live load.

Design column C3, using ACI magnifier method. Use $f_c' = 28\ MPa$ and $f_y = 414\ MPa$.




**Solution:**

1. The column will first be designed as a short column, assuming no slenderness effect. With the application of usual load factors,

$$P_u = 1.2 \cdot P_D + 1.6\, P_L = 1.2 \cdot 1000 + 1.6 \cdot 750 = 2400\, KN$$

$$M_u = 1.2 \cdot M_D + 1.6\, M_L = 1.2 \cdot 3 + 1.6 \cdot 145 = 235.6\, KN \cdot m$$

2. Try square column $450\ \times 450\, mm$. We will use the tied-column interaction diagrams with bars in four faces (A-9).
   - Compute the ratio $e/h$:

$$e = \frac{M_u}{P_u} = \frac{235.6}{2400} = 98.2\, mm$$

$$\frac{e}{h} = \frac{98.2}{450} = 0.218 \approx 0.22$$

To construct the $\frac{e}{h}$ line, take value 0.22 on $\frac{\phi M_n}{bh^2}$ axis and value 1.0 on $\frac{\phi P_n}{bh}$ axis.

   - Compute the ratio γ:

γ − the ratio of the distance between the centers of the outside layers of bars to the overall depth of the column. Assume ∅ 25 for bars.

$$\gamma = \frac{d - d'}{h} = \frac{450 - 2 \cdot 40 - 2 \cdot 10 - 25}{450} = 0.722$$

- Use interaction diagram A-9a ($\gamma = 0.6$) and A-9b ($\gamma = 0.75$) to determine $\rho_g$ for the selected dimensions : $h = b = 450\ mm$. The interaction diagrams are entered with

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{2400\ \cdot 10^3}{450^2} \times 0.145 = 1.72 \quad ksi$$

Diagram A-9a (for $\gamma = 0.6$ ), $\rho_g = 0.027$
Diagram A-9b (for $\gamma = 0.75$ ), $\rho_g = 0.02$

$$\rho_g\ (\gamma = 0.722\ ) = 0.027 - \left(\frac{0.027 - 0.02}{0.75 - 0.6}\right)(0.722 - 0.6) = 0.021 > \rho_{min} = 0.01\ - OK$$

$\rho_g = 0.021$ is low enough that an increase in steel area could be made, if necessary, to allow for slenderness, and the $450\ \times 450\ mm$ concrete dimensions will be retained.

**Note:** the previous step for short column design was done to show the slenderness effect on the reinforcement ration in the end of this example.

3. Check for slenderness: a column in a nonsway frame is short if

$$\frac{kl_u}{r} \le 34 - 12\left(\frac{M_1}{M_2}\right) \quad \le 40$$

$$M_2 = 1.2 \cdot 3 + 1.6 \cdot 145 = 235.8\ KN \cdot m$$

$$M_1 = 1.2 \cdot (-3) + 1.6 \cdot 135 = 212.4\ KN \cdot m$$

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12 \cdot \left(\frac{212.4}{235.8}\right) = 23.19 < 40$$

$$r = 0.3h = 0.3 \cdot 0.45 = 0.135\ m$$

$$\psi = \frac{\sum E_c I_c / l_c\ \ of\ columns\ at\ joint}{\sum E_b I_b / l_b\ \ of\ beams\ at\ joint}$$

Because $E_c$ is the same for column and beams ($E_c = E_b$), it will be canceled in the stiffness calculations. For this step:

- the column moment of inertia will be

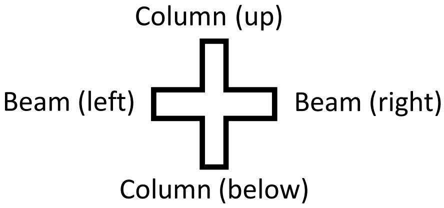
$$I_c = 0.7 I_g = 0.7 \cdot \frac{bh^3}{12} = 0.7 \frac{0.45^4}{12} = 2.392 \cdot 10^{-3}\ m^4$$

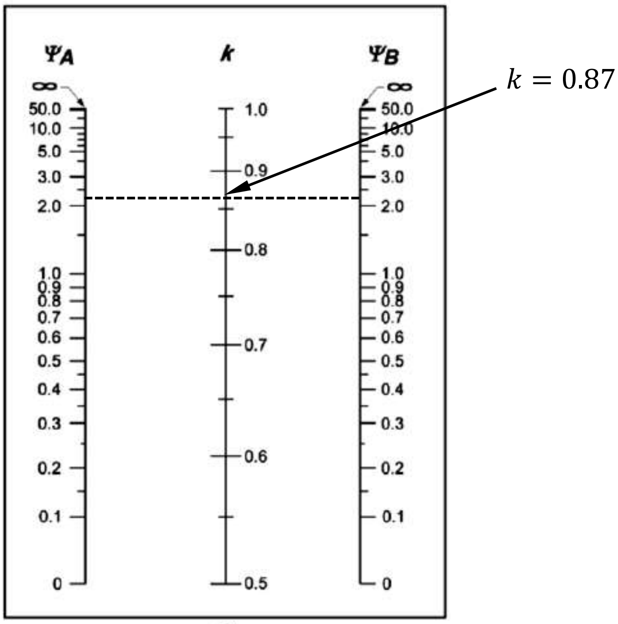
- The moment of inertia of T-beams should be based on the effective flange width $b_e$ defined in Section 8.12.2. It is generally sufficiently accurate to take $I_g$ of a T-beam as two times the $I_g$ of the web, $2(b_w h^3/12)$. Thus, the moment of inertia of a flanged section will be

$$I_b = 0.35 I_g = 0.35 \cdot 2 \cdot \frac{b_w h^3}{12} = 0.35 \cdot 2 \cdot \frac{1.2 \cdot 0.3^3}{12} = 1.89 \cdot 10^{-3}\ m^4$$

Rotational restraint factors at the top and bottom of column C3 are the same and are

$$\psi_A = \psi_B = \frac{\sum I_c / l_c}{\sum I_b / l_b} = \frac{\dfrac{2.392 \cdot 10^{-3}}{4.25} + \dfrac{2.392 \cdot 10^{-3}}{4.25}}{\dfrac{1.89 \cdot 10^{-3}}{7.3} + \dfrac{1.89 \cdot 10^{-3}}{7.3}} = \frac{1.126 \cdot 10^{-3}}{5.178 \cdot 10^{-4}} = 2.17$$



(a)
Nonsway Frames

$$\frac{kl_u}{r} = \frac{0.87 \cdot 3.95}{0.135} = 25.45 > 23.19$$

The column is long and the slenderness must be considered.

4. Check whether the moments are less than the minimum.

The minimum eccentricity $e_{min}$:

$$e_{min} = (15 + 0.03h) = 15 + 0.03 \cdot 450 =$$
$$= 28.5\ mm$$

$$M_{2,min} = P_u \cdot e_{min} = 2400 \cdot 0.0285 = 68.4\ KN \cdot m < M_2 = 235.8\ KN \cdot m$$

The design will be done for the moments from step 3.

5. Compute $EI$.

$$E_c = 4700\sqrt{f_c{}'} = 4700\sqrt{28} = 24870\ MPa$$

$$I_g = \frac{bh^3}{12} = \frac{450^4}{12} = 3.417 \cdot 10^9\ mm^4$$

$$\beta_{dns} = \frac{1.2\ D\ (sustained)}{1.2\ D + 1.6\ L} = \frac{1.2 \cdot 1000}{2400} = 0.5$$

$$EI = \frac{0.4 E_c I_g}{1 + \beta_{dns}} = \frac{0.4 \cdot 24870 \cdot 3.417}{1 + 0.5} = 22662\ \ KN \cdot m^2$$

6. Determine the Euler buckling load, $P_c$:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot 22662}{(0.87 \cdot 3.95)^2} = 18939.4\ KN$$

7. Calculate the moment magnifier factor $\delta_{ns}$:

$$\left(\frac{M_1}{M_2}\right) = \left(\frac{212.4}{235.8}\right) = 0.9$$

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4 \cdot 0.9 = 0.96$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} = \frac{0.96}{1 - \frac{2400}{0.75 \cdot 18939.4}} = 1.16 > 1.0$$

The magnified eccentricity and moment:

$$M_c = \delta_{ns} M_2 = 1.16 \cdot 235.8 = 273.5\ KN \cdot m$$

The magnified moments are less than 1.4 times the first-order moments, as required by ACI Code Section 10.10.2.1.

8. Select the column reinforcement. We will use the tied-column interaction diagrams with bars in four faces (A-9).
   - Compute the ratio $e/h$:

$$e = \frac{M_u}{P_u} = \frac{273.5}{2400} = 114\ mm$$

$$\frac{e}{h} = \frac{114}{450} = 0.25$$

To construct the $\frac{e}{h}$ line, take value 0.25 on $\frac{\phi M_n}{bh^2}$ axis and value 1.0 on $\frac{\phi P_n}{bh}$ axis.

- Compute the ratio γ:

$$\gamma = 0.722$$

- Use interaction diagram A-9a ($\gamma = 0.6$) and A-9b ($\gamma = 0.75$) to determine $\rho_g$ for the selected dimensions : $h = b = 450\ mm$. The interaction diagrams are entered with

$$\frac{\phi P_n}{A_g} = \frac{P_u}{A_g} = \frac{2400\ \cdot 10^3}{450^2} \times 0.145 = 1.72\quad ksi$$

Diagram A-9a (for $\gamma = 0.6$ ), $\rho_g = 0.034$
Diagram A-9b (for $\gamma = 0.75$ ), $\rho_g = 0.027$

$$\rho_g\ (\gamma = 0.722\ ) = 0.034 - \left(\frac{0.034 - 0.027}{0.75 - 0.6}\right)(0.722 - 0.6) = 0.0283 > \rho_{min} = 0.01 - OK$$

It is seen that the required reinforcement ratio is increased from 0.021 to 0.0283 because of slenderness.

- Select the reinforcement:

$$A_{st} = \rho_g A_g = 0.0283 \cdot 450 \cdot 450 = 5730.75\ mm^2$$

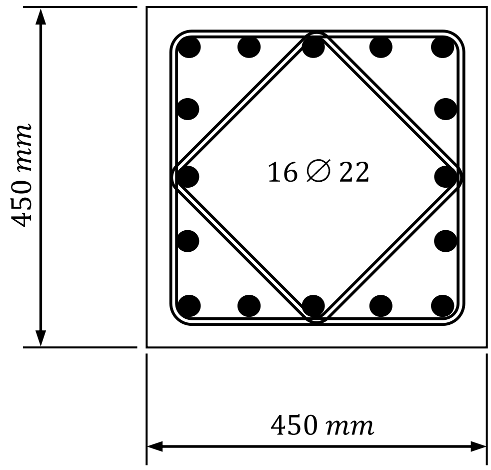
Take 16 ∅ 22 with $A_s = 6081.6\ \ mm^2 > A_{st} = 5730.75\ mm^2$.

- Design of Ties:

Use ties ∅ 10 with spacing of ties shall not exceed the smallest of:

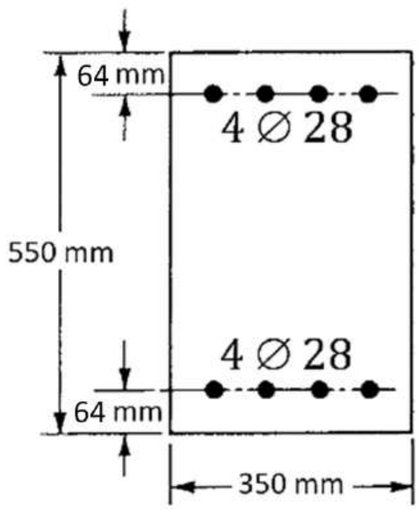
4. 48 times the tie diameter,
5. 16 times the longitudinal bar diameter,
6. the least dimension of the column.

- Check for code requirements:
  7. Clear spacing between longitudinal bars:
  8. Gross reinforcement ratio, $0.01 < \rho_g < 0.08$
  9. Number of bars.
  10. Minimum tie diameter.
  11. Spacing of ties.
  12. Arrangement of ties.




**Example:**

The column section in the figure carries an axial load $P_D = 600\ KN$ and a moment $M_D = 155\ KN \cdot m$ due to dead load and an axial load $P_L = 490\ KN$ and a moment $M_L = 125\ KN \cdot m$ due to live load. The column is part of a frame that is braced against sidesway and bent in single curvature about its major axis. The unsupported length of the column is $l_u = 5.8\ m$ and the moments at both ends of the column are equal. Check the adequacy of the column using $f_c' = 28\ MPa$ and $f_y = 414\ MPa$.




**Solution:**

1. Compute the factored loads and moments.

$P_D = 600\ KN,$ $\qquad M_D = 155\ KN \cdot m$

$P_L = 490\ KN,$ $\qquad M_L = 125\ KN \cdot m$

$$P_u = 1.2 \cdot P_D + 1.6\, P_L = 1.2 \cdot 600 + 1.6 \cdot 490 = 1504\ KN$$

$$M_u = 1.2 \cdot M_D + 1.6\, M_L = 1.2 \cdot 155 + 1.6 \cdot 125 = 386\ KN \cdot m$$

$$e = \frac{M_u}{P_u} = \frac{386}{1504} = 256.6\ mm$$

$$M_2 = M_1 = 386\ KN \cdot m \quad \Longrightarrow \quad \frac{M_1}{M_2} = 1 \qquad (+ve\ for\ single\ curvature)$$

2. Check for slenderness: a column in a nonsway frame is short if

$$\frac{kl_u}{r} \le 34 - 12\left(\frac{M_1}{M_2}\right) \le 40$$

Assume $k = 1.0 -$ for columns in nonsway frames.

$r = 0.3h = 0.3 \cdot 0.55 = 0.165\ m.$ $\quad l_u = 5.8\ m$

$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12 \cdot 1 = 22 < 40$

$$\frac{kl_u}{r} = \frac{1 \cdot 5.8}{0.165} = 35.2 > 22 - long\ column.$$

Slenderness effect must be considered.

3. Compute $EI$.

The area of reinforcement is known. So the equation $EI = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}}$ can be used.

$$E_c = 4700\sqrt{f_c{}'} = 4700\sqrt{28} = 24870\ MPa$$

$$E_s = 200\ 000\ MPa$$

$$I_g = \frac{bh^3}{12} = \frac{350 \cdot 550^3}{12} = 4.852 \cdot 10^9\ mm^4$$

$$A_{s(4\varnothing 28)} = 2463\ mm^2$$

$$I_{se} = 2 \cdot A_{s(4\varnothing 28)} \cdot \left(\frac{d-d'}{2}\right)^2 = 2 \cdot 2463 \cdot \left(\frac{550 - 2 \cdot 64}{2}\right)^2 = 0.2193 \cdot 10^9\ mm^4$$

$$\beta_{dns} = \frac{1.2\ D\ (sustained)}{1.2\ D + 1.6\ L} = \frac{1.2 \cdot 600}{1504} = 0.479$$

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}} = \frac{(0.2 \cdot 24870 \cdot 4.852 + 200\ 000 \cdot 0.2193)}{1 + 0.479} = 45973\ KN \cdot m^2$$

4. Determine the Euler buckling load, $P_c$:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot 45973}{(1.0 \cdot 5.8)^2} = 13488\ KN$$

5. Calculate the moment magnifier factor $\delta_{ns}$:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4 \cdot 1 = 1.0$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} = \frac{1}{1 - \frac{1504}{0.75 \cdot 13488}} = 1.175 > 1.0$$

6. Determine the balanced load and balanced eccentricity $e_b$.

$$c_b = \left(\frac{600}{600+f_y}\right)d = \left(\frac{600}{600+414}\right)486 = 287.6\ mm$$

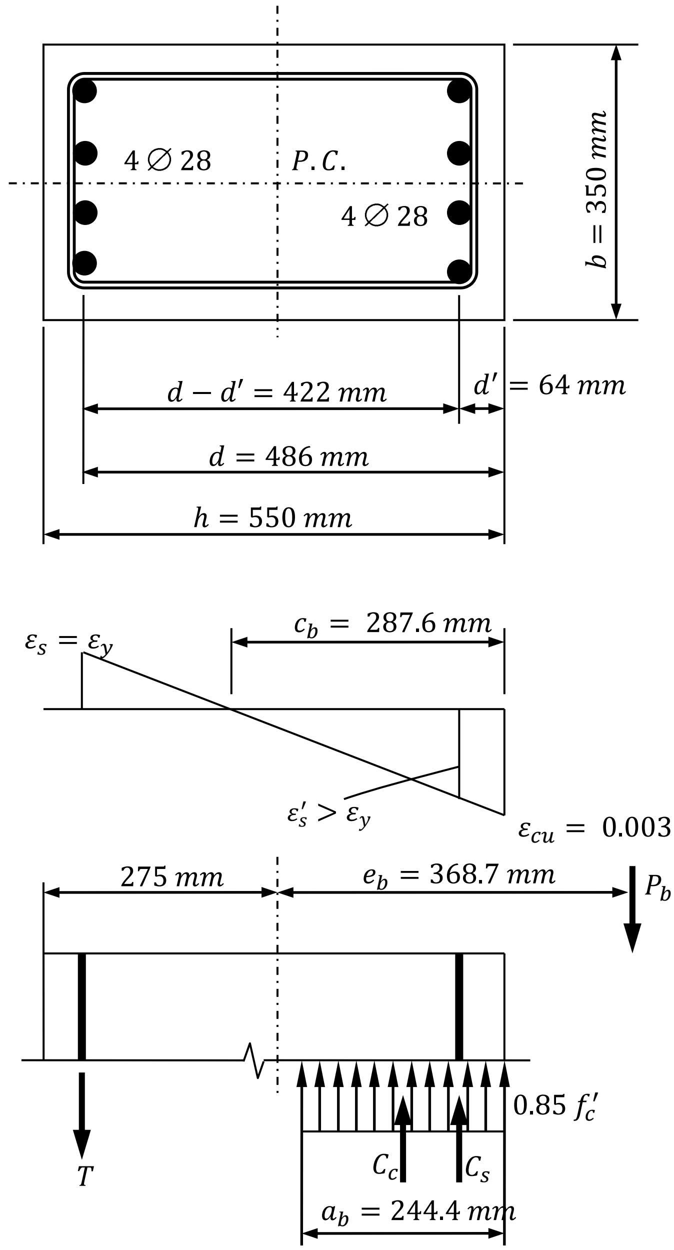
$$a_b = \beta_1 c_b = 0.85 \cdot 287.6 = 244.4\ mm$$

$$C_c = 0.85\ f_c'ab = 0.85 \cdot 28 \cdot 244.4 \cdot 350 \cdot 10^{-3} = 2035.9\ KN,$$

$$T = A_s f_y = 2463 \cdot 414 \cdot 10^{-3} = 1019.7\ KN$$

Check if compression steel yields

$$f_s' = 600\left(\frac{287.6 - 64}{287.6}\right) = 466.5\ MPa > f_y = 414\ MPa$$




Compression steel yields:

$$f_s' = f_y = 414\ MPa$$

$$C_s = A_s'(f_y - 0.85\ f_c') =$$

$$= 2463(414 - 0.85 \cdot 28) \cdot 10^{-3} =$$

$$= 961.1\ KN$$

$$P_b = C_c + C_s - T =$$

$$= 2035.9 + 961.1 - 1019.7 =$$

$$= 1977.3\ KN$$

For rotational equilibrium about the plastic centroid

$$P_b e_b = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) +$$

$$+T\left(\frac{h}{2} - d'\right)$$

$$1977.3 \cdot e_b = 2035.9\left(\frac{550}{2} - \frac{244.4}{2}\right) +$$

$$+961.1\left(\frac{550}{2} - 64\right) +$$

$$+1019.7\left(\frac{550}{2} - 64\right)$$

$$e_b = \frac{729034.3}{1977.3} = 368.7\ mm$$

7. Calculate the magnified eccentricity and the design force $P_n$.

$$e_c = \delta_{ns} \cdot e = 1.175 \cdot 256.6 = 301.5\ mm$$

$e_c = 301.5\ mm < e_b = 368.7\ mm -$ the section is Compression-controlled. $\phi = 0.65$

$$P_n = \frac{P_u}{\phi} = \frac{1504}{0.65} = 2314\ KN$$

8. Determine the nominal load strength of the section using $e = 301.5\ mm$. Here we can solve this problem as a statics problem as in sections (6.8, 6.9) to determine $P_n$, or using approximate formulas to calculate $P_n$.

The section is compression-controlled with symmetrical reinforcement. To determine nominal load strength $P_n$, we will use Whitney formula:

$$P_n = A_g\left[\frac{f_c'}{\left(\frac{3}{\xi^2}\right)\left(\frac{e}{h}\right) + 1.18} + \frac{\rho_g f_y}{\left(\frac{2}{\gamma}\right)\left(\frac{e}{h}\right) + 1}\right]$$

$$A_g = bh = 350 \cdot 550 = 192500\ mm^2$$

$$\rho_g = \frac{2A_s'}{A_g} = \frac{2 \cdot 2463}{192500} = 0.026$$

$$\xi = \frac{d}{h} = \frac{486}{550} = 0.884$$

$$\gamma = \frac{d - d'}{h} = \frac{550 - 2 \cdot 40 - 2 \cdot 10 - 28}{550} = 0.767$$

$$P_n = 192500 \left[ \frac{28}{\left(\frac{3}{0.884^2}\right)\left(\frac{300.2}{550}\right) + 1.18} + \frac{0.026 \cdot 414}{\left(\frac{2}{0.767}\right)\left(\frac{300.2}{550}\right) + 1} \right] \cdot 10^{-3} = 2500.7\ KN$$

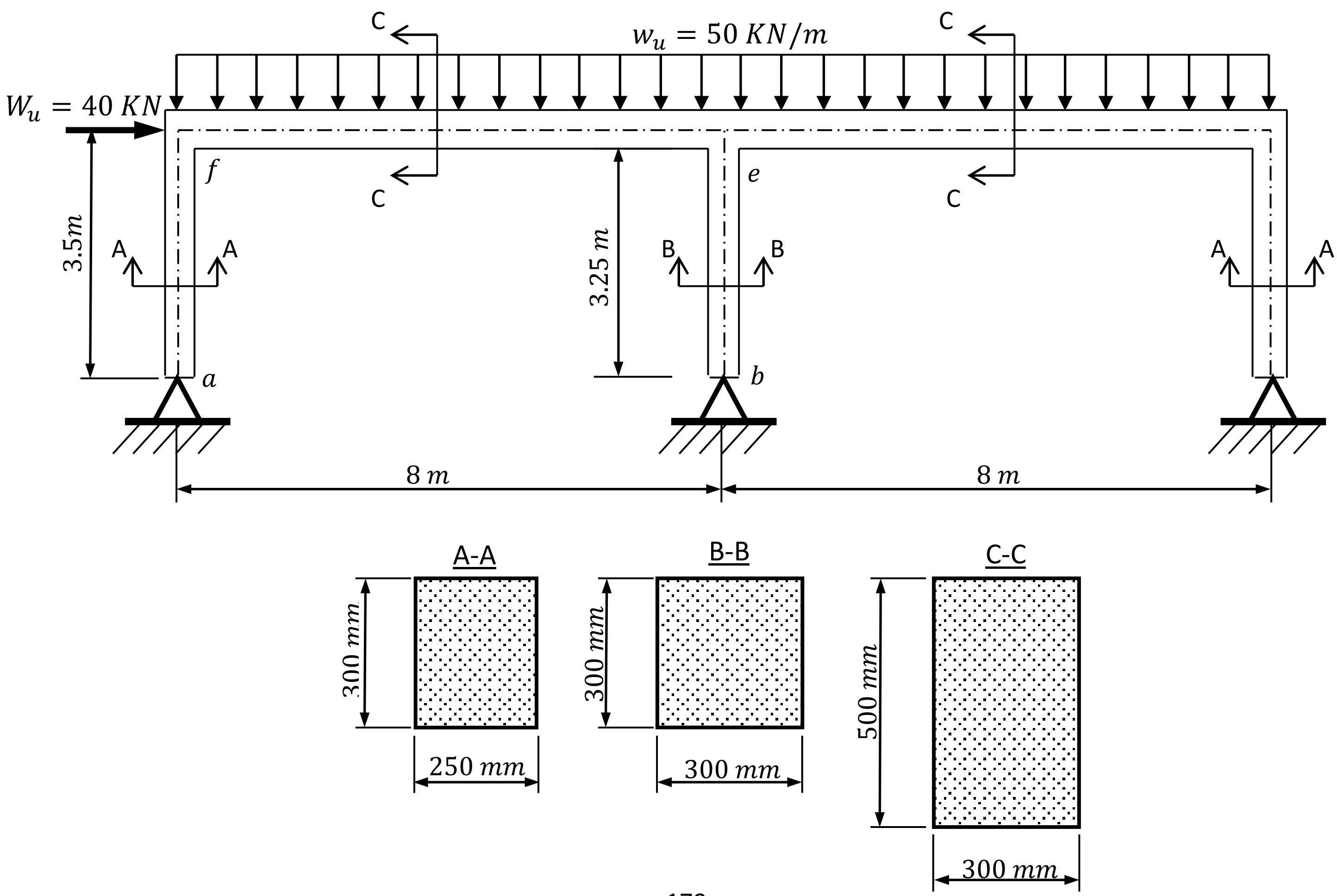
$$\frac{P_u}{\phi} = 2314\ KN < P_n = 2500.7\ KN$$

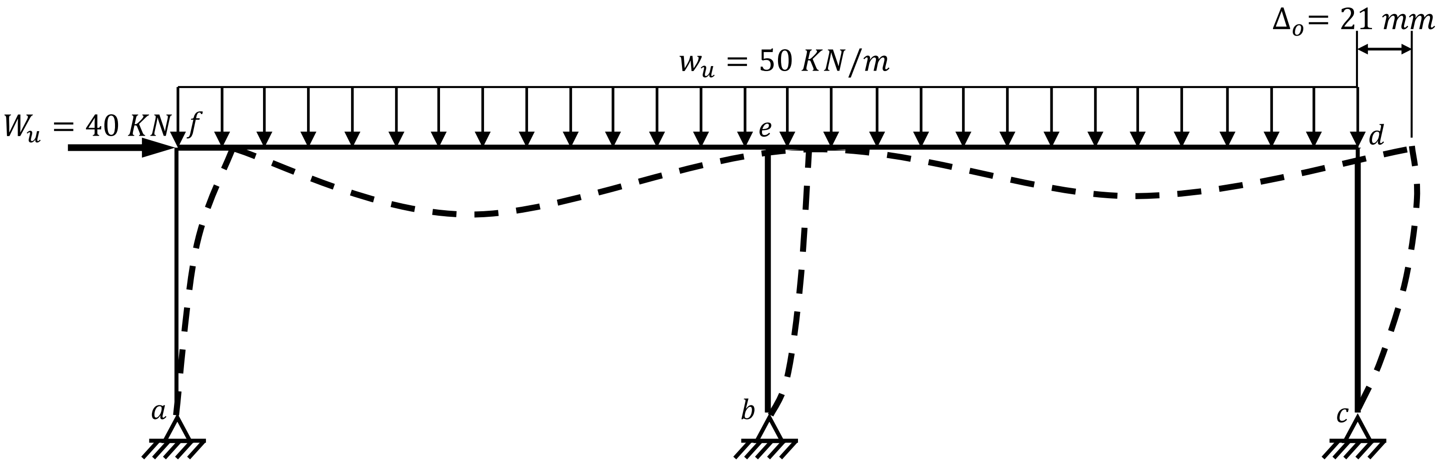
The section is adequate enough.

**Example:**

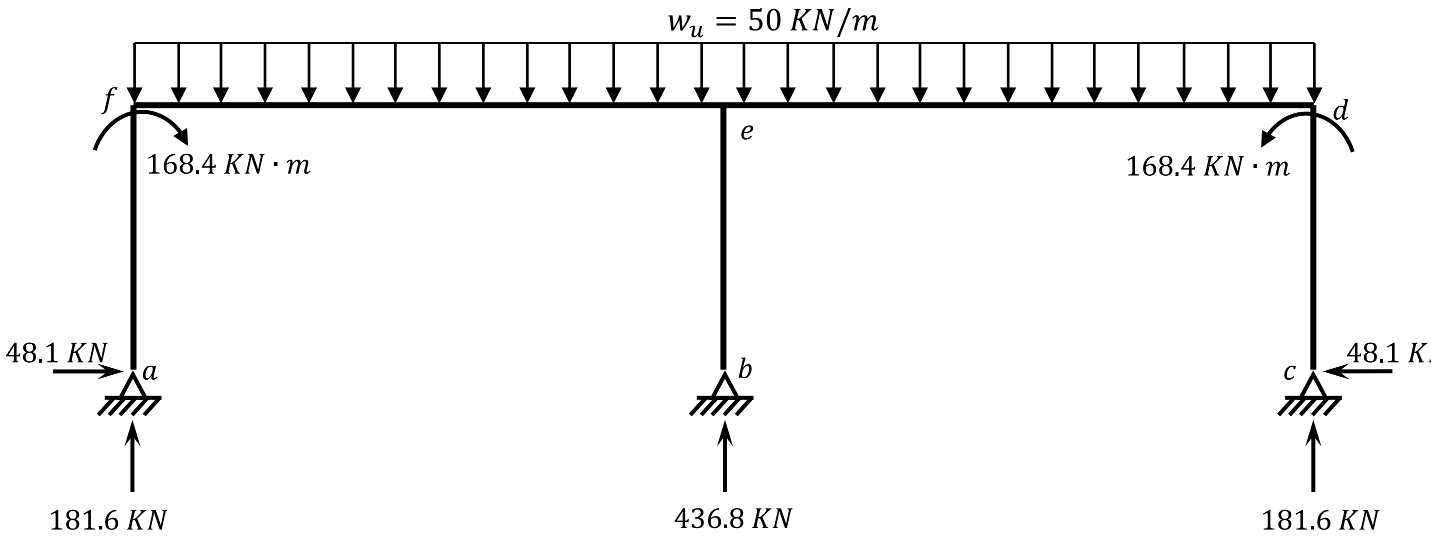
The frame which carries a total uniform factored gravity load of 50 $KN/m$ and a horizontal factored wind load at joint $f$, is unbraced in its own plane. The wind load in not a sustained load. Determine the axial load and moment for which coulmn must be designed. The dimensions of rectangular cross sections of beams and columns are given.

Take $f_c' = 28\ MPa$ and $f_y = 414\ MPa$.

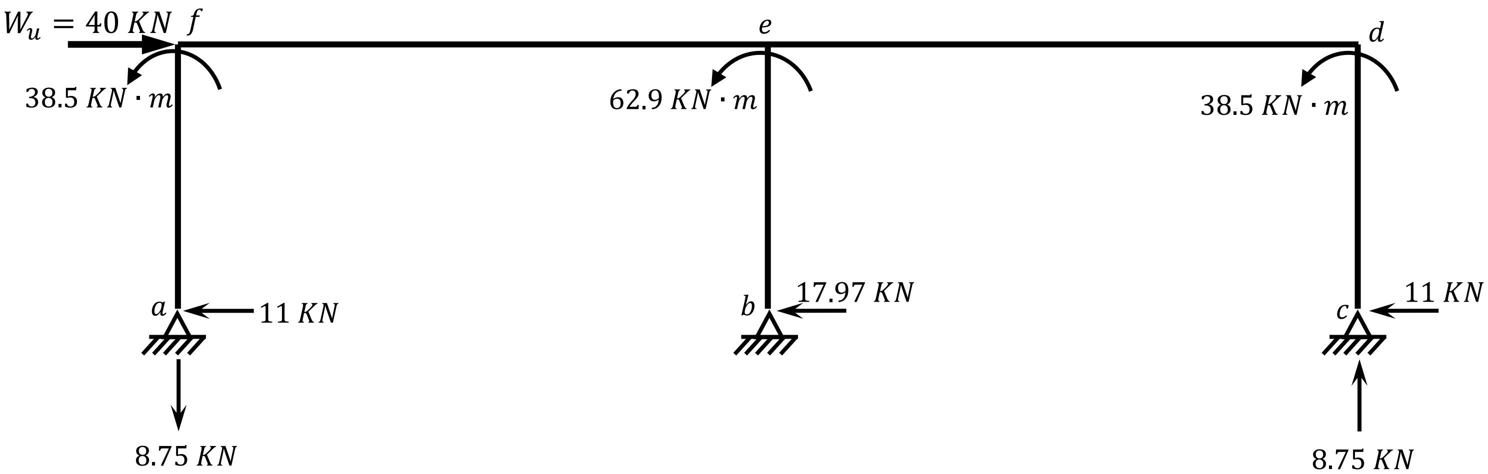




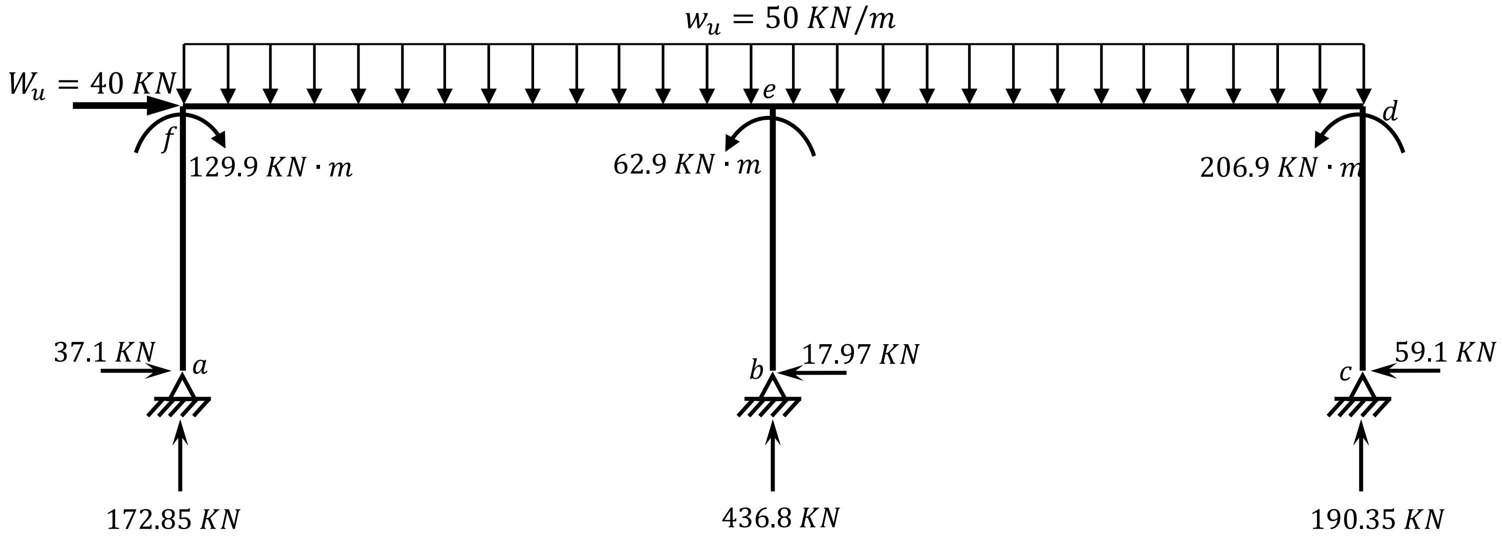

Deflected frame shape from all loads. Frame analysis by first-order computer program.




Support reactions and end moments produced by gravity loads.




Support reactions and end moments produced by lateral wind loads.




Support reactions and end moments produced by both gravity and lateral wind loads.

**Solution:**

1. The frame is unbraced against sidesway, the stability index

$$Q = \frac{\sum P_u \Delta_o}{V_{us} l_c} \le 0.05$$

$$Q = \frac{(172.85 + 436.8 + 190.35) \cdot 0.021}{40 \cdot 3.5} = 0.12 > 0.05$$

The frame is unbraced as given.

2. Check for slenderness for column "$cd$": a column in a sway frame is short if

$$\frac{kl_u}{r} \le 22$$

$r = 0.3h = 0.3 \cdot 0.25 = 0.075\ m.\quad l_u = 3.25\ m$

$$\psi = \frac{\sum E_c I_c / l_c \ \ of\ columns\ at\ joint}{\sum E_b I_b / l_b \ \ of\ beams\ at\ joint}$$

Because $E_c$ is the same for column and beams ($E_c = E_b$), it will be canceled in the stiffness calculations. For this step:

* the column moment of inertia will be

$$I_c = 0.7 I_g = 0.7 \cdot \frac{bh^3}{12} = 0.7 \frac{0.3 \cdot 0.25^3}{12} = 2.734 \cdot 10^{-4}\ m^4$$

* the beams moment of inertia will be

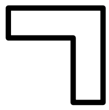
$$I_b = 0.35 I_g = 0.35 \cdot \frac{bh^3}{12} = 0.35 \cdot \frac{0.3 \cdot 0.5^3}{12} = 1.094 \cdot 10^{-3}\ m^4$$

Rotational restraint factors at the top and bottom of column "$cd$" are:

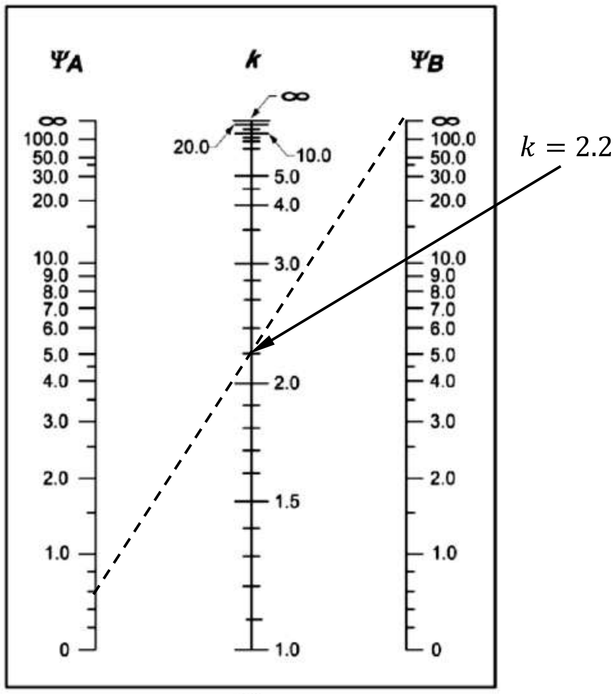
$$\psi_A(at\ top) = \frac{\sum \frac{I_c}{l_c}}{\sum \frac{I_b}{l_b}} = \frac{\frac{2.734 \cdot 10^{-4}}{3.5}}{\frac{1.094 \cdot 10^{-3}}{8}} = \frac{7.8125 \cdot 10^{-5}}{1.3675 \cdot 10^{-4}} = 0.57$$

$\psi_B(at\ bottom) = \infty$ – pin end.

Beam (left)



Column (below)




(b)
Sway Frames

From alignment chart $k = 2.2$

$$\frac{kl_u}{r} = \frac{2.2 \cdot 3.25}{0.075} = 95.335 > 22$$

The column is long and the slenderness must be considered.

3. Compute $EI$.

The area of reinforcement is not known. So the equation $EI = \frac{0.4E_cI_g}{1+\beta_{ds}}$ can be used.

$E_c = 4700\sqrt{f_c'} = 4700\sqrt{28} = 24870\ MPa$

$\beta_{ds} = 0$ – since the wind loads act for a short time period.

$$I_g(for\ column\ \ cd) = \frac{bh^3}{12} = \frac{300 \cdot 250^3}{12} = 0.391 \cdot 10^9\ mm^4$$

$$EI = \frac{0.4E_cI_g}{1+\beta_{ds}} = \frac{0.4 \cdot 24870 \cdot 0.391}{1+0} = 3889.7\ KN \cdot m^2$$

4. Determine the Euler buckling load, $P_c$:

For columns $cd, af$:

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot 3889.7}{(2.2 \cdot 3.25)^2} = 751\ KN$$

For column $be$:

$$I_g(for\ column\ \ be) = \frac{bh^3}{12} = \frac{300^4}{12} = 0.675 \cdot 10^9\ mm^4$$

$$EI = \frac{0.4E_cI_g}{1+\beta_{ds}} = \frac{0.4 \cdot 24870 \cdot 0.675}{1+0} = 6715\ KN \cdot m^2$$

Determine the effective length factor $k$ for column $be$:

$$\psi = \frac{\sum E_cI_c/l_c\ \ of\ columns\ at\ joint}{\sum E_bI_b/l_b\ \ of\ beams\ at\ joint}$$

- the column moment of inertia will be

$$I_c = 0.7I_g = 0.7 \cdot \frac{bh^3}{12} = 0.7\frac{0.3^4}{12} = 4.725 \cdot 10^{-4}\ m^4$$

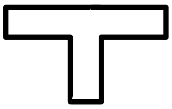
- the beams moment of inertia will be

$$I_b = 0.35I_g = 0.35 \cdot \frac{bh^3}{12} = 0.35 \cdot \frac{0.3 \cdot 0.5^3}{12} = 1.094 \cdot 10^{-3}\ m^4$$

Rotational restraint factors at the top and bottom of column "$be$" are:

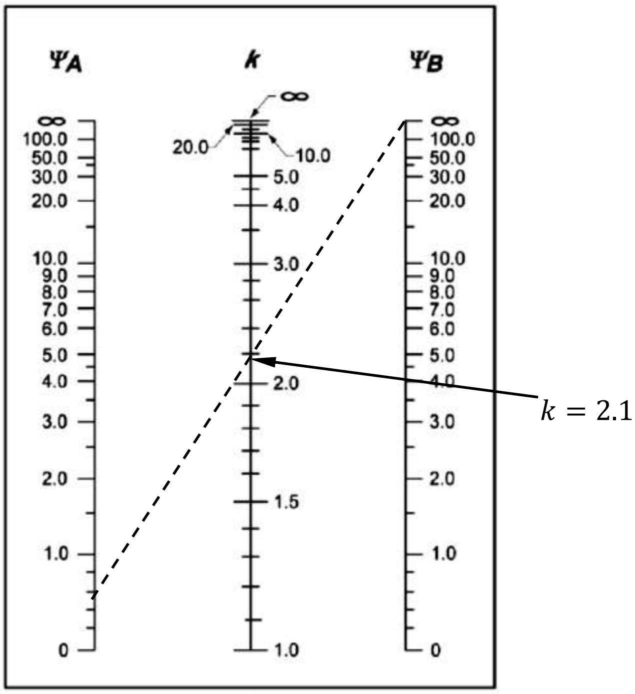
$$\psi_A(at\ top) = \frac{\sum \frac{I_c}{l_c}}{\sum \frac{I_b}{l_b}} = \frac{\frac{4.725 \cdot 10^{-4}}{3.5}}{\frac{1.094 \cdot 10^{-3}}{8} + \frac{1.094 \cdot 10^{-3}}{8}} = \frac{1.35 \cdot 10^{-4}}{2.735 \cdot 10^{-4}} = 0.49$$

$\psi_B(at\ bottom) = \infty -$ pin end.




From alignment chart $k = 2.17$

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot 6715}{(2.17 \cdot 3.25)^2} = 1332.5\ KN$$




(b)
Sway Frames

5. Calculate the moment magnifier factor $\delta_s$:

$\sum P_u = 172.85 + 436.8 + 190.35 = 800\ KN$

$\sum P_c = 751 + 1332.5 + 751 = 2834.5\ KN$

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} = \frac{1}{1 - \frac{800}{0.75 \cdot 2834.5}} = 1.6 > 1$$

6. The magnified moment at the top of the column $cd$:

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

$M_{2ns} = 168.4\ KN \cdot m -$ end top moment due to gravity loads only (nonsway condition).

$M_{2s} = 38.5\ KN \cdot m -$ end top moment due to lateral wind loads only (sway condition).

$$M_2 = 168.4 + 1.6 \cdot 38.5 = 230\ KN \cdot m$$

The magnified moments are less than 1.4 times the first-order moments, as required by ACI Code Section 10.10.2.1.

7. Check whether the moments are less than the minimum $M_2 < M_{2,min}$:

The minimum eccentricity $e_{min}$:

$$e_{min} = 15 + 0.03h = 15 + 0.03 \cdot 250 = 22.5\ mm$$

$$M_{2,min} = P_u \cdot e_{min} = 190.35 \cdot 0.0225 = 4.3\ KN \cdot m$$

$$M_2 = 230\ KN \cdot m > M_{2,min} = 4.3\ KN \cdot m$$

Therefore $M_{2,min}$ does not control.

8. Check to verify that the maximum moment does no occur between ends of column:

$$\frac{l_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c' A_g}}}$$

$$\frac{l_u}{r} = \frac{3.25}{0.3 \cdot 0.25} = 43.3 < \frac{35}{\sqrt{\frac{190.35 \cdot 10^3}{28 \cdot 300 \cdot 250}}} = 116.3$$

then, maximum moment envelops at top of column.

9. Design column $cd$ for $M_c = M_2 = 230\ KN \cdot m$ and $P_u = 190.35\ KN$.

# Palestine Polytechnic University

Collage of Engineering and Technology

Department of Civil and Architectural Engineering

# Reinforced Concrete I

Dr. Nasr Younis Abboushi

2013-2014